Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT
UNIVERSITY OF OREGON
Ph.D. Qualifying Examination, Part III
Friday, September 19, 2003, 1:00 p.m. to 5:00 p.m.

The examination pages are numbered in the upper left-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed.** Dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished the exam, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^+ rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J-s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Stefan-Boltzmann constant (σ)	5.67×10^{-8} J/(m^2-s-K^4)
Gravitational constant (G)	6.67×10^{-11} N-m^2/kg^2
Permeability of free space ($\mu_\circ$)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space ($\epsilon_\circ$)	8.85×10^{-12} F/m
Mass of Earth (M_E)	5.98×10^{24} kg
Equatorial radius of Earth (R_E)	6.38×10^{6} m
Radius of Sun (R_S)	6.96×10^{8} m
Mass of Sun (M_S)	1.99×10^{30} kg
Temperature of surface of the Sun (T_S)	$5,800$ K
Earth-Sun distance (R_{ES})	1.50×10^{11} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
atomic mass unit	1.7×10^{-27} kg

Stirling's Formula

$$\ln(x!) = x\,\ln(x) - x - \ln\sqrt{2\pi x} + \mathcal{O}(1/x) \tag{1}$$

Integrals

$$\int_{-\infty}^{\infty} dx\; x^{2n}\exp(-ax^2) = \frac{1\times 3\times 5\times\cdots\times(2n-1)}{2^n a^n}\sqrt{\frac{\pi}{a}} \tag{2}$$

$$\int_0^{\infty} \frac{dx}{x}\; x^n \exp(-x) = \Gamma(n) \tag{3}$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x+\sqrt{x^2+a^2}) \tag{4}$$

$$\text{if Re(a)} \;>\; 0 \text{ and Im(b)} \;=\; 0 \text{ then } \int_{-\infty}^{\infty} e^{-ay^2} e^{iby} dy \;=\; e^{-\frac{b^2}{4a}}\sqrt{\frac{\pi}{a}} \tag{5}$$

Problem 1

A classical **ideal gas** exists in a constant gravitational field with acceleration g. The constituent particles have mass m, and the gas is in equilibrium at a constant absolute temperature T.

(a) Find the density distribution $n(z)$ as a function of height z, relative to the density $n(0)$ at some arbitrary reference level.

(b) Now assume the gas is a plasma of electrons (mass m_e) and protons (mass m_p). We treat all particles as non-interacting, but assume that there is a non-vanishing electric field $\vec{E}$ whose effect is to maintain the densities of electrons and protons **locally equal**, thus maintaining charge neutrality. What is the magnitude and direction of $\vec{E}$? Show in particular that E is independent of z.

Problem 2

A large number N of indistinguishable, non-interacting particles occupies the energy levels of a quantum system with exactly M energy levels of energy E_ν. The E_ν are non-degenerate and equi-distant. A state of the system is uniquely characterized by the set of occupation numbers n_ν of energy level ν ($\nu = 1, ..., M$); this is because the particles within the same level are indistinguishable (whereas configurations with different level occupation numbers are distinct).

(a) Write an expression for the probability P_0 of finding the system in a particular state in which level ν is occupied by n_ν particles. Hint: P_0 is proportional to the number of ways in which the N particles can be arranged among M *containers* (the levels), disregarding the order of particles within the containers. The proportionality constant is just a normalization factor which you don't need to compute.

(b) Let the total energy of the system be W. The simplest energy-neutral ($W = const$) change of occupation is when **one** particle changes from a state i to a state $i + k$ while **another** makes the transition from a level $j > k$ to $j - k$. Here, k is the same integer in both transitions. Use the expression from (a) to write down the new probability P^* for this state in terms of the **original** occupation numbers $\{n_\nu\}$.

(c) Express P^* in terms of the probability P_0 for the original state and the occupation numbers (n_i, n_j, n_{i+k}, n_{j-k}) of the levels between which the transitions occur (fixed by the numbers i, j, k).

(d) What is the relation between the occupation numbers n_i, n_j, n_{i+k}, n_{j-k} if we require $P_0 = P^*$? Show that this relation is satisfied approximately if the occupation numbers obey the Boltzmann distribution, assuming large occupation numbers.

Problem 3

Photons in an electromagnetic cavity are to be modeled as a system of non-interacting bosons which can occupy only one possible energy level, but with the total particle number N not being a conserved quantity. The Hamiltonian of the many-particle system is $\mathcal{H} = \hbar\omega(\hat{N} + \frac{1}{2})$ where $\hat{N}$ is the (hermitian) number operator. The eigenstates of $\hat{N}$ are called $|n\rangle$, where n is the integer eigenvalue in $\hat{N}|n\rangle = n|n\rangle$. It turns out that the grand canonical partition function $\mathcal{Q}$ in this case can be viewed as the **canonical** partition function of a system whose Hilbert space is spanned by the set of $|n\rangle$.

(a) Write the density matrix ρ of the system in the basis of $|n\rangle$. Calculate the partition function $\mathcal{Q}$.

(b) Calculate the expectation value of the particle number, $\langle N\rangle$, in thermal equilibrium. Since there is no chemical potential in this problem, you should express $\langle N\rangle$ as a derivative of $\mathcal{Q}$ with respect to $x = \hbar\omega/(k_B T)$.

(c) Using the result of (b), write the probability $P(n)$ for finding n photons purely in terms of the quantities n and $\langle N\rangle$. What is the most probable value of n in this system?

Problem 4

A one-dimensional harmonic oscillator in the ground state is subjected to a uniform force $F = F_\circ e^{-t/\tau}$, beginning at $t = 0$.

Calculate the probability of finding the oscillator in the first excited state at $t \to \infty$ using first order perturbation theory. You can use $\langle 1|x|0\rangle = \sqrt{\frac{\hbar}{2m\omega}}$.

Problem 5

A model potential for a molecule is given by

$$U(r) = -2D\left[\frac{a}{r} - \frac{1}{2}\left(\frac{a}{r}\right)^2\right] ,$$

with a a characteristic length, and $D > 0$ a characteristic energy.

(a) Sketch and briefly discuss $U(r)$ as well as the effective potential $U_{eff}(r)$ that incorporates the centrifugal barrier. Compare the result with V_{eff} for the Coulomb potential

$$V(r) = -\frac{e^2}{r} .$$

(b) Given that the energy spectrum for a particle of mass m in a Coulomb potential is

$$E^c_{n,l} = -\frac{E_R}{(n+l+1)^2} \quad , \; E_R = \frac{me^4}{2\hbar^2}$$

with the radial quantum number, $n = 0, 1, 2, \cdots$, and the orbital quantum number, $l = 0, 1, 2, \cdots$, show that the energy spectrum for a particle in the U potential is

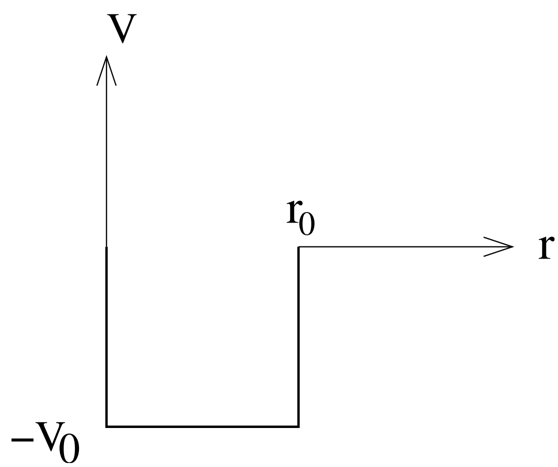
$$E_{n,l} = -\frac{\hbar^2}{2ma^2}\frac{\gamma^4}{(n+\lambda+1)^2} \quad , \; \gamma = \sqrt{2Dm}\; a/\hbar$$

with $\lambda = \sqrt{(l+1/2)^2 + \gamma^2} - 1/2$.

(c) Expand this result to lowest order in γ for small γ. Discuss the degeneracy of the energy levels in comparison with the Coulomb potential.

Problem 6

Consider the 3-dimensional spherically symmetric square-well potential shown in the figure below.



For a given potential radius r_0, find the minimum V_0 which will admit a bound state of energy $E = -E_b$ ($E_b > 0$) and zero angular momentum for a particle of mass m. Assume $E_b \ll V_0$.