Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART II

Quantum Mechanics and Statistical Mechanics

Thursday, September 21, 2000, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed**. Dictonaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, remain in your seat and a proctor will collect your exam.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60219×10^{-19} C
Electron volt (eV)	1.60219×10^{-19} J
Electron rest mass (m_e)	$0.51100\,\mathrm{MeV/c^2}$
Proton rest mass (m_p)	$938\,\mathrm{MeV/c^2}$
Neutron rest mass (m_n)	$940\,\mathrm{MeV/c^2}$
W^+ rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	$6.6262 \times 10^{-34}\,\mathrm{J \cdot s}$
Reduced Planck's constant times c ($\hbar c$)	$1.97 \times 10^{-4}\,\mathrm{MeV \cdot nm}$
Speed of light in vacuum (c)	$2.997925 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.3807 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.672 \times 10^{-11}\mathrm{N \cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.854 \times 10^{-12}\,\mathrm{F/m}$
Mass of Earth (M_E)	$5.98 \times 10^{24}\,\mathrm{kg}$
Equatorial radius of Earth (R_E)	$6.378 \times 10^{6}\,\mathrm{m}$

Problem 1

Consider a spinless electron in a magnetic field described by the vector potential

$$A_x = -By, \qquad A_y = A_z = 0,$$

corresponding to a uniform field along the z-axis.

a. Write down the Hamiltonian of the system.
b. Show that p_x and p_z are good quantum numbers, and find the part of the wavefunction that depends on p_x and p_z.
c. Find the eigenenergies of the system.

Problem 2

Consider a system of two spin 1/2 particles. Suppose that the Hamiltonian of this spin system is given by

$$H = A + \frac{B\boldsymbol{S}_1 \cdot \boldsymbol{S}_2}{\hbar^2} + \frac{C(S_{1z} + S_{2z})}{\hbar}.$$

Find the eigenfunctions and eigenvalues of this system under this Hamiltonian.

Problem 3

Positronium is a short-lived "atomic" system consisting of an electron bound electrically to a positron. This atom is described in a first approximation by an equation of the same form as the equation for the hydrogen atom, except that the reduced mass is $\mu = m_e/2 = 0.255\text{MeV}$.

a. Calculate the ionization energy of positronium.
b. The $n = 2$ state splits into six states which can be labelled by their orbital angular momentum (L), total spin (S), and total angular momentum (J)—for example, as in $n\,^{2S+1}L_J = 2\,^3S_1$. What are the labels $n\,^{2S+1}L_J$ for each of the six states?
c. Consider the transitions from each of the six states of b. to the ground state, $1\,^1S_0$, by electric dipole radiation. Which transitions are disallowed and why?

Problem 4

Consider a classical ideal gas of molecules that have an electric dipole moment $\vec{\mu}$. Let there be N such molecules in a volume V in a uniform electric field $\vec{E}$. Let $|\vec{\mu}| = \mu$, $|\vec{E}| = E$, and let the temperature of the gas be T.

a. What is the potential energy of a molecule whose dipole moment forms an angle θ with $\vec{E}$?
b. What is the probability that the direction of $\vec{\mu}$ for a particular molecule lies within a solid angle $d\Omega$ that makes an angle θ with $\vec{E}$.
c. Find the average electrical polarization P of the gas—i.e. the average dipole moment per unit volume. Express your answer in terms of N, V, μ, E, and T.

Problem 5

In a MOSFET, the electronic density of states, $D(E)$, as a function of the energy E, can be approximated by the step function

$$D(E) = D\Theta(E),$$

with D a constant. (The zero of energy is at the bottom of the band.)

a. Find the Fermi energy of the system as a function of the particle number N.
b. *Derive* a condition for the temperature T such that the electrons are *non-degenerate*; i.e., obey classical Boltzmann statistics. (No credit will be given for simply quoting a condition.)
c. Show that in the *degenerate* limit (i.e., the opposite limit to the one you just considered) the chemical potential is temperature independent, except for terms that vanish as $T \to 0$ like $\exp(-\text{const.}/T)$.
HINTS: 1) Integrate by parts. 2) $\int_{-\infty}^{\infty} dx \cosh^{-2} x = 2$.

Problem 6

A neutrino gas of N particles in volume V is, to a very good approximation, a quantum gas that follows the relationship $U = 3PV$ between its pressure P, its volume V, and its internal energy U.

a. Consider a reversible adiabatic expansion from an initial state P_i, V_i to a final state P_f, V_f, with $V_f = 10V_i$. What is the ratio of final to initial pressure, P_f/P_i?

b. Show that the ratio of the specific heat per unit volume at constant pressure to that at constant volume, c_P/c_V, is given by

$$\frac{c_P}{c_V} = 1 + \frac{T\alpha}{3}$$

where $\alpha = \frac{1}{V}\left.\frac{\partial V}{\partial T}\right|_P$ is the coefficient of thermal expansion.
HINT: Consider the total differential of the Helmholtz free energy $F = U - TS$.