

Exam #: _____

Printed Name: _____

Signature: _____

PHYSICS DEPARTMENT
UNIVERSITY OF OREGON
Unified Graduate Examination

PART II

Tuesday, September 25, 2012, 12:30 to 16:30

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

You are encouraged to use the constants and other information on the following two pages where appropriate to help you solve the problems.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed.** Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed.** **No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand in your exam paper on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	$1.60 \times 10^{-19} \text{ C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31} \text{ kg}$ ($0.511 \text{ MeV}/c^2$)
Proton rest mass (m_p)	$1.673 \times 10^{-27} \text{ kg}$ ($938 \text{ MeV}/c^2$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27} \text{ kg}$ ($940 \text{ MeV}/c^2$)
Atomic mass unit (AMU)	$1.7 \times 10^{-27} \text{ kg}$
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	$6.63 \times 10^{-34} \text{ J}\cdot\text{s}$
Speed of light in vacuum (c)	$3.00 \times 10^8 \text{ m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23} \text{ J/K}$
Gravitational constant (G)	$6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$
Permeability of free space (μ_0)	$4\pi \times 10^{-7} \text{ H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12} \text{ F/m}$
Mass of earth (M_E)	$5.98 \times 10^{24} \text{ kg}$
Equatorial radius of earth (R_E)	$6.38 \times 10^6 \text{ m}$
Mass of sun (M_S)	$1.99 \times 10^{30} \text{ kg}$
Radius of sun (R_S)	$6.96 \times 10^8 \text{ m}$
Classical electron radius (r_0)	$2.82 \times 10^{-15} \text{ m}$
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	9.8 m/s^2
1 atmosphere	$1.01 \times 10^5 \text{ Pa}$

Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Trigonometric identities

$$1 - \cos \theta = 2 \sin^2(\theta/2)$$

$$1 + \cos \theta = 2 \cos^2(\theta/2)$$

$$\sin \theta = 2 \sin(\theta/2) \cos(\theta/2)$$

Stirling's formula

$$\log(N!) \approx N \log N - N$$

Problem 1

Two identical spin $1/2$ fermions of mass m are constrained to move in one dimension with coordinate x and are confined by a one-dimensional potential energy $V(x)$ given by

$$V(x) = \begin{cases} 0 & \text{if } 0 < x < a \\ \infty & \text{otherwise} \end{cases}$$

Assume that the fermions do not interact with each other. You may use units with $\hbar = 1$ if you like.

- (a) In the ground state of the two particle system, is the spin state a spin 0 (singlet) state, a spin 1 (triplet) state, or a linear combination of these? Explain.
- (b) Find an expression for the (normalized) ground state spatial wave function, $\psi(x_1, x_2)$, where x_1 and x_2 are the positions of the particles.
- (c) Find an expression for the ground state energy.
- (d) If a third identical fermion is added, what is the ground state energy of the 3-particle system?

Problem 2

Consider a system of two spin $1/2$ particles (considering spin only and thus ignoring the dependence of the wave function on spatial positions). Suppose that the hamiltonian of this spin system is given by

$$H = A + B \mathbf{S}_1 \cdot \mathbf{S}_2 + C(S_{1,z} + S_{2,z}) ,$$

where \mathbf{S}_1 and \mathbf{S}_2 are the respective spin operators for the two particles and A , B , and C are constants. You may use units with $\hbar = 1$ if you like.

Find the energy eigenfunctions and eigenvalues for this system. (Hint: you may want to consider the operator that represents the total spin angular momentum.)

Problem 3

Consider a spinless charged particle with mass m and charge q in a magnetic field described by the vector potential

$$\begin{aligned}A_x &= -By , \\A_y &= 0 , \\A_z &= 0 .\end{aligned}$$

You may use units with $\hbar = c = 1$ if you like.

- (a) Write down the hamiltonian of the system.
- (b) Explain why the energy eigenfunctions can be simultaneously eigenfunctions of the momentum operators p_x and p_z . Taking the energy eigenfunctions to be simultaneously eigenfunctions of p_x and p_z with eigenvalues $\hbar k_x$ and $\hbar k_z$ respectively, find the x and z dependence of the wave function $\psi(x, y, z)$ that represents an energy eigenfunction.
- (c) For given values of k_x and k_z , find the energy eigenvalues of the system.

Problem 4

This problem deals with phase changes in water. The triple point of H_2O is at a pressure $P_T \approx 600 \text{ N/m}^2$ and temperature $T \approx 0 \text{ C}$.

- (a) Sketch the P - T phase diagram for H_2O . The diagram need not be to scale, but must be qualitatively correct.
- (b) Suppose the pressure is slowly lowered, at a fixed temperature of -1 C , on a chamber filled with what is initially liquid water, in thermal equilibrium. Describe any phase changes that occur as the pressure is lowered to zero, and calculate the pressures at which they occur, assuming that water vapor can be treated as an ideal gas. Make reasonable approximations.

Problem 5

A spherically symmetric black hole with mass M has Schwarzschild radius

$$r_{BH} = \frac{2GM}{c^2} .$$

It radiates blackbody radiation at a temperature

$$T_{BH} = \frac{\hbar c^3}{8\pi GkM} ,$$

and has entropy

$$S_{BH} = \frac{\pi k r_{BH}^2 c^3}{G\hbar} .$$

- (a) Given these relations, determine the temperature of such a black hole with the mass of the Sun. How many different microscopic configurations would such a black hole have?
- (b) Derive a differential equation for the rate of change of the mass of the black hole. (Hint: use energy considerations.)
- (c) Solve the differential equation to determine the time it takes for a black hole of initial mass M to evaporate away. Evaluate your expression numerically for a black hole whose initial mass is the mass of the Sun.

Problem 6

A large number N of equivalent, non-interacting impurity atoms in a solid can each be in either its ground state, or an excited state, with energies 0 and ϵ , respectively. This system is in thermal equilibrium at a temperature T .

- (a) Write an expression for the partition function for these impurity atoms.
- (b) Obtain an expression for the probability that a given atom is excited.
- (c) Find an expression for the temperature at which the probability that *all* of the atoms are in their ground state is $1/2$.
- (d) Evaluate your answer to (c) numerically (accuracy of 1 percent will do) for $N = 10^{12}$ and $(\epsilon/k_B) = 280\text{K}$, where k_B is Boltzmann's constant. (Hint: First simplify your analytic expression from (c) in the limit of $N \gg 1$.)