Exam #: ____________________

Printed Name: ____________________

Signature: ____________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

PART I

Monday, September 26, 2011, 12:30 to 16:30

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

You are encouraged to use the integrals, constants, and other information on the following two pages where appropriate to help you solve the problems.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room, put all problems in numerical order and staple them together with this sheet on top. Then hand your examination paper to the proctor.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

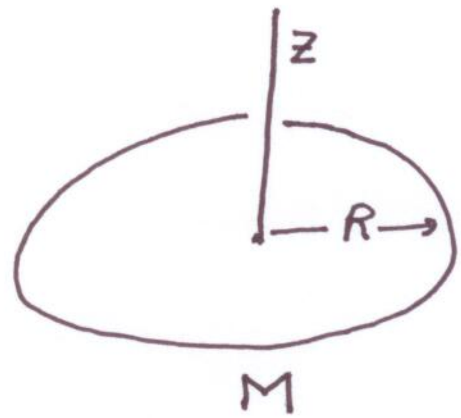
Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	1.673×10^{-27} kg ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	1.675×10^{-27} kg ($940\,\mathrm{MeV/c^2}$)
Atomic mass unit (AMU)	1.7×10^{-27} kg
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	$6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of earth (M_E)	5.98×10^{24} kg
Equatorial radius of earth (R_E)	6.38×10^{6} m
Radius of sun (R_S)	6.96×10^{8} m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
1 atmosphere	1.01×10^{5} Pa

Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \ , \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ . \tag{1}$$

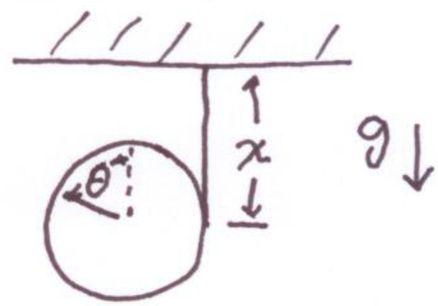
Problem 1

Write down Newton's law of universal gravitation being sure to define any symbols you use. Approximating a galaxy as a uniform thin circular disk of radius R and total mass M, use this law to calculate the escape speed for an object starting on the axis of the disk, a distance z from the center of the disk.



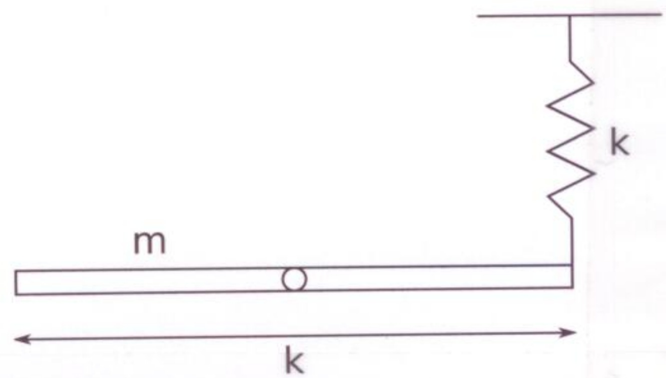
Problem 2

One end of a mass-less string is attached to a fixed support and the rest of the string is wrapped around a thin hoop of mass M and radius R. The initial condition is such that the hoop falls straight down under the force of gravity, and as it does, the string unwinds as shown in the Figure. Write down the Lagrangian for this system in terms of the length x of string unwound, and the angle θ that the hoop has rotated. Impose the constraint that the string does not slip on the surface of the hoop, and minimize the action to find the distance the hoop falls in time t. Also find the tension on the string.



Problem 3

A thin rod of mass m and length l is constrained to rotate about an axis in the middle of the rod as shown. An ideal spring k is connected to one end of the rod, such that the equilibrium position of the rod is horizontal. Show that for small displacements, the angle of the rod with respect to horizontal acts as a simple harmonic oscillator, and find the frequency of these oscillations.



Problem 4

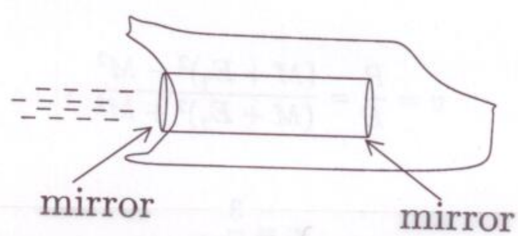
A rocket ship has a photon drive that works as follows (see drawing). The rocket ship contains a cylinder with mirrors at either end. The cylinder is filled with a large number of photons that move back and forth along the axis of the cylinder, reflecting from the mirrors. Before the photon drive is turned on, equal numbers of photons are always moving in each direction. To provide acceleration for the rocket ship, the captain engages the photon drive: the reflectivity of the mirror at the back end of the cylinder is reduced so that it transmits a fraction of the photons striking it, still reflecting the rest. Thus photons are emitted backwards from the back of the rocket when the photon drive is engaged.

Suppose that the rocket ship starts from rest with a filled photon drive. The mass of the rocket ship not counting the photons is M and the energy of the photons in the filled photon drive is E_γ. The photon drive is engaged until all of the photons have been emitted from the back of the rocket.

You may use units with $c = 1$ if you wish.

(a) Find the final velocity v of the rocket ship in terms of E_γ and M.

(b) Give a numerical value for v if $E_\gamma = Mc^2$.



Problem 5

Let $\rho(r)$ be a spherically symmetric charge distribution ($r = |\vec{x}|$).

(a.) Express the electric field $\vec{E}(\vec{x})$ that results from this charge distribution in terms of a one-dimensional integral.

(b.) Show that a potential, ϕ , exists such that $\vec{E}(\vec{x}) = -\vec{\nabla}\phi(\vec{x})$, and express ϕ in terms of an integral involving $\vec{E}$, given the boundary condition $\phi(\infty) = 0$.

(c.) Calculate and sketch $\vec{E}$ and ϕ for the special case of a homogeneously charged sphere with radius r_0,

$$\rho(r) = (Q/V)\Theta(r_0 - r),$$

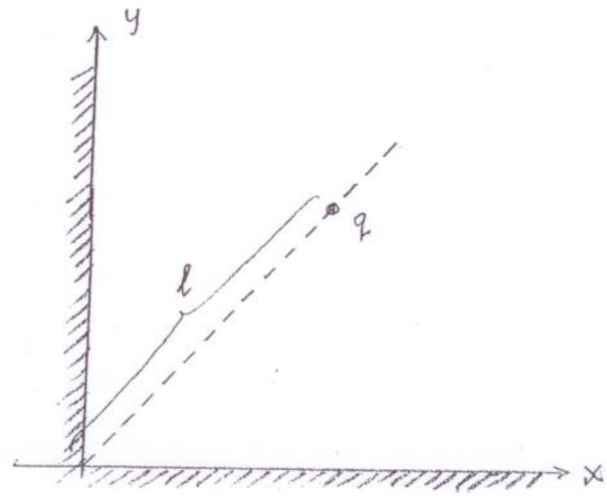
where Θ is the step function, Q is the total charge, and $V = 4\pi r_0^3/3$ is the volume of the sphere.

Problem 6

Two semi-infinite conducing plane surfaces intersect at an angle $\pi/2$. A charge q is located halfway between the planes at a distance ℓ from the line of intersection. (See Figure.)

(a.) Calculate the magnitude and location of the image charge(s).

(b.) Evaluate the work needed to move q to infinity along the line (dashed in the figure) connecting q to the line of intersection.



Problem 7

A source generates identical particles of mass m, charge q, and momentum (same for all particles) $\vec{p}$. The mass m and the direction of $\vec{p}$ are known. The quantities q and $|\vec{p}|$ are not known, and you want to measure them. To accomplish that, you have at your disposal only one instrument that has the following characteristics:

It can receive the particles (to be studied) in a volume, in which it can generate a magnetic field $\vec{B}$ and/or an electric field $\vec{E}$, both fields being uniform and homogeneous. The direction of both fields, and the magnitude of $\vec{E}$ can be chosen at will, but the magnitude of $\vec{B}$ is fixed. The instrument can record the trajectory of any particle crossing the volume, so that you can study the shape of the trajectories. How would you use this instrument to measure the unknown quantities of interest, expressing them in terms of the $\vec{E}$ and $\vec{B}$ fields?

(Remember that measurements are usually most accurate, if each unknown is directly measured separately from the others, rather than measuring various combinations of the unknowns.)

Problem 8

The axis of an axisymmetric mirror is the z axis. Its profile is characterized by the equation

$$z = r^2/2R.$$

(See Figure.) Show that any ray of light incident on this mirror that is traveling parallel to the z-axis, will, after reflection, be focused on one particular point, F, (called the "focus" of the mirror), and give the coordinates of F.