Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART III

Modern and Applied Physics

Friday, September 22, 2000, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers. **Answer any six of the eight questions.** If you turn in complete or partial solutions to more than six questions, then **only the first six solutions will be graded.**

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed**. Dictonaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, remain in your seat and a proctor will collect your exam.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60219×10^{-19} C
Electron volt (eV)	1.60219×10^{-19} J
Electron rest mass (m_e)	$0.51100\,\mathrm{MeV/c^2}$
Proton rest mass (m_p)	$938\,\mathrm{MeV/c^2}$
Neutron rest mass (m_n)	$940\,\mathrm{MeV/c^2}$
W^+ rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	$6.6262 \times 10^{-34}\,\mathrm{J\cdot s}$
Reduced Planck's constant times c ($\hbar c$)	$1.97 \times 10^{-4}\,\mathrm{MeV\cdot nm}$
Speed of light in vacuum (c)	$2.997925 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.3807 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.672 \times 10^{-11}\mathrm{N\cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.854 \times 10^{-12}\,\mathrm{F/m}$
Mass of Earth (M_E)	$5.98 \times 10^{24}\,\mathrm{kg}$
Equatorial radius of Earth (R_E)	$6.378 \times 10^{6}\,\mathrm{m}$

Problem 1

The highest energy cosmic rays are thought to be protons. In principle, a cosmic ray proton can strike a proton in a hydrogen atom in the upper atmosphere and make a W boson in the process $p + p \to p + n + W^+$.

a. What is the minimum energy for the cosmic ray proton in order for this process to be allowed?
b. If the cosmic ray proton has just this minimum energy, what are the energies of the proton p, the neutron n, and the W boson in the final state?

Problem 2

Photons from a helium-neon laser ($\lambda = 632.82\,\text{nm}$) collide head on with incident electrons of energy $E_1 = 100\,\text{MeV}$. Some of the photons are scattered back in the direction from which they came. What is the wavelength of the back-scattered light?

Problem 3

An x-ray beam with a continuous spectrum is diffracted from the (100) planes of a sodium chloride crystal with a lattice plane spacing of 5.64 Å. The beam is incident at an angle of $\theta_i = 30°$ from the (100) planes—i.e., 60° from the normal to the planes.

a. At what angle θ_f to the (100) planes is the beam reflected?
b. What wavelengths are diffracted?
c. Due to elastic reflection and also inelastic absorption, the x-ray beam travels only 100μm into the crystal. Estimate the energy width, $\delta E/E$, of the reflected x-ray beam.

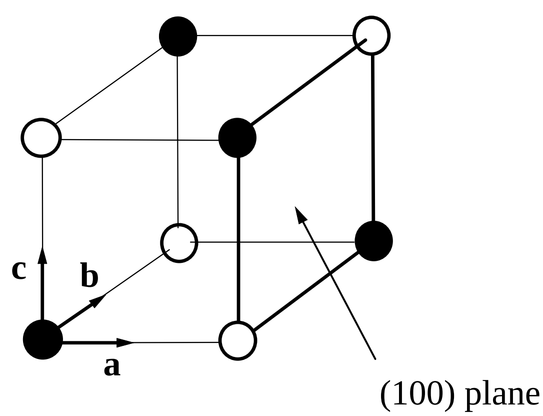

Figure 1: **Figure for Problem 3**

Problem 4

In the interferometer shown below, a narrow, single frequency laser beam, with wavelength λ and power P_0, is split into two beams and recombined after the beams reflect from mirrors. The goal is to measure a very small displacement, x, of one of the mirrors. The mean power P_1 striking the detector is given by $P_1 = P_0 \cos^2(2\pi x/\lambda)$. The fundamental noise limiting the measurement precision arises from the fact that the light beam presents to the detector a random stream of photons. The number, n, of photons detected in time T is distributed according to the Poisson distribution $p(n) = \bar{n}^n \exp(-\bar{n})/n!$, where $\bar{n}$ is the mean value of n. Assume $P_0 = 1\,\mathrm{W}$, $\lambda = 1.06\mu\mathrm{m}$ ($\hbar\omega = 1.88 \times 10^{-19}\,\mathrm{J}$), and $T = 10\,\mathrm{s}$. Find how the error in the x-measurement varies with x. Find the value(s) of x at which the error is minimum and evaluate numerically this minimum error.

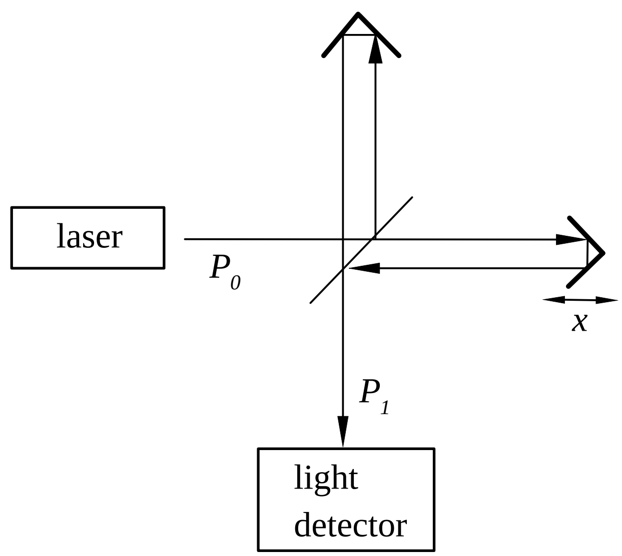

Figure 2: **Figure for Problem 4**

Problem 5

The B meson (mass $= 5.3\,\mathrm{GeV/c^2}$) has a lifetime of about $1.5\,\mathrm{ps}$.

a. Estimate the *theoretical* limit on the mass resolution for reconstructing this particle following a decay—i.e, what is its total width?
b. Calculate the mean distance a B meson with $30\,\mathrm{GeV}$ energy will travel before it decays.
c. The rate for the B meson to decay to $J/\Psi\, K_s$ is different from the rate for the $\bar{B}$ to decay to $J/\Psi\, K_s$ (J/Ψ and K_s are the names of two distinct particles),

$$\Gamma(B \to J/\Psi\, K_s) \neq \Gamma(\bar{B} \to J/\Psi\, K_s).$$

On the other hand, the CPT Theorem requires that the lifetimes of particles and anti-particles be equal. Explain how this can be consistent with the above relationship.

Problem 6

A slab of dielectric with sides of length a, b, c is doped in such a way that it effectively has only one type (i.e. sign) of charge carrier. A voltage V_x is applied as shown, and it is observed that when a magnetic field $\vec{H} = (0, 0, H_z)$ is applied, a voltage V_y appears as in the figure. If the magnetic field is removed, the resistance of the slab using the x-leads (i.e., parallel to the a dimension) is found to be R.

a. Show that the carrier density, n, is given by

$$n = -\frac{\mu_0}{Rce}\frac{V_x H_z}{V_y}.$$

(Assume the semiconductor has the permeability of the vacuum.)

b. A flip-coil fluxmeter consisting of a coil of radius r is used to measure the magnetic field. The value of the flip-coil measurement is effectively determined by the area of the coil. Estimate the precision with which the carrier density can be measured if all of the following hold:
(i) the radius of the coil is known to 2%,
(ii) the voltage measurements have an error of 1%,
(iii) the dimensions of the slab are known to 2%,
(iv) the resistance is measured to 1%.
(Assume μ_0 and e are known to great precision.)

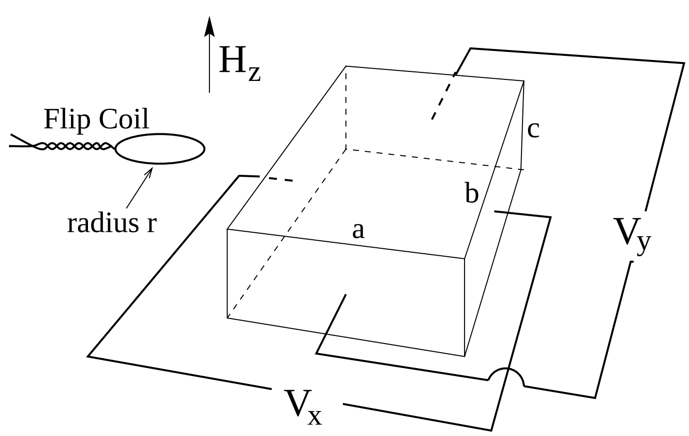

Figure 3: **Figure for Problem 6**

Problem 7

The plot below (see over) shows some radio telescope data. The radio amplitudes are measured in 32 separate frequency channels from 1383 MHz to 1423 MHz, each as a function of the signal arrival time, as shown. The series of large amplitudes indicated by the arrows is due to a pulsar. Dispersion due to free electrons in the interstellar medium causes the pulsar signal arrival times to vary with frequency. We wish to determine the dispersion relation and use it to estimate the distance to the pulsar, assuming the electron density is known.

a. Consider an element of dilute interstellar free electron gas subject to pulsar electromagnetic waves. Let z be the direction of propagation. In the x direction, the electric field at frequency ω is $E_x = E_0 e^{i(kz-\omega t)}$, where E_0 is a constant. The electric polarization of this element takes the form $P_x = -nex(t)$, where e is the magnitude of the electron charge, n is the electron gas density, and $x(t) = x_0 e^{-i\omega t}$ describes the motion of one electron in the electric field. Use this information, plus the wave equation

$$\nabla^2 E_x = \frac{\epsilon}{\epsilon_0 c^2}\frac{\partial^2 E_x}{\partial t^2},$$

to determine the dispersion relation

$$\omega^2 = k^2 c^2 + \omega_p^2,$$

where $\omega_p = (ne^2/\epsilon_0 m_e)^{1/2}$ is the plasma frequency and m_e is the electron mass.

b. Show that the difference in signal propagation velocities at two frequencies ω_1 and ω_2 is

$$v_2 - v_1 \approx \frac{\omega_p^2 c \Delta\omega}{\bar{\omega}^3},$$

where $\Delta\omega \equiv \omega_2 - \omega_1$ and $\bar{\omega} \equiv (\omega_1 + \omega_2)/2$. Use the approximations $\Delta\omega \ll \bar{\omega}$ and $\omega_p \ll \bar{\omega}$.

c. Use the result of b. to find an expression which relates the distance to the pulsar, D, to the propagation delay Δt between signals of different frequency. If the average density of the electron gas is $n = 3 \times 10^4\,\mathrm{m}^{-3}$, estimate the distance D.

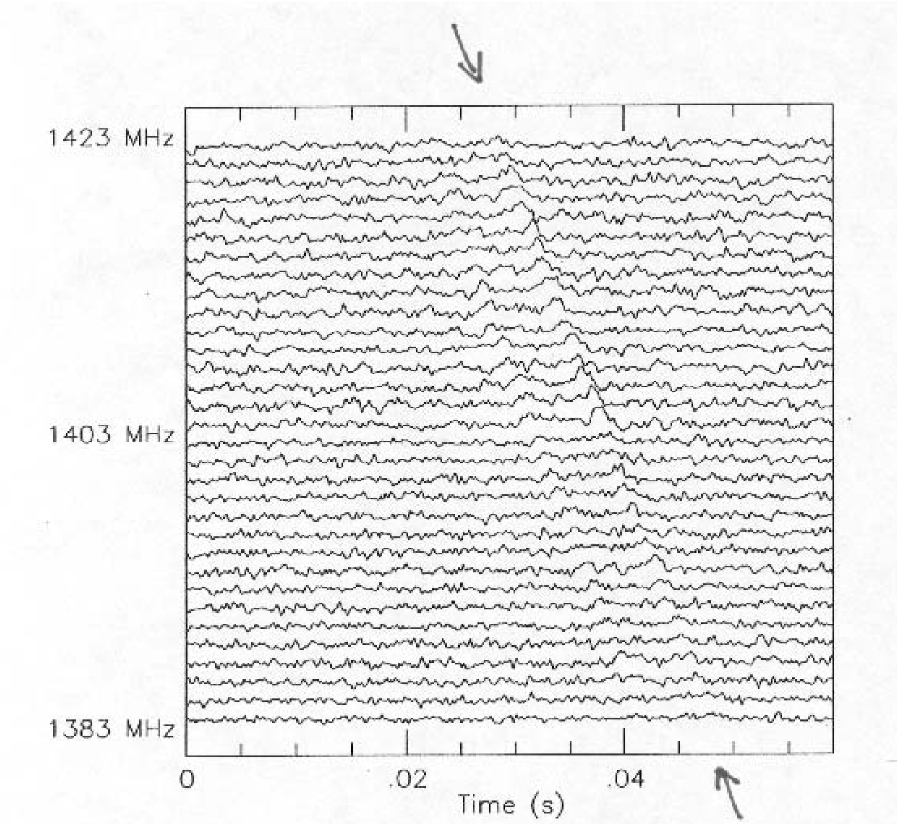

Figure 4: **Figure for Problem 7**

Problem 8

Suppose you study the light from a distant star that is moving away from Earth with a velocity v, not necessarily small compared to the speed of light c. As viewed by a hypothetical observer on a planet at rest with respect to this star, the starlight is approximately black body radiation with a temperature T_0. (There are various emission and absorption lines superimposed on the black-body spectrum, which we ignore.) An observer on Earth observes the number of photons per unit time, per unit area, in frequency interval $d\omega$, in the light coming from the star to be $n(\omega)d\omega$. Determine the function $n(\omega)$ up to an overall normalization—i.e., what is the *shape* of the function $n(\omega)$? Does it still have the shape of a black-body spectrum? If so, how is the effective temperature related to T_0?