Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

PART I

Monday, September 24, 2012, 12:30 to 16:30

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

You are encouraged to use the constants and other information on the following two pages where appropriate to help you solve the problems.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed.** Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand in your exam paper on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
Atomic mass unit (AMU)	1.7×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11} N·m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of earth (M_E)	5.98×10^{24} kg
Equatorial radius of earth (R_E)	6.38×10^{6} m
Mass of sun (M_S)	1.99×10^{30} kg
Radius of sun (R_S)	6.96×10^{8} m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	9.8 m/s^2
1 atmosphere	1.01×10^{5} Pa

Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \ , \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ .$$

Trigonometric identities

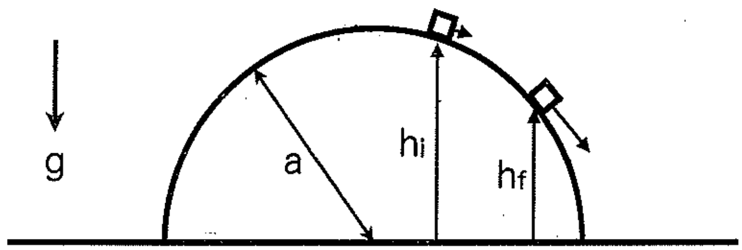
$$1 - \cos\theta = 2\,\sin^2(\theta/2)$$
$$1 + \cos\theta = 2\,\cos^2(\theta/2)$$
$$\sin\theta = 2\,\sin(\theta/2)\,\cos(\theta/2)$$

Stirling's formula

$$\log(N!) \approx N \log N - N$$

Problem 1

A particle is placed on a smooth, frictionless, hemispherical surface of radius a at a height $h_i < a$ above the horizontal surface. The particle is released at rest and begins sliding down the surface. At what height h_f does the particle leave the surface? Simplify your answer as far as possible.



Problem 2

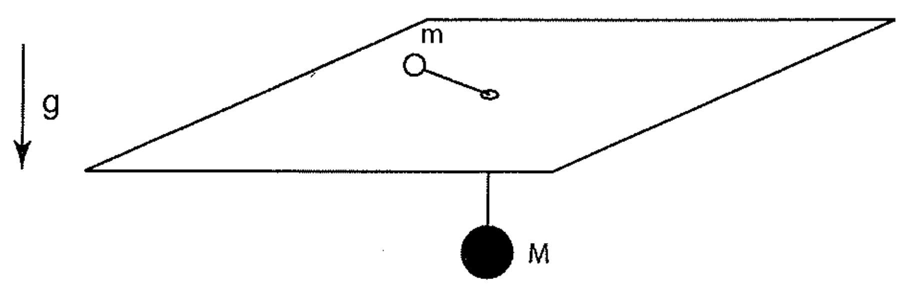
Consider a mass m sliding without rolling or friction on a flat, horizontal, and perfectly smooth table with a small hole. An inextensible string passes through the hole and connects the mass m to a second mass M which hangs suspended vertically under the influence of gravity. The mass M can only move in the vertical direction.

(a) Write the Lagrangian and Hamiltonian for this system in terms of the polar coordinates (r, ϕ), where r is the radial distance of mass m from the hole and ϕ is the azimuthal angle of m in the plane of the table.

(b) Find the Lagrange equations. What conservation laws, and thus conserved quantities, can be identified?

(c) Suppose the mass m moves in a circular path with radius $r = r_0$. Calculate the angular frequency $\dot{\phi} = \omega_0$ and radius r_0 in terms of conserved quantities [found in part (b)] and other constants given in the problem.

(d) The mass m, initially in circular motion, is given a small radial push inward. Is the system unstable or stable? If it is unstable, calculate the time it takes for m to reach the small hole in terms of $1/\omega_0$. If it is stable, calculate the frequency of small oscillations about a circular path in terms of ω_0.

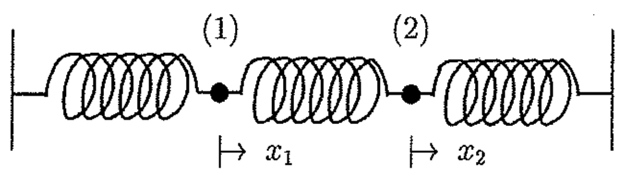


Problem 3

Two particles having equal mass M are connected together with three identical ideal springs with spring constants k whose ends are connected to immovable walls on each side as shown in the figure. The particles' displacements from their equilibrium positions are given by $x_1(t)$ and $x_2(t)$. The masses and springs are constrained to move only in the horizontal direction with no friction.

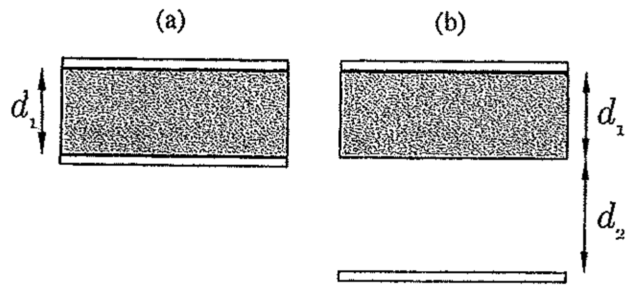
(a) Find the two eigenfrequencies of the system.

(b) Find the displacements of the particles, $x_1(t)$ and $x_2(t)$, for each eigenfrequency found in part (a). Assume both particles are in motion and that $x_1(0) = 0$ and $x_2(0) = 0$.



Problem 4

A parallel plate capacitor of plate separation d_1 is filled with a solid dielectric material whose dielectric constant is ϵ, as shown in Figure (a) below. The capacitor is charged to voltage V_1. The capacitor is then disconnected from the battery and pulled apart so that the plate separation becomes $d_1 + d_2$. The dielectric does not expand and the dielectric-free region has size d_2 [see Figure (b)]. Assuming that the length and width of the plates are large compared to both d_1 and d_2, compute the voltage V_2 after the capacitor is pulled apart.



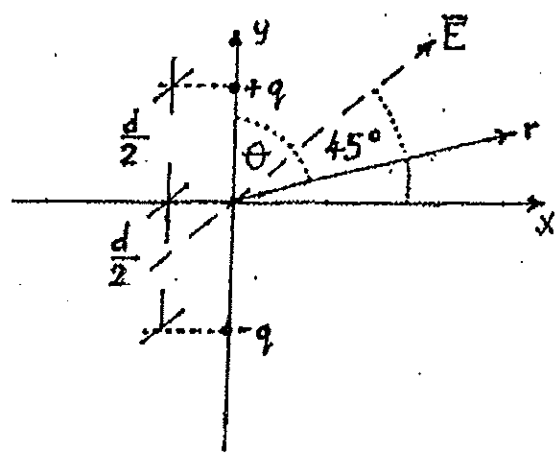
Problem 5

Two electric charges in the $[x, y]$ plane are separated by a rigid non-conducting rod: charge q is located at $x = 0$, $y = d/2$, and charge $-q$ at $x = 0$, $y = -d/2$.

(a) Write down the potential and the electric field generated by these charges in the $[x, y]$ plane at a general point (r, θ) , when $r \gg d$. The angle θ is measured clockwise from the y-axis, and $r = 0$ corresponds to $x = y = 0$. (Express your result in terms of the polar coordinates r and θ.)

(b) If the charges are embedded in a constant homogeneous electric field, $\mathbf{E}$, making an angle of 45° with the y-axis, what is the torque on this pair of charges?

(c) How much energy must be supplied to rotate the rod connecting the charges to be orthogonal to the electric field?



Problem 6

A uniform electric field, E, points along the x axis, and is perpendicular to a uniform magnetic field, B, pointing along the y axis. A particle of positive charge q is released from rest at time $t = 0$, at the point $x = a$, $y = 0$, $z = 0$. Give the x, y, and z coordinates of the particle as a function of t. Assume that $| c \cdot \mathrm{B} | \gg | \mathrm{E} |$ (measured in in S.I. units), and that all velocities are non-relativistic.