Exam #: ____________________

Printed Name: ____________________

Signature: ____________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART III

Friday, September 20, 2002, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^+ rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J$\cdot$ s
Speed of light in vacuum (c)	3.00×10^8 m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N$\cdot$ m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth (M_{Earth})	5.98×10^{24} kg
Mass of Moon (M_{Moon})	7.35×10^{22} kg
Radius of Earth (R_{Earth})	6.38×10^6 m
Radius of Moon (M_{Moon})	1.74×10^6 m
Radius of Mars (R_{Mars})	3.39×10^6 m
Radius of Sun (R_{Sun})	6.96×10^8 m
Temperature of surface of Sun (T_{Sun})	5.8×10^3 K
Earth - Sun distance (R_{ES})	1.50×10^{11} m
Mars - Sun distance (R_{MS})	2.28×10^{11} m
Density of iron at low temperature (ρ_{Fe})	7.88×10^3 kg/m^3
Classical electron radius (r_0)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
Atomic mass unit	1.66×10^{-27} kg
Specific heat of oxygen (c_V)	21.1 J/mole$\cdot$ K
Specific heat of oxygen (c_P)	29.4 J/mole$\cdot$ K

Moments of Inertia

For a hoop of mass M and radius R, about its symmetry axis: MR^2.

For a disk of mass M and radius R, about its symmetry axis: $(1/2)\,MR^2$.

For a solid sphere of mass M and radius R, about any symmetry axis: $(3/5)\,MR^2$.

Spherical harmonics

The spherical harmonics Y_{lm} have the normalization property

$$\int_{-1}^{1} d\cos\theta \int_{0}^{2\pi} d\phi \; Y^{\star}_{lm}(\theta,\phi)\, Y_{l'm'}(\theta,\phi) = \delta_{ll'}\,\delta_{mm'}.$$

The first few are

$$\begin{aligned}
Y_{00} &= \frac{1}{\sqrt{4\pi}} \\
Y_{11} &= -\sqrt{\frac{3}{8\pi}}\,\sin\theta\, e^{i\phi} \\
Y_{10} &= \sqrt{\frac{3}{4\pi}}\,\cos\theta \\
Y_{22} &= \sqrt{\frac{15}{32\pi}}\,\sin^2\theta\, e^{2i\phi} \\
Y_{21} &= -\sqrt{\frac{15}{8\pi}}\,\sin\theta\,\cos\theta\, e^{i\phi} \\
Y_{20} &= \sqrt{\frac{5}{16\pi}}\,[3\cos^2\theta - 1]
\end{aligned}$$

with $Y_{l,-m}(\theta,\phi) = -Y^{*}_{lm}(\theta,\phi)$.

Spherical Bessel functions

The spherical Bessel functions $j_l(x)$ have the normalization property

$$\int_0^{\infty} r^2 dr \; j_l(kr)\, j_{l'}(kr) = \frac{\pi}{2k^2}\,\delta(k-k')$$

The first few are

$$\begin{aligned}
j_0(x) &= \frac{\sin x}{x} \\
j_1(x) &= \frac{\sin x}{x^2} - \frac{\cos x}{x} \\
j_2(x) &= \left(\frac{3}{s^2} - \frac{1}{x}\right)\sin x - \frac{3\cos x}{x^2}.
\end{aligned}$$

Problem 1

Consider the hydrogen atom and suppose that the proton, instead of being a point positive charge, is instead a spherical shell of radius R with total positive charge $+e$ spread uniformly over the shell. This will modify the potential energy function to give

$$V(r) = -\frac{e^2}{4\pi\epsilon_0}\,\frac{1}{r}$$

for $r > R$ but

$$V(r) = -\frac{e^2}{4\pi\epsilon_0}\,\frac{1}{R}$$

for $r < R$. Now suppose that $R = a/1000$, where a is the Bohr radius. Estimate the energy correction (in eV) to the ground state energy of hydrogen.

You may find it useful to recall that the 1s ground state wave function for hydrogen with a point charge at the center is

$$\Psi_{100}(r,\theta,\phi) = \frac{2}{a^{3/2}}\,\exp(-r/a)\,Y_{00}(\theta,\phi).$$

Problem 2

Which of the following functions are acceptable as solutions to the time-independent Schrödinger equation for a freely moving particle and which are not? Give a reason for your answer in each case. (The parameters A, B, k, and α are non-zero constants.)

$$\psi(r,\theta,\phi) = A\left[\frac{\sin(kr)}{(kr)^2} - \frac{\cos(kr)}{kr}\right](3\cos^2\theta - 1) \quad (1)$$

$$\psi(r,\theta,\phi) = A\left[\frac{\sin(kr)}{(kr)^2} + \frac{\cos(kr)}{kr}\right]\cos\theta \quad (2)$$

$$\psi(r,\theta,\phi) = A\,\frac{\sin(kr)}{kr}\sin\theta\, e^{-i\phi} \quad (3)$$

$$\psi(r,\theta,\phi) = A\,\frac{\sin(kr)}{kr} + B\left[\frac{\sin(kr)}{(kr)^2} - \frac{\cos(kr)}{kr}\right]\cos\theta \quad (4)$$

$$\psi(r,\theta,\phi) = A\,\frac{\sin(kr+\alpha)}{kr}. \quad (5)$$

Problem 3

A nonrelativistic electron of mass m_e is confined in a cube $0 < x < d$, $0 < y < d$, $0 < z < d$. The potential energy function of the electron is zero inside the cube.

a) What is the ground state wave function and energy for the electron?

b) A second electron is added. What is the ground state wave function (including spin) and energy for the pair of electrons, neglecting the electric interaction between the electrons?

c) Now, suppose that the interaction between electrons is given by the "contact" potential

$$V(\vec{r}_1 - \vec{r}_2) = V_0 \, d^3 \, \delta(\vec{r}_1 - \vec{r}_2).$$

Estimate the change in the ground state energy for the system if V_0 is small compared to the kinetic energy of each electron.

Problem 4

This problem concerns the possibility of having an atmosphere on the moon.

a) What is the escape velocity v_e for an object of mass m near the surface of the moon? That is, what is the minimum speed that the object would have to have to escape the moon's gravitational field?

b) The moon has a daytime surface temperature of $T \approx 400$ K. Suppose that the moon had a CO_2 atmosphere in equilibrium at this temperature. What would the mean speed of the CO_2 molecules be? (Take the mass of a CO_2 molecule to be 44 times the mass of a proton.) How does that speed compare to v_e?

c) Suppose that the moon originally had a thin CO_2 atmosphere. What fraction of the molecules would have a speed bigger than the escape velocity? (You don't need to perform any difficult integrals here: an estimate that is correct to within a factor of 3 will be good enough.)

Problem 5

Consider a system in one dimension consisting of N noninteracting particles, each of mass m and potential energy $V(x_i) = \frac{1}{2}m\omega^2 x_i^2$, where ω is a constant and x_i is the coordinate of the particle. The system is in contact with a heat reservoir of temperature T.

a) Write down the possible quantum mechanical energy eigenvalues for each particle.

b) Determine the partition function for a particle.

c) Calculate the average energy $\overline{E}$ of the N-particle system.

d) Show that in the classical limit your result for $\overline{E}$ is consistent with what is expected for the classical equipartition theorem.

Problem 6

The ground state of a certain alloy of N atoms ($N \gg 1$) of type A and N atoms of type B is a simple cubic lattice in which every atom of one kind has six nearest neighbors of the other kind. (See the diagram below.) At temperatures above $0\,\mathrm{K}$, atoms can occasionally move and so a few of the atoms will be surrounded by atoms of the same kind.

a) Derive an expression for the change in the entropy of the alloy compared to the ground state if one A atom and one B atom end up in wrong sites (*i.e.*, surrounded by atoms of their same type).

b) Thinking of each atom as bonded to its six nearest neighbors, suppose that the energy of a wrong bond (between two like atoms) is D greater than the energy of a bond between different atoms. Calculate the change in the free energy of the alloy if one A atom and one B atom end up in wrong sites.

c) Now suppose that the number of A atoms in wrong sites (which equals the number of B atoms at wrong sites) is K. Take both N and K to be large, but assume that $K/N \ll 1$. What is the difference of the free energy of this configuration compared to the free energy of the ground state? Use your result for the free energy to compute the equilibrium value of K/N if the system is maintained at temperature T.