Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART III

Friday, September 21, 2001, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	1.673×10^{-27} kg ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	1.675×10^{-27} kg ($940\,\mathrm{MeV/c^2}$)
W^+ rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	6.63×10^{-34} J$\cdot$ s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	$6.67 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth (M_E)	5.98×10^{24} kg
Equatorial radius of Earth (R_E)	6.38×10^{6} m
Density of iron at low temperature (ρ_{Fe})	$7.88 \times 10^{3}\,\mathrm{kg/m^3}$
Classical electron radius (r_0)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
Atomic mass unit	1.7×10^{-27} kg
Specific heat of oxygen (c_V)	21.1 J/mole$\cdot$ K
Specific heat of oxygen (c_P)	29.4 J/mole$\cdot$ K

Integrals

$$\int_{-\infty}^{\infty} dx\, x^{2n}\, e^{-ax^2} = \frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{2^n\, a^n} \sqrt{\frac{\pi}{a}}$$

$$\int_{0}^{\infty} \frac{dx}{x}\, x^n\, e^{-x} = \Gamma(n)$$

Problem 1

Consider the one-dimensional harmonic oscillator with mass m and spring constant k. The first few normalized energy eigenfunctions are u_n with

$$\begin{aligned} u_0(x) &= \frac{\alpha^{1/2}}{\pi^{1/4}}\, e^{-\alpha^2 x^2/2} \\ u_1(x) &= \frac{\alpha^{1/2}}{2^{1/2}\pi^{1/4}}\, 2\alpha x\, e^{-\alpha^2 x^2/2} \\ u_2(x) &= \frac{\alpha^{1/2}}{8^{1/2}\pi^{1/4}}\, (4\alpha^2 x^2 - 2)\, e^{-\alpha^2 x^2/2}, \end{aligned} \tag{1}$$

with

$$\alpha^4 = \frac{km}{\hbar^2} \tag{2}$$

and with energy eigenvalues

$$E_n = \left(\frac{k}{m}\right)^{1/2} \left(n + \frac{1}{2}\right) \hbar. \tag{3}$$

a) Show that u_0 satisfies the Schrödinger equation.

b) Evaluate $\langle u_1|x|u_1\rangle$ and $\langle u_1|x^2|u_1\rangle$.

c) Given the results $\langle u_1|p|u_1\rangle = 0$ and $\langle u_1|p^2|u_1\rangle = (3/2)\hbar^2\alpha^2$, evaluate the uncertainty product $\Delta x\, \Delta p$ for the state u_1.

Problem 2

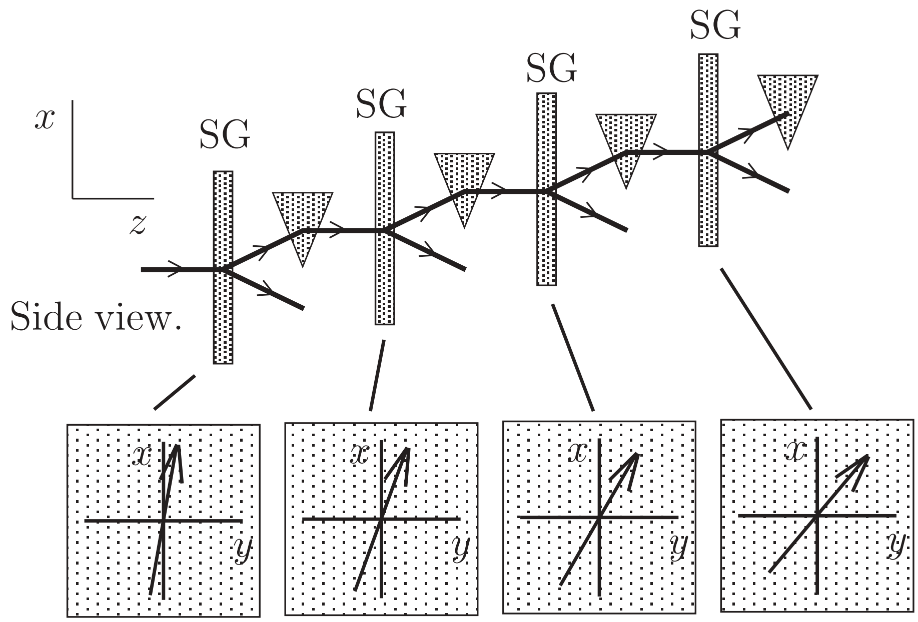
A hydrogen atom is subject to a uniform static electric field in the z-direction, $\vec{E} = E\hat{z}$.

a) Write the appropriate perturbation hamiltonian operator for the atom, assuming that the field is weak enough so that you can neglect quadratic terms.

b) What is the principle quantum number $n = n_0$ of the lowest energy level that is perturbed, and what is the degeneracy of this level? (Spin is irrelevant for this problem, so treat the electron as if it had spin 0.)

c) For this lowest perturbed energy level n_0, write the perturbation hamiltonian in matrix form using the $\{n, l, m\}$ basis. Although you should indicate all vanishing matrix elements, you are not required to compute any integrals explicitly.

d) Into how many levels does the perturbed energy level split? Sketch the perturbed spectrum and relate the magnitude of the observed splitting to the perturbation hamiltonian (without computing any integrals).



End view.
Arrow indicates direction of magnetic field gradient.

Problem 3

A spin 1/2 particle initially has spin in the $+\hat{x}$ direction and is traveling in the $+\hat{z}$ direction when it passes through a series of N Stern-Gerlach (SG) apparatuses. The jth SG apparatus creates a magnetic field with gradient making an angle θ_j with the x-axis, splitting the particle trajectory into components containing particles with spin-parallel to the magnetic field gradient and with spin-antiparallel to the magnetic field gradient. Following each SG apparatus is an apparatus that deflects the trajectory of the spin-parallel component back along the $+\hat{z}$ direction. (Note that in the "side view" drawing the deflection is drawn for the case that the θ_j are small. For larger values of the θ_j, the deflection can be sideways or downward instead of upward as drawn.)

a) Find expressions for the probabilities that the particle remains in the spin-parallel trajectory after one, two, and N such SG apparatuses.

b) Now assume that the SG angles are given by $\theta_j = j\,\pi/N$ for $j = 1, 2, \ldots, N$. In the limit $n \to \infty$, what is the probability that the particle will travel through all of the SG apparatuses in the spin-parallel trajectory (and so emerge with spin pointing in the $-\hat{x}$ direction)?

Problem 4

A certain solid contains N mutually non-interacting nuclei of spin 1. Each nucleus can therefore be in any of three quantum states labeled by the quantum number m, where $m = 0, +1, -1$. Because of electric field gradients within the solid that interact with the electric quadrupole moment of these nuclei, a nucleus in the sate $m = +1$ or $m = -1$ has the same energy, $\epsilon > 0$, while its energy in the state $m = 0$ is zero.

a) Write down the partition function for the nuclei in this solid.

b) Find an expression for the entropy of the nuclei in this solid.

c) Give physical explanations for the values the entropy has in the limit of very high or very low temperatures. Explain why (or why not) either of these values would be altered if ϵ were negative instead of positive.

Problem 5

The differential work required to magnetize a substance is $\delta W = H\delta M$, where H is the applied magnetic field and M is the magnetization.

a) Derive expressions for $(\partial S/\partial H)_{T,P}$ and $(\partial V/\partial H)_{T,P}$ that could be evaluated if $M(T,P,H)$ were known. Hint: consider the quantity $X = U - TS - HM + PV$.

b) What inference can be drawn from the third law of thermodynamics about $(\partial M/\partial T)_{H,P}$?

c) For an ideal paramagnet at low field and not too high or too low temperature, the magnetization is observed to follow the law $M = \gamma H/T$ where the constant γ is called the Curie constant. If this relation held for all H less than some fixed field strength and all T less than some fixed temperature, would it be consistent with the third law of thermodynamics? Why or why not?

Problem 6

The Stefan-Boltzmann law gives the energy per unit time and per unit area emitted by a surface at a given temperature T when the radiation is in thermal equilibrium with the surface in question. This law can be derived by evaluating the photon occupation numbers for the allowed states of an electromagnetic field in equilibrium with the surface. Instead of using the procedure described above, derive the Stefan-Boltzmann law, up to an overall multiplicative constant, from the assumption that the power radiated per unit surface area depends on the temperature of the surface but not on any property of the material of which the surface is made. Hint: try dimensional analysis.