Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART I

Mechanics and Electromagnetism

Wednesday, September 20, 2000, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed**. Dictonaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, remain in your seat and a proctor will collect your exam.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60219×10^{-19} C
Electron volt (eV)	1.60219×10^{-19} J
Electron rest mass (m_e)	$0.51100\,\mathrm{MeV/c^2}$
Proton rest mass (m_p)	$938\,\mathrm{MeV/c^2}$
Neutron rest mass (m_n)	$940\,\mathrm{MeV/c^2}$
W^+ rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	$6.6262 \times 10^{-34}\,\mathrm{J \cdot s}$
Reduced Planck's constant times c ($\hbar c$)	$1.97 \times 10^{-4}\,\mathrm{MeV \cdot nm}$
Speed of light in vacuum (c)	$2.997925 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.3807 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.672 \times 10^{-11}\mathrm{N \cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.854 \times 10^{-12}\,\mathrm{F/m}$
Mass of Earth (M_E)	$5.98 \times 10^{24}\,\mathrm{kg}$
Equatorial radius of Earth (R_E)	$6.378 \times 10^{6}\,\mathrm{m}$

Problem 1

A simple harmonic oscillator with spring constant k and mass m is damped with a force $-bv$, where v is the velocity of the mass and b is a constant. The mass is also driven by a harmonic force $F(t) = F_0 \cos \omega t$.

a. Find the angular frequency, ω, at which the amplitude of the displacement of the mass is a maximum.
b. Find the angular frequency at which the power needed to drive the mass is a maximum.

Problem 2

A homogeneous disk of radius R and mass M rolls without slipping on a horizontal surface and is attached by a massless, ideal spring of spring constant k to a point which lies at a distance d below the plane of the surface.

a. Show that the extension of the spring for a rotation of the disk through an angle θ from its equilibrium position is

$$D = \sqrt{(d+R)^2 + R^2\theta^2}.$$

b. Find the frequency of oscillation about the equilibrium position.

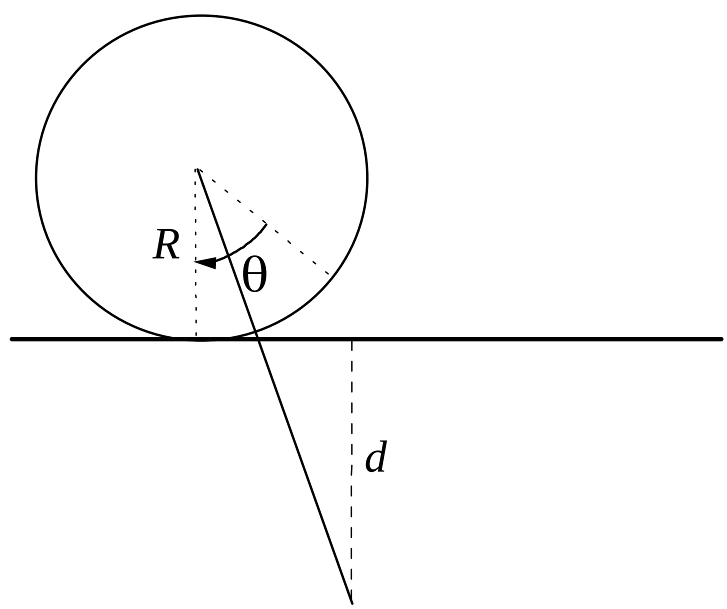

Figure 1: **Figure for Problem 2**

Problem 3

A particle of mass m interacts with a fixed point via a central force $\boldsymbol{F}(\boldsymbol{r}) = F(r)\hat{r}$.

a. Write the Lagrangian for the system in polar coordinates.
b. What are the constants of motion?
c. Let $F(r) = -kr^{-n}$ where k and n are positive constants. Derive an expression for the *effective radial* potential. Derive the condition under which a stable circular orbit can exist.

Problem 4

Two metal objects of arbitrary shape are embedded in weakly conducting material of uniform conductivity σ.

a. Derive a relationship between the resistance, R, between the objects and the capacitance, C.

b. The two objects are charged to a potential difference V_0. If the battery is then disconnected, derive an expression for the potential difference as a function of time in terms of σ and ϵ_0.

Problem 5

Consider a wave guide consisting of two large parallel metal mirrors with separation d. For simplicity, assume monochromatic electromagnetic waves which propagate along the z-direction inside the wave guide. The electric field is along the x-direction—i.e., we consider Transverse Electric (TE) modes.

a. Starting with the wave equation for monochromatic waves, derive the Helmholtz equation. What are the boundary conditions for the electric field of the electromagnetic waves propagating inside the wave guide?
b. Determine the spatial profile of the electric field of the eigenmodes in the wave guide.
c. For electromagnetic waves of frequency ω, determine the total number of modes that can be supported by the wave guide.

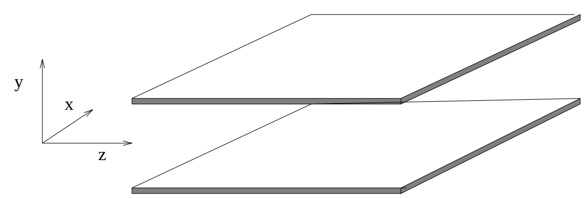

Figure 2: **Figure for Problem 5**

Problem 6

Suppose magnetic monopoles with charge q_M exist. By analogy with the Coulomb force law, the magnetic charges carry B-fields given by

$$\boldsymbol{B} = \frac{\mu_o}{4\pi}\frac{q_M \hat{r}}{r^2}.$$

a. Write down appropriate forms for Maxwell's equations which include such magnetic monopoles. Make sure that both magnetic and electric charges are conserved.

b. A superconducting loop of radius a is placed in the xy-plane. The z-axis lies along the axis of the loop. A magnetic monopole traveling with constant velocity, $\boldsymbol{v} = v_0\hat{z}$, along the z-axis passes through the loop at time $t = 0$. The monopole approaches from $z = -\infty$. Find the current $I(t)$ induced in the superconducting loop. Plot $I(t)$.

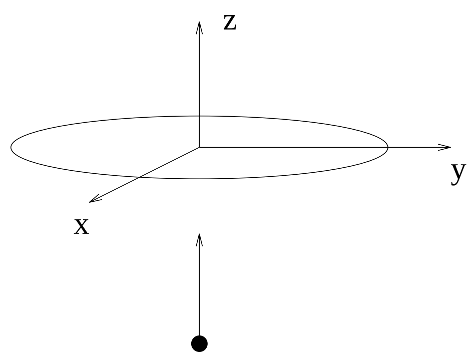

Figure 3: **Figure for Problem 6**