Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Master's Final Examination and Ph.D. Qualifying Examination, Part I

Monday, September 27, 2010, 9:00 a.m. to 1:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic, and will be provided. **Personal calculators of any type are not allowed.** Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries will not be allowed. No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

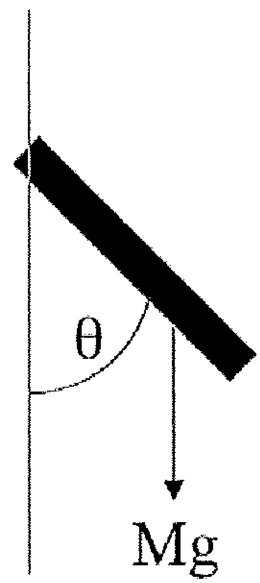
Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand in your exam paper on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31}\,\mathrm{kg}$ ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	$1.673 \times 10^{-27}\,\mathrm{kg}$ ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27}\,\mathrm{kg}$ ($940\,\mathrm{MeV/c^2}$)
W^- rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J \cdot s}$
Speed of light in vacuum (c)	$3.00 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Heat Capacity of Water	$4.19\ \mathrm{J/K/cm^3}$
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N \cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Mass of Earth ($M_\oplus$)	$5.98 \times 10^{24}\,\mathrm{kg}$
Mass of Moon (M_{Moon})	$7.35 \times 10^{22}\,\mathrm{kg}$
Mass of Sun ($M_\odot$)	$1.99 \times 10^{30}\,\mathrm{kg}$
Radius of Earth ($R_\oplus$)	$6.38 \times 10^{6}\,\mathrm{m}$
Radius of Moon (M_{Moon})	$1.74 \times 10^{6}\,\mathrm{m}$
Radius of Sun ($R_\odot$)	$6.96 \times 10^{8}\,\mathrm{m}$
Earth - Sun distance ($R_{\oplus,\odot}$)	$1.50 \times 10^{11}\,\mathrm{m}$
Classical electron radius ($r_\circ$)	$2.82 \times 10^{-15}\,\mathrm{m}$
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
Atomic mass unit	$1.66 \times 10^{-27}\,\mathrm{kg}$
One atmosphere ($1\,\mathrm{atm}$)	$1.01 \times 10^{5}\,\mathrm{N/m^2}$

Problem 1

A pendulum consisting of a thin uniform rod of mass M and length L is suspended by a frictionless bearing in the Earth's gravitational field.

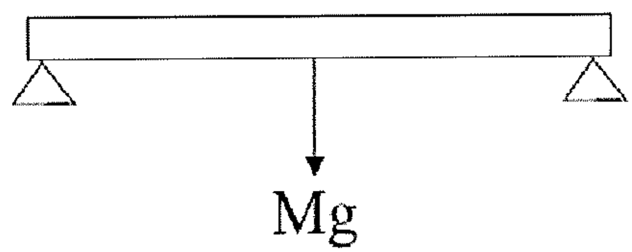


a. Determine the period of oscillation for small amplitude oscillations.

b. Compare this period to that of an ideal pendulum in which the entire mass M is located at a distance L from the bearing.

Problem 2

A uniform thin rod of mass M and length L is supported horizontally by two supports, one at each end. The acceleration of gravity, g, is constant and in the downward direction. At time $t = 0$, the left support is removed.



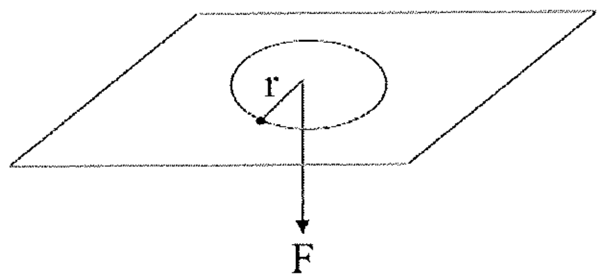
a. Determine the downward acceleration of the center-of-mass of the rod immediately after the support is removed in terms of M, L, and g.

b. Find the angular acceleration of the rod about the remaining support immediately after the support is removed in terms of M, L, and g.

c. Find the force exerted on the rod by the right support immediately after the left support is removed in terms of M, L, and g.

Problem 3

Consider the situation below: A mass m is attached to a massless string at some distance r



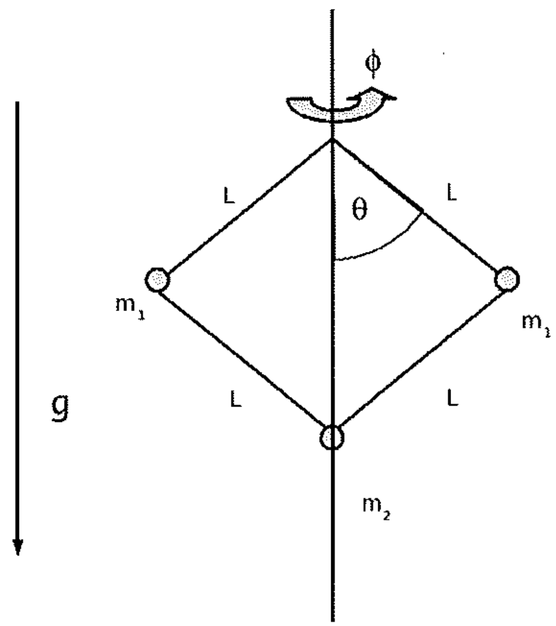
from a hole. The string is pulled down through the hole with a constant force $\mathbf{F}$.

a. At what speed must the mass m move to remain at distance r?

b. $\mathbf{F}$ is increased adiabatically so that the radius of the orbit of m changes from its initial radius r to a final radius of $r/2$. Determine the work $\mathbf{F}$ performed on m.

Problem 4

A bead of mass m_2 slides on a frictionless vertical shaft under gravity with gravitational acceleration $\mathbf{g}$. It is connected by massless rigid rods to two other masses of equal mass m_1. The masses m_1 are each connected by rigid rods to a fixed pivot mounted on the shaft. The rods are connected in such a way that all four points (the pivot, and the three masses) always lie in a common vertical plane. The entire apparatus is free to rotate about the shaft, and is illustrated in the figure below. The entire system is frictionless.

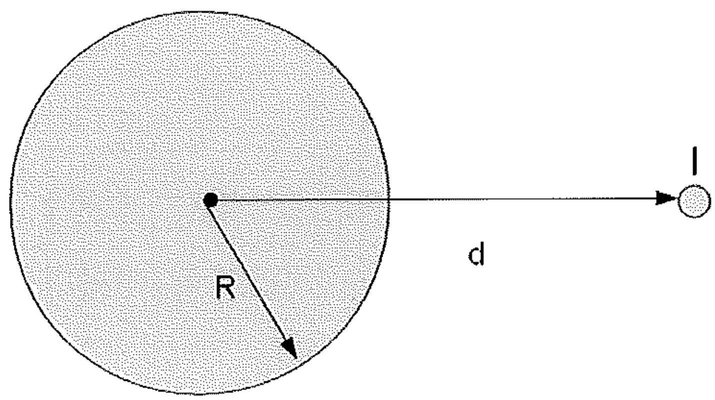


a. Write down the Lagrangian for this system in terms of the variables θ and ϕ, shown in the figure.

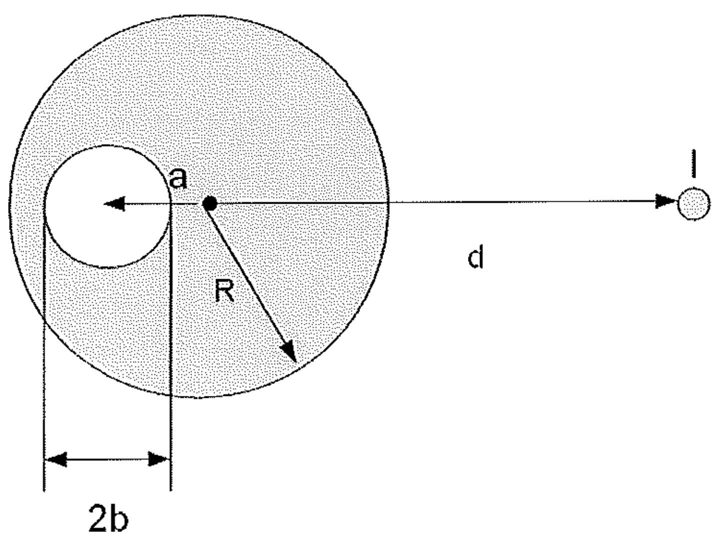
b. Find two independent conserved quantities.

c. If $\frac{d\phi}{dt}(t=0) \neq 0$ and $\theta(t=0) \neq 0$, can $\theta(t>0)$ *ever* $=0$? If not, why not?

Problem 5



a. An infinitely long cylindrical conductor carries uniform current density **J**, directed into the page. The radius of the cylindrical conductor is R. Determine the force per unit length on this conductor exerted by a parallel wire filament which carries current I also directed into the page. The filament is located a distance d from the axis of the conductor.



b. An infinitely long cylindrical cavity with radius b is cut into the cylindrical conductor of part (a). The axes of the cavity, cylindrical conductor, and wire filament are parallel, and lie in the same plane. The cavity axis is distance a from the axis of the conductor, where $a + b < R$. Determine the force per unit length on the conductor exerted by the parallel wire filament assuming that the new conductor carries the same current density **J** and the filament carries the same current I as in part (a).

Problem 6

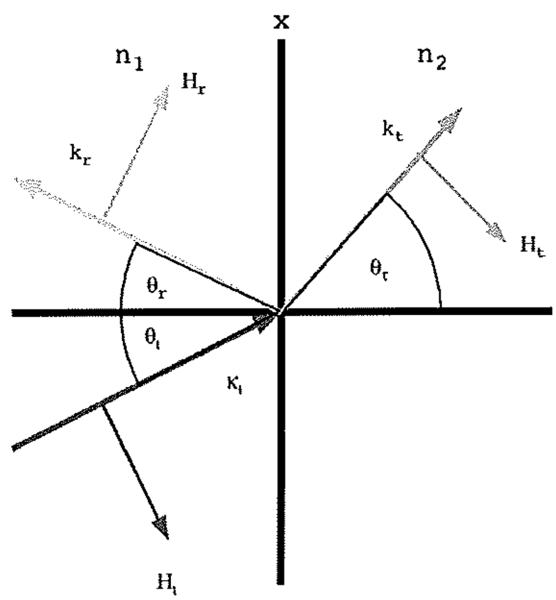
a. A conducting sphere of radius R_1 carries charge Q. Express the electric field of this conductor for distances $r > R_1$.

b. A second, initially uncharged conducting sphere of radius R_2, is placed at large distance from the first conducting sphere and then connected to the first sphere by a very long, fine wire. After the system of two conducting spheres reaches equilibrium, what will be the charge on the second conducting sphere?

c. If the distance between the two spheres is R, what will be the electric force between the two conducting spheres after the system has reached equilibrium? Assume $R_1 + R_2 \ll R$.

Problem 7

A monochromatic plane electromagnetic wave is incident on a planar glass/air interface with refractive indices n_1 and n_2, respectively, with $n_1 > n_2$. The wave approaches the interface with angle of incidence θ_i, where θ_i is greater than the critical angle θ_c for total internal reflection. The wave is polarized with its electric vector parallel to the interface.



a. Set up the boundary condition equations for the electric and magnetic fields.

b. Determine the amplitude reflection coefficient for the electric field.

c. What happens to this amplitude reflection coefficient as the angle of incidence approaches the critical angle? Find an expression for the critical angle in terms of n_1 and n_2.

Problem 8

A capacitor is made of two infinite coaxial metal cylinders with a dielectric material in the space between them. The cylinders have radii a and b, respectively. The material has dielectric constant which varies with radius ρ as $\mathrm{k} = \varepsilon/\varepsilon_\circ = \eta\rho$ where k is the dielectric constant, ε is the permittivity, $\varepsilon_\circ$ is the permittivity of free space, and η is a constant. Under the assumption that the metal cylinders have infinite conductivity, find the capacitance per unit length of the system by finding the electrostatic energy contained in the region between the metal cylinders.