Exam #: ______________________________

Printed Name: ______________________________

Signature: ______________________________

PHYSICS DEPARTMENT
UNIVERSITY OF OREGON
Masters Final Examination
and
Ph.D. Qualifying Examination, Part I
Wednesday, September 17, 2003, 1:00 p.m. to 5:00 p.m.

The examination pages are numbered in the upper left-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are twelve equally weighted questions, each beginning on a new page. Read all twelve questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed.** Dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished the exam, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
Planck's constant (h)	6.63×10^{-34} J-s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Stefan-Boltzmann constant (σ)	5.67×10^{-8} J/(m^2-s-K^4)
Gravitational constant (G)	6.67×10^{-11} N-m^2/kg^2
Permeability of free space ($\mu_\circ$)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space ($\epsilon_\circ$)	8.85×10^{-12} F/m
Mass of Earth (M_E)	5.98×10^{24} kg
Equatorial radius of Earth (R_E)	6.38×10^{6} m
Radius of Sun (R_S)	6.96×10^{8} m
Mass of Sun (M_S)	1.99×10^{30} kg
Temperature of surface of the Sun (T_S)	$5,800$ K
Equatorial radius of Saturn (R_{saturn})	6.03×10^{7} m
Mass of Saturn (M_{saturn})	5.69×10^{26} kg
Temperature of surface of Saturn (T_{saturn})	123 K
Earth-Sun distance (R_{ES})	1.50×10^{11} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
atomic mass unit	1.7×10^{-27} kg

Stirling's Formula

$$\ln(x!) = x\,\ln(x) - x - \ln\sqrt{2\pi x} + \mathcal{O}(1/x) \tag{1}$$

Integrals

$$\int_{-\infty}^{\infty} dx\; x^{2n}\exp(-ax^2) = \frac{1\times 3\times 5\times\cdots\times(2n-1)}{2^n a^n}\sqrt{\frac{\pi}{a}} \tag{2}$$

$$\int_0^{\infty} \frac{dx}{x}\; x^n\exp(-x) = \Gamma(n) \tag{3}$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x+\sqrt{x^2+a^2}) \tag{4}$$

Problem 1

A particle with mass m is constrained to move in one dimension along the x-axis. Its potential energy is given by

$$V(x) = V_\circ - \frac{1}{2}\beta x^2$$

where $V_\circ$ and β are positive constants. The particle experiences a frictional force that is linearly proportional to its velocity. At time $t = 0$, the particle has position $x_\circ$ and velocity $\dot{x} = 0$.

(a) Find an expression for the particle's position for all $t > 0$.

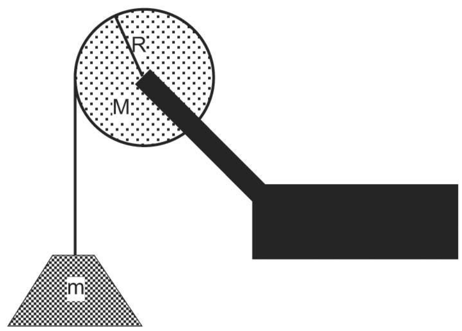
(b) Find the limiting behavior of the particle's position at large t.

Problem 2

A satellite is put into a circular orbit at radial distance $R_\circ$ from the center of the Earth. The thin upper atmosphere of the Earth produces a viscous drag such that the satellite's orbit is observed to decay at a very slow rate. By *slow* we mean that the loss of energy per orbital period is small compared to the satellite's total kinetic energy. Moreover, it is observed that the rate of change of the radial distance dR/dt is constant, $dR/dt = -C$ where $C > 0$. If the viscous drag force on the satellite has the form $F(v) = Av^\alpha$, obtain expressions or constraints on the possible values of A and α.

Problem 3

A disk of mass M and radius R acts as a pulley. A mass m is suspended (in the Earth's gravitational field) from the pulley by a massless, inextensible string wrapped around the outer edge of the pulley. Find the acceleration of the mass m as a function of m, M, R, and g, the gravitational acceleration at the surface of the Earth.



Problem 4

A coherent plane wave with wavelength λ is normally incident on four slits with width $a/3$ and with separation a between the centers of any two adjacent slits. The distance between the observation plane and the slits, L, is assumed to be large. As a function of θ, the angle measured from the axis

(a) determine the position of the interference maximum on the screen

(b) determine the position of the diffraction and interference minima on the screen

(c) draw schematically the interference-diffraction pattern on the screen.

Problem 5

An infinite straight wire carries current $I(t)$,

$$I(t) = \begin{cases} I_\circ, & \text{if } t < 0 \\ 0, & \text{if } t > 0 \end{cases}$$

where $I_\circ$ is a constant. Find the space and time dependence of the electric and magnetic fields generated by this current.

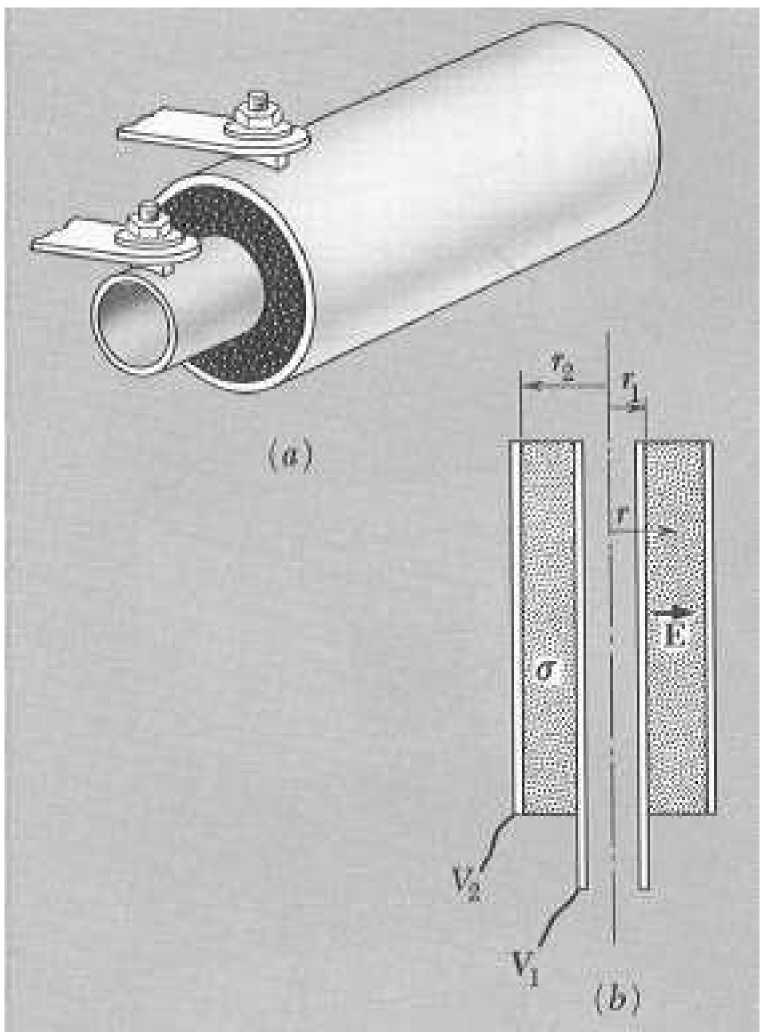
Problem 6

Two cylindrical copper sleeves are separated by a sheath of graphite. (see Figure [a]) A radial current flows between the cylinders.

(a) How does the radial electric field depend on the distance from the common axis of the cylinders?

(b) Express the radial electric field in terms of the potential difference between the outer and inner cylinders, V_0, and the radius of the inner and outer cylinders, r_1 and r_2, respectively (see Figure [b])

(c) Find the resistance of a section of length, L, of the cylindrical graphite when conducting a radial current between the two cylinders. Denote the conductivity of graphite by σ.



Problem 7

For an adiabatic **and** quasistatic expansion of an ideal gas, the pressure P and volume V satisfy the relation

$$P\,V^{\gamma} = const.$$

with $\gamma > 1$, a material-dependent constant. Assume that this relation can be used to model the fireball of a detonating atomic bomb. In this problem, the fireball is a sphere of hot gas with $\gamma = 1.4$, confined to a radius R that grows with time. Assume that the number of particles in the sphere stays constant. The pressure changes in time but is independent of position inside the fireball.

(a) Immediately after the detonation, the temperature of the fireball is $T_1 = 300,000$ K and its radius is $R_1 = 10$ m. Estimate the radius R_2 when the fireball has cooled to $T_2 = 3,000$ K.

(b) Derive an expression for the work W done by the bomb as the fireball expands from R_1 to R_2, always assuming spherical shape. Write the result in terms of the initial and final radii, and the final pressure P_2 that prevails when R_2 is reached.

Problem 8

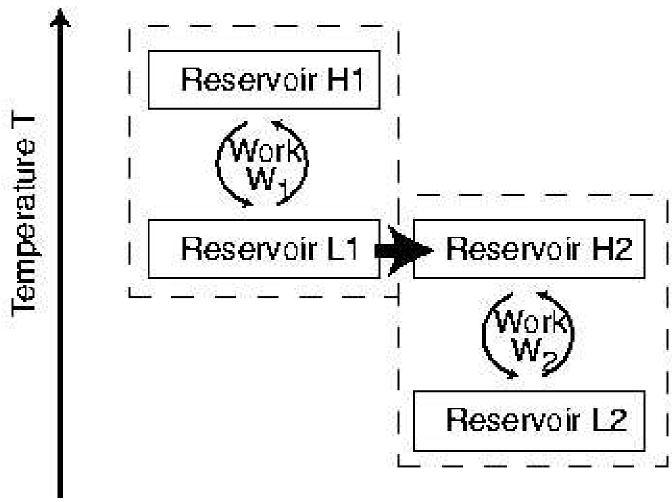
Suppose that the lowest possible energy of a conduction electron in a metal is $-V_0$ below the energy of a free electron at infinite distance from the metal. The conduction electrons have a Fermi energy, E_F. The minimum energy needed to remove an electron from the metal at zero temperature is thus $\Phi = V_0 - E_F$ (see diagram below). This Φ is called the "work function" of the metal.

(a) Cesium behaves to a good approximation as a degenerate free electron gas. The density of its conduction electrons is 9×10^{21} cm^{-3} and $V_0 = 3.6$ eV. Find Φ (in eV). Hint: the density of states in k space for a free three-dimensional (3D) electron gas is $g(\mathbf{k}) = 2\ Vol/(2\pi)^3$ where Vol is the volume.

(b) Consider an electron gas outside the metal that is in thermal equilibrium with the electrons within the metal at temperature $T = 500$ K. The density of conduction electrons is quite small for $k_B T \ll \Phi$ so that they may be considered a classical ideal gas. Find the density of electrons outside the metal. Hint: the free energy of a monatomic classical ideal gas is $F = N k_B T\,[\ln(n/n_Q) - 1]$ where $n = N/Vol$ and $n_Q = [m k_B T/(2\pi\hbar^2)]^{3/2}$.

Problem 9

In a steam power plant, steam engines work in pairs. The heat output from one is the approximate heat input of the second; see diagram.



The operating temperatures of the first engine are $T_{H1} = 670°$ C and $T_{L1} = 440°$ C, and of the second are $T_{H2} = 440°$ C and $T_{L2} = 290°$ C.

(a) If the heat of combustion of coal is 2.8×10^7 J/kg, at what rate must coal be burned if the plant is to put out $P_{total} = 1000$ MW of power? Assume the efficiency of the engines is 60% of the Carnot efficiency.

(b) Water is used to cool the power plant. If the water temperature is allowed to increase by no more than 6.0° C, estimate how much water must pass through the plant per hour. To get the specific heat of water, recall the definition of one calorie, 1 cal = 4.184 J.

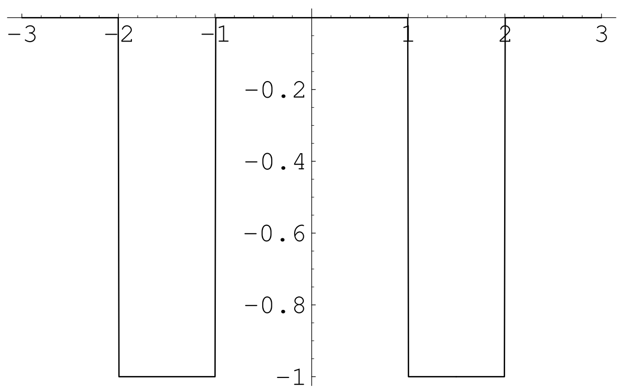
Problem 10

A particle of mass m moves in a one-dimensional double-well potential $V(x)$ (see figure below). The potential energy equals zero outside each well. The potential $V(x)$ can be written as the sum of two identically shaped wells

$$V(x) = V_1(x) + V_2(x) \quad .$$

Considered separately, each well V_i has a groundstate wavefunction ϕ_i. The two wells are separated sufficiently that the overlap between ϕ_i's is small.

Find the approximate energies of the solutions $\psi_{\pm}(x) = (\phi_1(x) \ \pm \ \phi_2(x))/\sqrt{2}$ by taking expectation values. Which has higher energy?



Problem 11

The following process involving nuclei occurs during carbon burning which can take place in the interiors of stars:

$$^{12}_{6}\mathrm{C} + ^{12}_{6}\mathrm{C} \rightarrow \; ^{24}_{12}\mathrm{Mg} + \gamma$$

The C and Mg masses are 12.0000 u and 23.9850 u, respectively, where 1 $u = 931.49342$ MeV/c^2.

(a) How much energy (in MeV) is released in this process?

(b) In a reference frame in which the initial momentum is zero, what is the speed of the Mg nucleus?

(c) In the same reference frame as (b), what is the wavelength of the photon? This would belong to which region of the electromagnetic spectrum?

Problem 12

In this problem some aspects of the Pound-Rebka experiment that determined the gravitational red-shift of photons are explored.

(a) Consider a photon emitted at the surface of the Earth and detected at height of 22.5 m above the Earth's surface. Using energy mass equivalence, estimate the fractional shift in frequency that is measured.

(b) Suppose the photon is emitted by an excited state of a nucleus. If the photon has energy 14.4 keV, estimate the lifetime that the nuclear state must have to give an energy smearing which is of the same order as the red shift calculated in (a).

(c) In the Pound-Rebka experiment, Mössbauer spectroscopy (in which the target is moving) is used so that the absorbed photon recoils against the entire crystal containing the nucleus. (If free atoms were used, the effects of the recoiling nucleus would be much larger than the gravitational red shift.) Ignoring effects of recoil, with what velocity must the target be moved in order to compensate for the expected gravitational red shift?