Exam #: ________________________

Printed Name: ________________________

Signature: ________________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

Part I

Analytical Mechanics

Monday, September 28, 2015, 10:00 to 12:40

The examination booklet is numbered in the upper right-hand corner of the cover page. Print and then sign your name in the spaces provided on the cover page. For identification purposes, be sure to submit this page together with your answers when the exam is finished.

There are four questions, each beginning on a new page. Read all four questions before attempting any answer. You may answer as many questions as you wish, however, only your top three scores will be used in the evaluation of your performance for a Ph.D. pass in this area, or your top two scores for a master's pass. (These scores will be added to your aggregate if taking the exam "under the old rules".)

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page. Place both the exam number and the question number on all pages you wish to have graded. You are encouraged to use the constants on the following page, where appropriate, to help you solve the problems.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
Atomic mass unit (AMU)	1.66×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Bohr Magneton (μ_B)	9.27×10^{-24} J/T
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11} N·m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	9.8 m/s^2
1 atmosphere	1.01×10^{5} Pa

Problem 1

A mass M is fixed at the right-angled vertex where a massless rod of length l is attached to a very long massless rod (see figure). A mass m is free to move frictionlessly along the long rod (assume that it can pass through M). Gravity near the surface of the Earth acts on the system. The rod of length l is hinged at a support, and the whole system is free to rotate, in the plane of the rods, about the hinge. Let θ be the angle of rotation of the system, and let x be the distance between m and M.

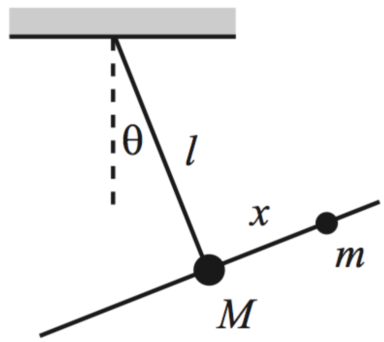


Figure 1:

a. (3 points) Find the Lagrangian for the system.

b. (2 points) Find the equations of motion using the Euler-Lagrange equation and the Lagrangian in Part a.

c. (2 point) Find the equations of motion when θ, x/l, $\dot{\theta}$, and $\dot{x}$ are very small.

d. (3 points) Find the eigenvalues and eigenfunctions for the equations of motion you found in Part c.

Problem 2

Let the Lagrangian of two interacting point particles be

$$L = \frac{1}{2}m_1\dot{\vec{r}}_1^2 + \frac{1}{2}m_2\dot{\vec{r}}_2^2 - \alpha\frac{1}{|\vec{r}_1 - \vec{r}_2|}$$

where m_1 and m_2 are the particle masses, $\dot{\vec{r}}_1$ and $\dot{\vec{r}}_2$ are the particle velocity vectors, $\vec{r}_1$ and $\vec{r}_2$ are the particle postition vectors, and α is a positive constant.

a. (1 point) Explain why the energy of this system is, or is not, conserved.

b. (1 point) Write down the expression for the center of mass $\vec{R}(t)$ and reduced mass μ of the system. How is the center of mass reference frame defined?

c. (2 points) Let $\vec{r} = \vec{r}_1 - \vec{r}_2$ and express the Lagrangian in the center of mass reference frame as a function of the coordinate $\vec{r}$, the velocity $\dot{\vec{r}}$, and the reduced mass μ.

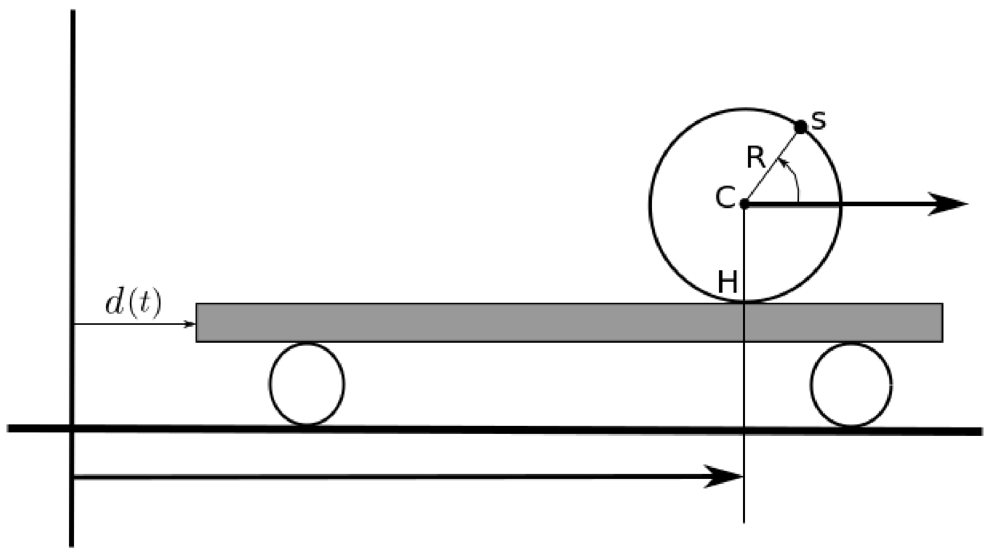
d. (2 points) Rewrite the resulting Lagrangian in spherical coordinates (polar angle θ, azimuthal angle ϕ, and magnitude $r = |\vec{r}|$).

e. (3 points) Write down the Euler-Lagrange equations for the two angles and identify from one of these equations a conserved quantity.

f. (1 point) Prove that the motion is confined to a plane.

Problem 3

A homogeneous cylinder of radius R, mass M and moment of inertia I (around its rotational axis C) rolls **without** slipping on a moving horizontal tray. H is the contact point between the cylinder and the tray and s is an arbitrary point on the cylinder. The translational motion of the tray is of constant acceleration and is perpendicular to the rotational axis of the cylinder. The position of the rear of the tray is given by the function of time $d(t)$.



a. (1 point) What is the velocity of the tray at the point of contact H (V_{Ht})? What is the velocity of point s when it is at the point of contact H (V_{sc})?

b. (1 point) How are V_{Ht} and V_{sc} related?

c. (1 point) What is the kinetic energy T of the cylinder?

d. (4 points) What is the angular acceleration of the cylinder?

e. (3 points) What is the linear acceleration of the cylinder's axis?

Problem 4

A uniform, solid cylinder of mass M, radius R, and length L, rotating with angular speed Ω about an axis through its center, is lowered gently onto a horizontal surface and released. The coefficient of kinetic friction of the surface is μ_k.

a. (1 point) Show that the moment-of-inertia of the cylinder about its axis is $\mathcal{I} = MR^2/2$.

b. (4 points) Find the translational acceleration of the cylinder, and the angular acceleration of rotation about the axis of the cylinder.

c. (3 points) The cylinder is initially slipping completely. Eventually, the cylinder rolls without slipping. Calculate the distance the cylinder moves before slipping stops.

d. (2 points) Calculate the energy dissipated by friction as the cylinder moves from where it is set down to where it begins to roll without slipping.