Exam #: ______________________________

Printed Name: ______________________________

Signature: ______________________________

PHYSICS DEPARTMENT
UNIVERSITY OF OREGON
Ph.D. Qualifying Examination, Part II
Thursday, September 18, 2003, 1:00 p.m. to 5:00 p.m.

The examination pages are numbered in the upper left-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed.** Dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished the exam, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^+ rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J-s
Speed of light in vacuum (c)	3.00×10^8 m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Stefan-Boltzmann constant (σ)	5.67×10^{-8} J/(m^2-s-K^4)
Gravitational constant (G)	6.67×10^{-11} N-m^2/kg^2
Permeability of free space ($\mu_\circ$)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space ($\epsilon_\circ$)	8.85×10^{-12} F/m
Mass of Earth (M_E)	5.98×10^{24} kg
Equatorial radius of Earth (R_E)	6.38×10^6 m
Radius of Sun (R_S)	6.96×10^8 m
Mass of Sun (M_S)	1.99×10^{30} kg
Temperature of surface of the Sun (T_S)	$5,800$ K
Earth-Sun distance (R_{ES})	1.50×10^{11} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
atomic mass unit	1.7×10^{-27} kg

Stirling's Formula

$$\ln(x!) = x\,\ln(x) - x - \ln\sqrt{2\pi x} + \mathcal{O}(1/x) \tag{1}$$

Integrals

$$\int_{-\infty}^{\infty} dx\; x^{2n}\exp(-ax^2) = \frac{1\times 3\times 5\times\cdots\times(2n-1)}{2^n a^n}\sqrt{\frac{\pi}{a}} \tag{2}$$

$$\int \frac{dx}{\sqrt{a+bx+cx^2}} = \frac{1}{\sqrt{-c}}\sin^{-1}\left(\frac{-2cx-b}{\sqrt{b^2-4ac}}\right),\ \text{if } c<0 \tag{3}$$

$$= \frac{1}{\sqrt{c}}\sinh^{-1}\left(\frac{2cx+b}{\sqrt{b^2-4ac}}\right),\ \text{if } c>0$$

$$\int_0^{\infty}\frac{dx}{x}\,x^n\exp(-x) = \Gamma(n) \tag{4}$$

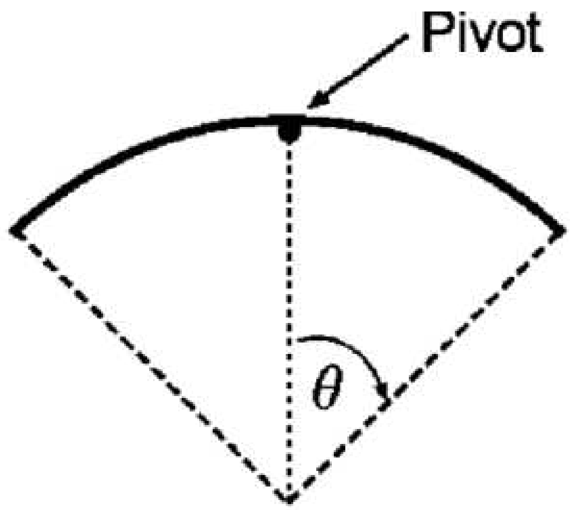
$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x+\sqrt{x^2+a^2}) \tag{5}$$

$$\int_{-\infty}^{\infty} e^{-ay^2}e^{iby}dy = e^{-\frac{b^2}{4a}}\sqrt{\frac{\pi}{a}},\ \text{if Re(a)} > 0 \text{ and Im(b)} = 0 \tag{6}$$

Problem 1

(a) A thin walled hoop of radius r is hung from a peg and oscillates as a physical pendulum. Show that the frequency of small oscillations is the same as that of a simple pendulum of length $2r$.

(b) A section of a thin walled hoop of radius r is hung balanced on a peg and oscillates as a physical pendulum (see Figure). Test the following assertion for correctness: The frequency of small oscillations is the same as that of part (a) above.



Problem 2

The orbital precession of the planet Mercury was an important first test of the General Theory of Relativity. The relativistic correction for the orbital motion of the inner planets can be treated classically using a modified potential function of the form

$$V_R(r) = V_g(r) - \frac{[E - V_g(r)]^2}{2mc^2}$$

where $V_g(r) = -GM_\odot m/r$ is the nonrelativistic gravitational potential, E is the total energy for a planet of mass m which orbits about the Sun, and $M_\odot$ is the mass of the Sun.

(a) The two-body, three-dimensional Lagrangian can be reduced to a one-body, two-dimensional Lagrangian of the form

$$\mathcal{L} = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2) - V(r)$$

for motion in a central force field. Here, μ is the reduced mass for the system and r is the distance between the two masses. θ is the aziumthal polar coordinate. Using this Lagrangian show that the equation of motion for a mass m in a central force field can be written

$$\frac{d^2u}{d\theta^2} + u = -\frac{\mu}{l_\circ^2}\frac{\partial[V(u^{-1})]}{\partial u}$$

where $u = 1/r$ and $l_\circ$ is a constant of the motion given by $l_\circ = \mu\dot{\theta}r^2$.

(b) Find an expression for the per orbit precession of the line-of-apsides for a planet of mass m that moves in the potential $V_R(r)$ given at the top. Using your expression, estimate the amount that the line-of-apsides of Mercury's orbit precesses per orbit. The mass of the Sun is 2.0×10^{33} g, the mass of Mercury is 3.3×10^{26} g, the orbital period of Mercury is 88 d, the semi-major axis of the orbit of Mercury is $a = 5.8 \times 10^{12}$ cm, and the eccentricity of the orbit of Mercury is 0.206. Note that the relativistic correction to the potential is small.

Problem 3

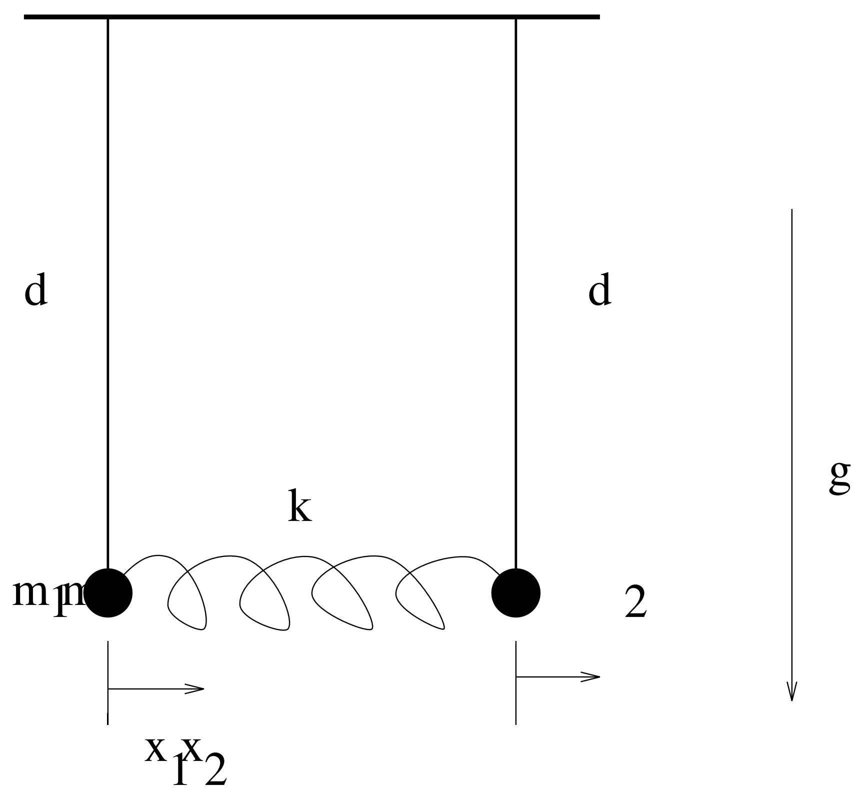
Consider a system of two identical frictionless pendula of length d and mass m_1 and m_2 coupled with a massless spring of force constant k and moving in a plane. The pivots for the two pendula are separated by a distance L. The unstretched spring has length L. Begin by considering the case where $m_1 = m_2 = m$ for parts (a), (b), and (c) below.

(a) Assuming small angle displacements, write the Lagrangian for this system in terms of x_1 and x_2. State all assumptions you make.

(b) What are the equations of motion for this system?

(c) Using the equations of motion, find the natural frequencies ω_i for this system.

(d) Now, suppose that $m_1 \ll m_2$. What are the natural frequencies ω_i for this new system?



Problem 4

(a) From Maxwell's equations, derive the wave equations for electric and magnetic fields propagating through free space.

(b) Calculate the average r.m.s. electric field intensity (i.e. the square root of the average of the square of the electric field) due to solar radiation at the surface of the Sun. The Sun's radius is 7×10^8 meters, and its radiated power is 3.8×10^{26} Watts.

Problem 5

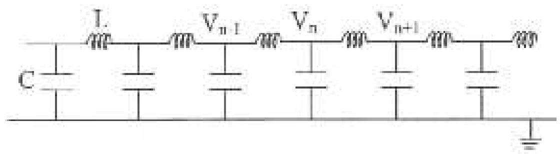
The electron cloud of a hydrogen atom in its ground state has charge density:

$$\rho = \frac{q}{\pi a^3} \exp(-\frac{2r}{a})$$

where q is the charge of the electron and a is the Bohr radius. Show that the dipole moment induced in this atom by an external electric field, $\mathbf{E}$, can be approximated by the relationship $\mathbf{p} = \alpha \mathbf{E}$, and derive an expression for α. (Assume that under the influence of an external electric field the electron cloud is displaced rigidly, that is, it remains spherical as it is displaced.)

Problem 6

Consider the infinite periodic circuit shown below:



All capacitors have the same capacitance C and all inductors have the same inductance L. The current flowing through the n^{th} inductor is I_n, and the potential across the n^{th} capacitor , V_n, oscillates harmonically with a frequency ω as $V_n = A_n \exp(i\omega t)$.

It can be shown that under these circumstances the V_n satisfy the equation

$$\frac{d^2 V_n}{dt^2} = \frac{(V_{n+1} - 2V_n + V_{n-1})}{(LC)}$$

Find the range of frequencies, ω, for which a periodically oscillating electric signal can propagate along the circuit without attenuation. (Hint: Try solutions that oscillate harmonically in time with frequency ω and as a function of position n with wave vector k; then see what k has to be.)