Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

PART I

Monday, September 29, 2014, 13:00 to 17:00

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

You are encouraged to use the constants and other information on the following page, where appropriate, to help you solve the problems.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand in your exam paper on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	1.673×10^{-27} kg ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	1.675×10^{-27} kg ($940\,\mathrm{MeV/c^2}$)
Atomic mass unit (AMU)	1.7×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11} N·m^2/kg^2
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
Mass of the Earth	5.97×10^{24} kg
Radius of the Earth	6.38×10^{6} m
Earth-Moon average distance	3.84×10^{8} m
Mass of the Sun	1.99×10^{30} kg
Radius of the Sun	6.96×10^{8} m
Earth-Sun average distance	1.49×10^{11} m
Mass of the Moon	7.35×10^{22} kg
1 atmosphere	1.01×10^{5} Pa

Vector Operators:

Cylindrical Coordinates

$$\begin{aligned}
\nabla f &= \hat{r}\frac{\partial}{\partial r}f + \hat{\phi}\frac{1}{r}\frac{\partial}{\partial \phi}f + \hat{z}\frac{\partial}{\partial z}f \\
\nabla \cdot \mathbf{A} &= \frac{1}{r}\frac{\partial}{\partial r}rA_r + \frac{1}{r}\frac{\partial}{\partial \phi}A_\phi + \frac{\partial}{\partial z}A_z \\
\nabla \times \mathbf{A} &= \hat{r}\left(\frac{1}{r}\frac{\partial}{\partial \phi}A_z - \frac{\partial}{\partial z}A_\phi\right) + \hat{\phi}\left(\frac{\partial}{\partial z}A_r - \frac{\partial}{\partial r}A_z\right) + \hat{z}\frac{1}{r}\left(\frac{\partial}{\partial r}rA_\phi - \frac{\partial}{\partial \phi}A_r\right)
\end{aligned}$$

Spherical Coordinates

$$\begin{aligned}
\nabla f &= \hat{r}\frac{\partial}{\partial r}f + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta}f + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}f \\
\nabla \cdot \mathbf{A} &= \frac{1}{r^2}\frac{\partial}{\partial r}r^2A_r + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\sin\theta A_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}A_\phi \\
\nabla \times \mathbf{A} &= \hat{r}\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}\sin\theta A_\phi - \frac{\partial}{\partial \phi}A_\theta\right) + \hat{\theta}\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \phi}A_r - \frac{\partial}{\partial r}rA_\phi\right) + \hat{\phi}\frac{1}{r}\left(\frac{\partial}{\partial r}rA_\theta - \frac{\partial}{\partial \theta}A_r\right)
\end{aligned}$$

Problem 1

A point particle of mass m is moving in a plane. Its polar coordinates are r and ϕ. It is acted on by a generalized potential

$$U = \frac{a}{r}(\frac{dr}{dt})^2, \tag{1}$$

where a is a constant and t denotes the time. In terms of the polar coordinates, write down for this system

a. the Lagrangian,

b. the Hamiltonian,

c. Hamilton's equations.

d. Are the forces acting on the particle conservative?

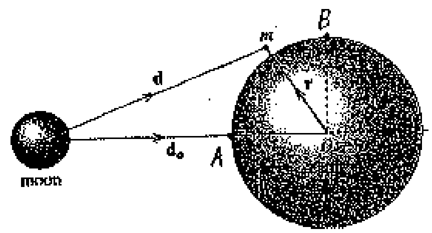
e. What is the relationship between the energy, E, and the Hamiltonian?

Problem 2

A frictionless spherical pendulum consists of a point mass m and a massless thread of length l. One end of the thread is tied to a fixed point P. The mass m moves so that the thread sweeps out the surface of a vertical cone, *i.e.*, the mass m moves along the circumference of a horizontal circle. Besides gravitation and the force exerted by the thread, no other forces act on m. Calculate the period of the pendulum (*i.e.,* the going around time of the mass) if the half angle of the cone is ϕ.

Problem 3

The purpose of this problem is to consider the gravitational effects of the Moon and the ocean tides of the Earth. Assume the Earth is a spherical, non-rotating (as seen from an inertial frame) and rigid body that is completely covered by an ocean of water a few kilometers deep. Ignore the gravitational effects of the Sun, so that the Earth and Moon orbit around their common center of mass. Consider an element of water, ϵ, of mass m, at the location shown in the figure. Choose a coordinate frame, K, rigidly attached to the Earth, such that the origin of K is at the location of ϵ.



a. Is K an inertial frame?

b. Write all of the relevant forces acting on ϵ as seen from K in terms of r, the mass of the Earth, M_E, the mass of the Moon, M_m, the center-to-center Earth-Moon distance, $d_\circ$, and d, the distance between the center of the Moon and ϵ. Evaluate the effective tidal force, calculating to lowest order in the quantity $r/d_\circ$.

c. The surface of the ocean is an equipotential surface. Sketch the shape of this surface, justifying the shape using your results from part b.

d. Calculate the difference in depth between the highest and lowest tide.

Problem 4

Consider an uniformly polarized, electrically neutral dielectric sphere with radius R and polarization $\mathbf{P}$ along the z-axis.

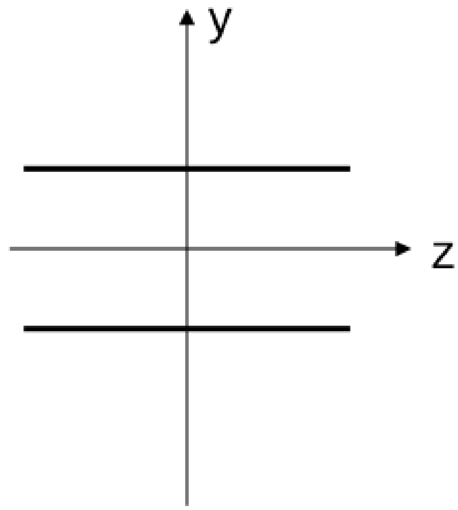
a. Determine the bound charge densities in the sphere and on the surface of the sphere.

b. Using the mathematical approach of separation of variables, find the electric potential inside and outside the polarized sphere.

c. Find the electric field inside and outside the polarized sphere.

Problem 5

Consider a waveguide consisting of two large parallel metal mirrors with separation d. Assume that monochromatic electromagnetic waves propagate along the z-direction inside the waveguide and the electric field is along the x-direction.



a. Starting with the wave equation for monochromatic waves, derive the Helmholtz equation, *i.e.*, the equation for the spatial dependence of the monochromatic waves. What are the boundary conditions for the electric field of the electromagnetic waves propagating inside the wave guide?

b. Find the solutions of the Helmholtz equation, in particular, the dependence of the electric field along the y-direction.

c. For electromagnetic waves of frequency ω, determine the total number of modes (*i.e.*, the number of solutions to the Helmholtz equation) that can be supported by the wave guide.

Problem 6

A long straight wire connects the two poles of a battery and carries a time independent current I. The wire has a circular cross section of radius a and has a conductivity σ.

a. Calculate the Poynting vector at the surface of the wire.

b. How much energy flows per unit length of the wire? What happens to this energy?

c. What is the immediate source for the energy flow?