Exam #: ____________________

Printed Name: ____________________

Signature: ____________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

PART II

Tuesday, September 27, 2011, 12:30 to 16:30

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

You are encouraged to use the integrals, constants, and other information on the following two pages where appropriate to help you solve the problems.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed.** Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room, put all problems in numerical order and staple them together with this sheet on top. Then hand your examination paper to the proctor.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

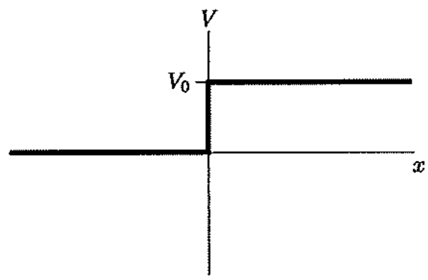
Constants

Electron charge (e)	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31}\,\mathrm{kg}$ ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	$1.673 \times 10^{-27}\,\mathrm{kg}$ ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27}\,\mathrm{kg}$ ($940\,\mathrm{MeV/c^2}$)
Atomic mass unit (AMU)	$1.7 \times 10^{-27}\,\mathrm{kg}$
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J\cdot s}$
Speed of light in vacuum (c)	$3.00 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.67 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Mass of earth (M_E)	$5.98 \times 10^{24}\,\mathrm{kg}$
Equatorial radius of earth (R_E)	$6.38 \times 10^{6}\,\mathrm{m}$
Radius of sun (R_S)	$6.96 \times 10^{8}\,\mathrm{m}$
Classical electron radius (r_0)	$2.82 \times 10^{-15}\,\mathrm{m}$
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Specific heat of oxygen (c_V)	$21.1\,\mathrm{J/mole\cdot K}$
Specific heat of oxygen (c_P)	$29.4\,\mathrm{J/mole\cdot K}$
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
1 atmosphere	1.01×10^{5} Pa

Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \;,\; \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \;,\; \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \;. \tag{1}$$

Problem 1



An electron of energy E is incident from the left on a potential step from potential energy 0 to potential energy V_0, as sketched in the figure above.

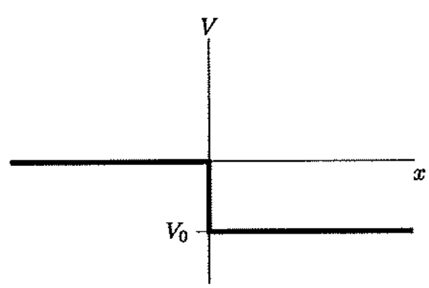
(a) Calculate the probability that the electron is transmitted, eventually reaching large positive x.

(b) Find a numerical value for the transmission probability and comment on the agreement or disagreement with classical behavior for the following three cases:

(1) The potential step is $V_0 = +4$ eV and the electron energy is $E = +5$ eV.

(2) The potential step is $V_0 = +4$ eV and the electron energy is $E = +3$ eV.

(3) The potential step is $V_0 = -400$ eV and the electron energy is $E = +3$ eV. (See the figure below, in which the vertical scale is compressed by a factor of 100.)



Problem 2

A particle of mass m and charge q moves under the influence of a one-dimensional harmonic oscillator potential and a constant electric field, $\mathcal{E}$, so that the Hamiltonian is given by

$$H = \frac{1}{2m} P^2 + \frac{m\omega^2}{2} X^2 - q\mathcal{E} X \ .$$

Here X is the position operator and P is the momentum operator.

(a) Derive an expression for the energy spectrum of the system (i.e. the energies E_n of the energy eigenstates $|n\rangle$). Hint: Consider a change of variables $X' = X + c$ with some choice for the constant, c.

(b) Calculate the mean electric dipole moment in state $|n\rangle$, i.e. $D_n = q\langle n|X|n\rangle$.

Problem 3

Consider a particle of mass m confined to a one-dimensional potential $V(x)$ such that $V = 0$ for $0 < x < a$ and $V = \infty$ otherwise. The particle is in the ground state for this potential.

(a) Obtain an expression for the normalized wave function describing this particle.

(b) Obtain an expression for the energy E_0 of the particle.

(c) Let $\rho(p)dp$ denote the the probability that the confined particle has momentum in the range $(p, p + dp)$. Find $\rho(p)$.

(d) At a certain instant in time the walls of the potential are moved to $x = \pm\infty$. If the energy of the particle is now measured, is it always the value E_0 that you found in part (b)?

Problem 4

Consider a quantum mechanical system based on two orthonormal basis vectors, $|1\rangle$ and $|2\rangle$. Let the Hamiltonian for this system be

$$H = 2\omega\left(|1\rangle\langle 1| + |2\rangle\langle 2|\right) + i\omega|1\rangle\langle 2| - i\omega|2\rangle\langle 1| \ ,$$

where ω is a real number. Suppose that we prepare a system in the state

$$|\psi\rangle = \frac{1}{\sqrt{5}}\left(|1\rangle + 2i|2\rangle\right)$$

and measure the energy E of the system. We do this many times. Because we are dealing with a quantum mechanical system, we do not get the same result for the energy every time.

(a) Calculate the average of the results for E that we will get if we do this experiment many times.

(b) Each time that we do the experiment, we get some particular energy E. What are the possibilities for what energy E we can find?

Problem 5

Consider the addition of 200 grams of ice at $-10°C$ to an insulated container of 1.0 kg of water at $15°C$.

(a) When the combination reaches thermodynamic equilibrium, how much ice, if any, remains?

(b) What is the final temperature of the combination?

(c) Suppose the addition of ice is only 100 grams (at $-10°C$). What is the final temperature in this case?

Problem 6

Consider a gas whose equation of state is

$$P(V - nb) = nRT$$

where P is the pressure, V is the volume and T the temperature; n is the number density, and b is a constant related to the volume excluded by the gas particles. In the following you may use the thermodynamic identity

$$dS = \frac{C_V}{T} dT + \left.\frac{\partial P}{\partial T}\right|_V dV.$$

where C_V is the heat capacity.

(a) Use standard methods of thermodynamics to prove that at constant temperature the internal energy does not depend on volume.

(b) Compute the entropy of the gas, up to an additive constant, assuming the heat capacity is independent of temperature.

Problem 7

Consider a gas of N free electrons (mass m) in two dimensions, confined to a square of area A.

(a) Assume vanishing probability density on the boundary, and energy E large enough such that the spectrum can be considered continuous. Calculate the density of states, $\mathcal{D}(E)$ (including spin) of the electron gas.
Hint: Given a large M, the number of integers n_x, n_y with $n_x^2 + n_y^2 \leq M^2$ is approximately equal to the area of a circle of radius M.

(b) Using the result of (a), obtain an expression for the Fermi energy E_F in terms of N and A.

(c) At temperature $T = 0$, obtain an expression for the average energy per particle in terms of E_F only.

Problem 8

Consider a system with a total of four single-particle energy levels at energies $E = 0$ and $E = \varepsilon > 0$: the ground state is non-degenerate, and the excited state ε is three-fold degenerate. Each of these four levels can be occupied by at most one particle. This system is held at constant temperature T and chemical potential μ by allowing it to exchange energy and particles with a reservoir of much larger size.

(a) Calculate the partition function $\mathcal{Z}(T, \mu)$ of the many-particle system in the grand canonical ensemble.

In the following, assume that the chemical potential is adjusted such that $\mu = \varepsilon$.

(b) Find the average number $\langle N \rangle$ of particles in the system.

(c) Calculate the average number of particles that are in the excited state at the given T and μ.

(d) What will the number of particles in the ground state be in the limits $T \to 0$ and $T \to \infty$?