Exam #: ____________________

Printed Name: ____________________

Signature: ____________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Master's Final Examination and Ph.D. Qualifying Examination, Part II

Tuesday, September 28, 2010, 9:00 a.m. to 1:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic, and will be provided. **Personal calculators of any type are not allowed**. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries will not be allowed. No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand in your exam paper on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	1.673×10^{-27} kg ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	1.675×10^{-27} kg ($940\,\mathrm{MeV/c^2}$)
W^- rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J \cdot s}$
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Heat Capacity of Water	$4.19\ \mathrm{J/K/cm^3}$
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N \cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth ($M_\oplus$)	5.98×10^{24} kg
Mass of Moon (M_{Moon})	7.35×10^{22} kg
Mass of Sun ($M_\odot$)	1.99×10^{30} kg
Radius of Earth ($R_\oplus$)	6.38×10^{6} m
Radius of Moon (M_{Moon})	1.74×10^{6} m
Radius of Sun ($R_\odot$)	6.96×10^{8} m
Earth - Sun distance ($R_{\oplus,\odot}$)	1.50×10^{11} m
Classical electron radius ($r_\circ$)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
Atomic mass unit	1.66×10^{-27} kg
One atmosphere (1 atm)	$1.01 \times 10^{5}\,\mathrm{N/m^2}$

Stirling's Approximation

$$\log N! \;\approx\; N \log N - N$$

Useful Definitions, Commutators, Expectation Values, and Pauli Spin-Matrices

$$J_\pm = J_x \pm iJ_-$$

$$[J_i, J_k]_- = i\epsilon_{ikl} J_l$$

$$< j, m-1 \mid J_- \mid j, m >= \hbar[(j+m)(j-m+1)]^{1/2}$$

$$< j, m+1 \mid J_+ \mid j, m >= \hbar[(j-m)(j+m+1)]^{1/2}$$

$$R_{\hat{n}} = e^{(i/\hbar)\cdot(\vec{S}\cdot\hat{n}\phi)}$$

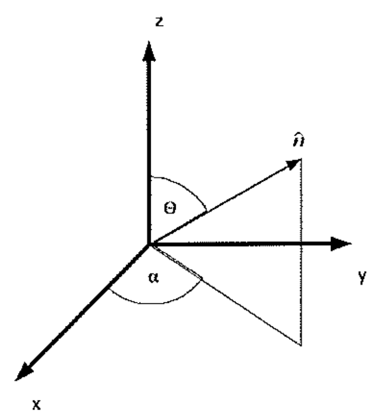
$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Problem 1

The spin of an electron points along the + x direction.

a. Write down this state using the notation (representation) in which $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represent the spin state of an electron whose spin is parallel or anti-parallel to the z-axis, respectively.

b. In this same representation write down the spin state of the electron in question, Ψ', after its spin has been rotated by an angle φ around an axis that is parallel to the unit vector n̂. The components of n̂ are expressed in terms of the usual spherical coordinates (see figure) as

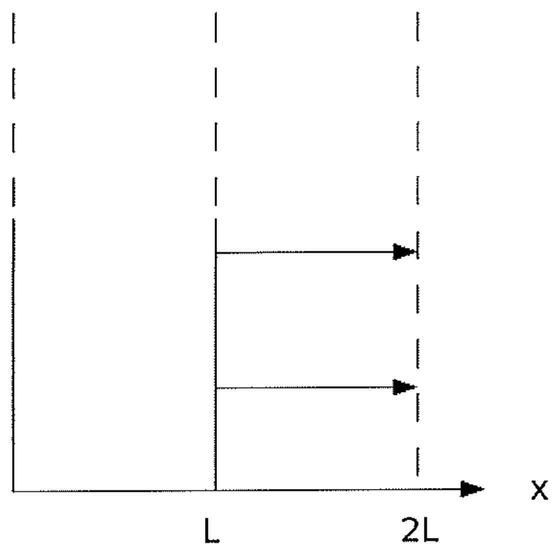


$$
\begin{aligned}
n_x &= sin(\Theta)cos(\alpha) \quad (1)\\
n_y &= sin(\Theta)sin(\alpha) \quad (2)\\
n_z &= cos(\Theta) \quad (3)
\end{aligned}
$$

c. Write down Ψ' in the special case when $\alpha = 0$, $\Theta = \pi/2$, and $\varphi = 2\pi$.

Problem 2

A spinless particle of mass m and electric charge e occupies the ground state of an infinite square well of width L. Precisely at the time $t = t_{\circ}$, the right side wall of the square well (see figure) is instantaneously moved to a new position so that the resulting infinite square well has width $2L$. Write down the wave function, Ψ, of the particle at times $t > t_{\circ}$; i.e. specify how Ψ depends on x and t at times $t > t_{\circ}$. (You may express the answer as a sum of many terms.)



Problem 3

A particle at time t is in a state $\Psi(t)$. The $\Psi(t)$ obeys the Schroedinger equation with H as the Hamiltonian: $i\hbar\partial_t\Psi(t) = H\Psi(t)$. We wish to describe the the time evolution of $\Psi(t)$ in terms of a set of orthonormal basis states that change as time goes on. Therefore we describe the state of the particle by a new state $\overline{\Psi(t)} = \overline{U(t)} \cdot \Psi(t)$, where the unitary operator U *does depend* on t. The Hamiltonian in the new, time dependent representation is $\overline{H(t)}$. Express $\overline{H(t)}$ in terms of H and U. (Hint: Since $\partial_t(UU^+) = 0$, $U\partial_t U^+ = -(\partial_t U)U^+$.)

Problem 4

Consider a particle of mass m moving in a central potential given by

$$\Phi(r) = -\frac{C}{m r_\circ^\alpha}, \tag{4}$$

where $C > 0$ and $\alpha > 0$.

a. If the ground state wave function of the particle has a mean radius $< r >= r_\circ$, estimate the typical linear momentum of this state using the Heisenberg uncertainty principle.

b. Estimate the energy $E(r_\circ)$ of the ground state.

c. Minimize $E(r_\circ)$ as a function of $r_\circ$ to obtain an estimate of $r_\circ$. Express your answer in terms of $\hbar, m, C$, and α.

d. Find the critical value of $\alpha = \alpha_c$ such that, if $\alpha > \alpha_c$, no minimum exists. What happens in this case, and why?

Problem 5

Consider a system of N non-interacting spin $\frac{1}{2}$ particles. Each particle has a magnetic moment μ which can point either parallel or antiparallel to an external field H. All energies and degrees of freedom other than those of the spins can be neglected.

a. What is the allowed range of total energies E for this system?

Now suppose the system of spins as a whole is in equilibrium at a temperature T.

b. What is the partition function when $N = 1$?

c. What is the partition function when $N > 1$?

d. Find the average energy $\bar{E}$ as a function of the temperature T of the spins.

e. Now suppose we fix the total energy E of the spins, rather than their temperature T. For $N \gg 1$, calculate and plot the equilibrium temperature $T(E)$ as a function of E over the *entire* range of allowed energies E you found in part (a). Comment on any unusual features of your result.

Problem 6

Consider an ideal gas with a constant ratio $\gamma \equiv \frac{C_P}{C_V}$ of specific heats at constant pressure and volume. Derive the relation between pressure (P) and volume (V) for this gas when it undergoes adiabatic compression.

Problem 7

A classical ideal gas is in a constant gravitational field with acceleration g. The constituent particles have mass M, and the gas is in equilibrium at a constant absolute temperature T.

a. Find the number density $n(z)$ of the gas particles as a function of height z. Express your answer in terms of the quantities given above, fundamental constants, and $n(z = 0) \equiv n_0$.

b. Now assume the gas is a plasma of electrons (mass m_e) and protons (mass m_p). Treat all particles as non-interacting, and assume the system is in thermal equilibrium at a temperature T as before, but assume there is a non-vanishing electric field $\vec{E}$ whose effect is to keep the number densities of electrons and protons *locally equal*, thus maintaining charge neutrality. What is the magnitude and direction of $\vec{E}$? Do either depend on z? Express your answer in terms of m_e, m_p, g, T, the electric charge e of a proton, and fundamental constants.

Problem 8

An ideal classical monoatomic gas of N particles is initially in thermal equilibrium at a temperature $T_\circ$ and volume $V_\circ$. The piston is then allowed to move quasi-statically until its volume increases to V_f. During this process, the heat flow into the system is controlled in such a way that the T, V curve is a straight line:

$$T(V) = T_\circ - \alpha(V - V_\circ) \tag{5}$$

where α is a constant. In terms of N, $T_\circ$, $V_\circ$, V_f, and α, find:

a. the total work done *on* the gas in this process; and

b. the total heat transferred *to* the gas.