Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART II

Thursday, September 19, 2002, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^+ rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J$\cdot$ s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N$\cdot$ m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth (M_{Earth})	5.98×10^{24} kg
Mass of Moon (M_{Moon})	7.35×10^{22} kg
Radius of Earth (R_{Earth})	6.38×10^{6} m
Radius of Moon (M_{Moon})	1.74×10^{6} m
Radius of Mars (R_{Mars})	3.39×10^{6} m
Radius of Sun (R_{Sun})	6.96×10^{8} m
Temperature of surface of Sun (T_{Sun})	5.8×10^{3} K
Earth - Sun distance (R_{ES})	1.50×10^{11} m
Mars - Sun distance (R_{MS})	2.28×10^{11} m
Density of iron at low temperature (ρ_{Fe})	7.88×10^{3} kg/m^3
Classical electron radius (r_0)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
Atomic mass unit	1.66×10^{-27} kg
Specific heat of oxygen (c_V)	21.1 J/mole$\cdot$ K
Specific heat of oxygen (c_P)	29.4 J/mole$\cdot$ K

Moments of Inertia

For a hoop of mass M and radius R, about its symmetry axis: MR^2.

For a disk of mass M and radius R, about its symmetry axis: $(1/2)\,MR^2$.

For a solid sphere of mass M and radius R, about any symmetry axis: $(3/5)\,MR^2$.

Spherical harmonics

The spherical harmonics Y_{lm} have the normalization property

$$\int_{-1}^{1} d\cos\theta \int_{0}^{2\pi} d\phi \; Y^{\star}_{lm}(\theta,\phi)\, Y_{l'm'}(\theta,\phi) = \delta_{ll'}\, \delta_{mm'}.$$

The first few are

$$\begin{aligned}
Y_{00} &= \frac{1}{\sqrt{4\pi}} \\
Y_{11} &= -\sqrt{\frac{3}{8\pi}}\, \sin\theta\, e^{i\phi} \\
Y_{10} &= \sqrt{\frac{3}{4\pi}}\, \cos\theta \\
Y_{22} &= \sqrt{\frac{15}{32\pi}}\, \sin^2\theta\, e^{2i\phi} \\
Y_{21} &= -\sqrt{\frac{15}{8\pi}}\, \sin\theta\, \cos\theta\, e^{i\phi} \\
Y_{20} &= \sqrt{\frac{5}{16\pi}}\, [3\cos^2\theta - 1]
\end{aligned}$$

with $Y_{l,-m}(\theta,\phi) = -Y^{*}_{lm}(\theta,\phi)$.

Spherical Bessel functions

The spherical Bessel functions $j_l(x)$ have the normalization property

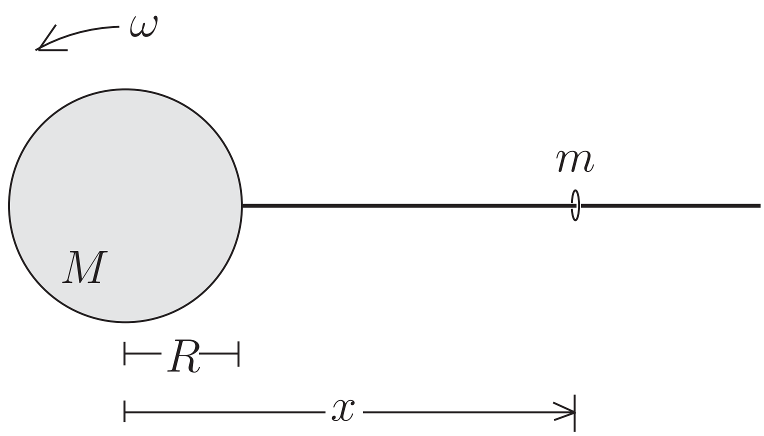
$$\int_{0}^{\infty} r^2 dr \; j_l(kr)\, j_{l'}(kr) = \frac{\pi}{2k^2}\, \delta(k - k')$$

The first few are

$$\begin{aligned}
j_0(x) &= \frac{\sin x}{x} \\
j_1(x) &= \frac{\sin x}{x^2} - \frac{\cos x}{x} \\
j_2(x) &= \left(\frac{3}{s^2} - \frac{1}{x}\right) \sin x - \frac{3\cos x}{x^2}.
\end{aligned}$$

Problem 1

Consider a rigid, frictionless wire attached vertically to a point on the equator of the earth. The earth has radius R, mass M, and is rotating at angular velocity ω. A bead of mass m can slide without friction on the wire. Let the distance of the bead from the center of the earth be $x(t)$. Assume that $m \ll M$. The bead's distance $(x - R)$ from the surface of the earth is not necessarily small compared to R.



View from above the north pole.

a) Write the equations of motion for the bead.

b) Is there a conserved quantity for the motion of the bead?

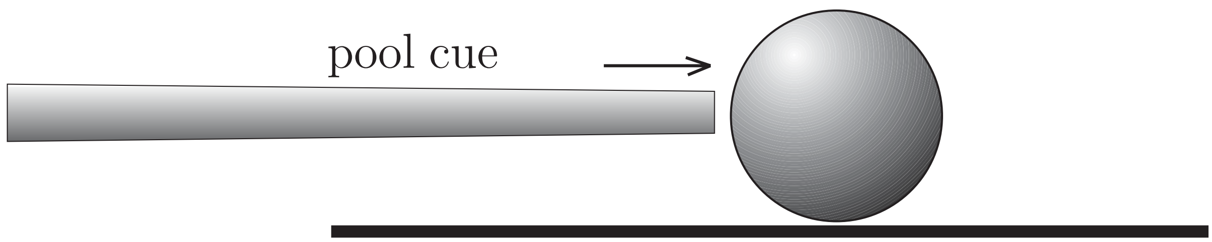
c) Show that there is a point $x = P$ on the wire at which, ignoring stability questions, the bead can sit forever. How far is this point from the center of the earth?

d) Consider the stability of the bead located very near to P. If the bead is stable at P, solve for the frequency of small oscillations. If it is unstable, at which velocity $\dot{x}$ will it hit the surface of the earth if it starts just below P?

Problem 2

A pole cue impulsively strikes a pool ball that is resting on a table, as shown in the picture. The pool ball is a homogeneous sphere of mass m and radius R. It gets an initial velocity of v_0 from being struck by the cue. The interface between the ball and the pool table is characterized by a coefficient of dynamic friction friction μ, which is independent of the relative velocity of the contacting surfaces. Initially, the ball skids on the surface without rolling at all, but eventually it will have purely rolling motion.

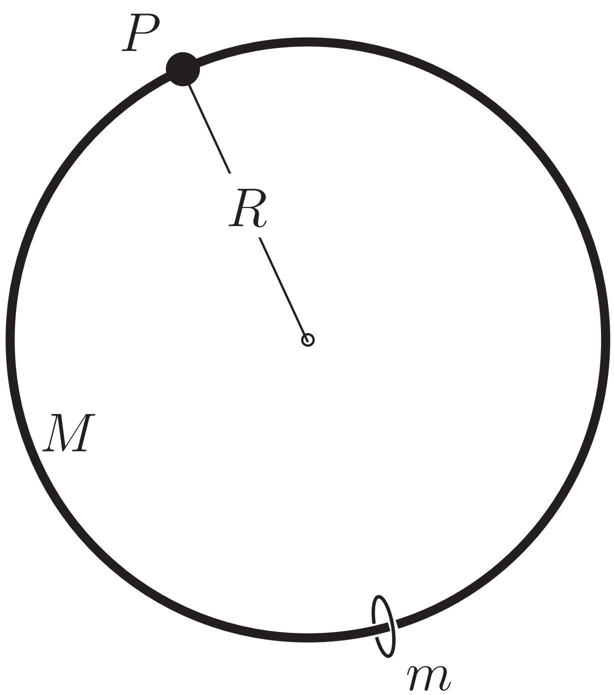
How far will the ball travel before pure rolling sets in?



Problem 3

A bead of mass m can slide freely along a rigid circular hoop of radius R and mass M. The hoop is suspended from a fixed pivot point P so that it can swing freely in the vertical plane.

Find the frequencies and normal modes for small oscillations about the equilibrium position of this system.



Problem 4

Consider a spherical shell of radius R with a uniform charge density η. The shell is centered at the origin and is rotating about the z axis with a constant angular velocity ω. The magnetic field at an observation point $\vec{r}$ can be written in terms of the vector potential as $\vec{B}(\vec{r}) = \vec{\nabla} \times \vec{A}(\vec{r})$.

a) Calculate the surface current density $\vec{j}(\vec{r'})$ at any point $\vec{r'}$ on the surface of the shell.

b) Find $\vec{A}(\vec{r})$ outside the shell. (Use Coulomb gauge for $\vec{A}$.)

c) Find $\vec{A}(\vec{r})$ inside the shell.

d) Sketch $\vec{A}(\vec{r})$ in the equatorial plane of the rotating shell, indicating its direction.

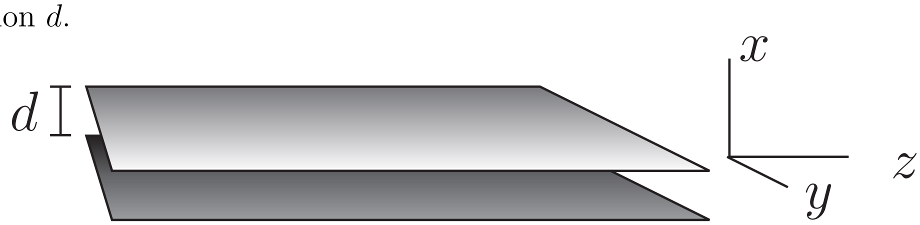
You may want to use the following formula for $1/|\vec{r} - \vec{r'}|$, where $\vec{r}$ has polar coordinates r, θ, ϕ and $\vec{r'}$ has polar coordinates r', θ', ϕ':

$$\frac{1}{|\vec{r} - \vec{r'}|} = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} \frac{4\pi}{2l+1} \frac{r_<^l}{r_>^{l+1}} Y_{lm}^{\star}(\theta', \phi')\, Y_{lm}(\theta, \phi).$$

Here $r_<$ is the lesser of r and r' and $r_>$ is the greater of r and r'.

Problem 5

Consider a waveguide along the z-direction consisting of two parallel metal plates with a separation d. Assume electromagnetic waves propagate along the z-direction inside the waveguide and the magnetic field is along the y-direction and is parallel to the plates (so that we have TM waves). The length and width of the plates are very large compared to the separation d.



a) Write down the differential (Helmholtz) equations and boundary conditions for the magnetic field of electromagnetic waves propagating inside the waveguide.

b) For a given wavelength of the electromagnetic field, find the spatial profile of the magnetic field of the eigenmodes in the waveguide and the total number of modes which can be supported by the waveguide.

c) Under what conditions can the waveguide support at least one TM mode?

Problem 6

A dielectric film of thickness D and index of refraction n_2 lies between media with indices of refraction n_1 and n_3. Assume that $n_1 \neq n_3$. All three media have magnetic permeabilities equal to that of the vacuum: $\mu_1 = \mu_2 = \mu_3 = \mu_0$. A plane electromagnetic wave with frequency ω is incident from the left, with its wave fronts parallel to the plane of the films. Show that there will be a reflected wave unless $n_2 = \sqrt{n_1 n_3}$ and the thickness of the dielectric film is related to the wavelength λ_2 of the light in the film by

$$D = \frac{m}{4}\lambda_2,$$

where m is an odd integer.