Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

PART I

Monday, September 30, 2013, 13:00 to 17:00

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

You are encouraged to use the constants and other information on the following two pages where appropriate to help you solve the problems.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand in your exam paper on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	1.673×10^{-27} kg ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	1.675×10^{-27} kg ($940\,\mathrm{MeV/c^2}$)
Atomic mass unit (AMU)	1.7×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	$6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of earth (M_E)	5.98×10^{24} kg
Equatorial radius of earth (R_E)	6.38×10^{6} m
Mass of sun (M_S)	1.99×10^{30} kg
Radius of sun (R_S)	6.96×10^{8} m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
1 atmosphere	1.01×10^{5} Pa

Trigonometric identities

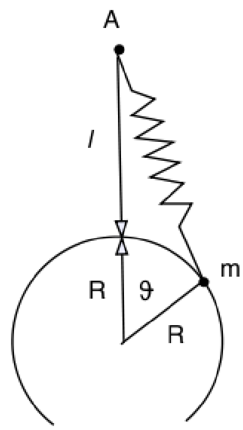
$$1 - \cos\theta = 2\,\sin^2(\theta/2)$$
$$1 + \cos\theta = 2\,\cos^2(\theta/2)$$
$$\sin\theta = 2\,\sin(\theta/2)\,\cos(\theta/2)$$

Integrals

$$\int \frac{dx}{\sqrt{(x^2 \pm a^2)^3}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$$
$$\int \frac{x\,dx}{\sqrt{(x^2 \pm a^2)^3}} = \frac{-1}{\sqrt{x^2 \pm a^2}}$$
$$\int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \pm\sqrt{x^2 \pm a^2}$$

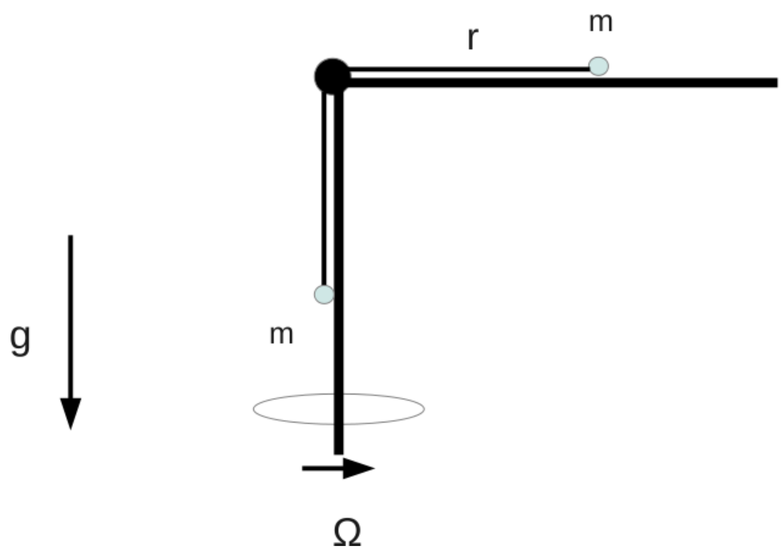
Problem 1

A particle of mass m is constrained to move along a circle of radius R (see figure below). The particle is attached to a spring whose end is fixed at a point A at distance l from the circle. A force $\mathbf{F}$ is needed to extend the spring to length l. Find the frequency of small oscillations about the vertical.



Problem 2

A rigid wire shaped like an upside-down L is spinning about its vertical segment as shown in the figure. The angular velocity of the motion is Ω. A bead of mass m is constrained to slide without friction on the horizontal segment of the wire and is connected by a massless string to an identical bead on the vertical segment. The string has constant length l and follows the shape of the wire without friction. The bead on the vertical segment is subjected to gravity ($\mathbf{g}$ is downward).



(a) State Hamilton's Principle for variational mechanics.

(b) In terms of the distance r of the revolving bead from the center of rotation, write down the Lagrangian for this system.

(c) Find the equation-of-motion for the bead by solving the variational problem.

(d) Solve the equation-of-motion for the trajectory of the bead.

Problem 3

A comet with mass m approaches the Sun from ∞. The comet has initial speed $v_\circ$ and approaches with impact parameter b.

(a) Find the energy E and the angular momentum $|\mathbf{L}|$ of the comet in terms of m, $v_\circ$, and b.

(b) Write down an expression for the energy of the comet.

(c) Find an expression for the impact parameter for which the comet just grazes the surface of the Sun when subject only to the force of gravity. The mass and radius of the Sun are M_s and R_s where $M_s \gg m$.

(d) Write down an expression for the capture cross-section for the comet by the Sun. What is the cross-section in the limit $v_\circ \to \infty$?

Problem 4

A very thin sheet of charge with constant charge per unit area σ lies in the x-y plane with a hole of radius R centered at the origin. Assuming the sheet has infinite extent, find the electric field $\mathbf{E}$ along the z-axis for any value of z. Explain that your solution makes sense for the limiting cases of $z \to \pm\infty$ and $z \to 0$.

Problem 5

An electromagnetic plane wave with field amplitude $E_\circ$ and frequency ν is incident on a glass slab with an angle of incidence θ. The slab has thickness d and index of refraction n. The wave goes through multiple reflections inside the slab. The amplitude transmission coefficients (*i.e.,* the ratios of the transmitted and incident electric field amplitudes) from air to glass and from glass to air are t and t', respectively. The amplitude reflection coefficient (*i.e.,* the ratio of the reflected and incident electric field amplitudes) in the glass is r. **Note**: $r^2 + tt' = 1$.

(a) Determine the phase difference between adjacent transmitted waves.

(b) Write down the amplitude of each successive transmitted wave.

(c) Determine the total intensity of the transmitted wave.

(d) Plot schematically the transmitted intensity as a function of the frequency of the incident wave (assuming the input intensity is held constant). What is the spectral separation between neighboring transmission resonances?

(e) What determine the width of the transmission resonance?

Problem 6

Suppose the magnetic field inside a large piece of magnetic material with uniform magnetization $\mathbf{M}$ is $\mathbf{B}_\circ$, so that $\mathbf{H}_\circ = \mathbf{B}_\circ/\mu_\circ$-$\mathbf{M}$.

(a) A small spherical cavity is hollowed out of the material. Find the magnetic field $\mathbf{B}$ at the center of the cavity in terms of $\mathbf{M}$ and $\mathbf{B}_\circ$. Also, find $\mathbf{H}$ at the center of the cavity in terms of $\mathbf{M}$ and $\mathbf{H}_\circ$. **Note**: It helps to first find the magnetic field of an uniformly magnetized sphere.

(b) Do the same for a long needle shaped cavity running parallel to $\mathbf{M}$.

(c) Do the same for a thin disk-shaped cavity perpendicular to $\mathbf{M}$.