A Formalization of Ricardo's *Essay on Profits*
Along Sraffian Lines

by

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I. Introduction.

It is usually considered that the great breakthrough in Ricardo scholarship of modern times is contained in Piero Sraffa's introduction to the first volume of his edition of Ricardo's works. It is there that Sraffa suggests that the "rational foundation" of Ricardo's dictum that "the profits of the farmer ... regulate the profits of all other trades" is the assumption that the inputs and outputs of agricultural production are roughly homogeneous and hence may be conceived, for the sake of analysis, as a single commodity. In Sraffa's often quoted words:

in agriculture the same commodity, namely corn, forms both the capital (conceived as composed of the subsistence necessary for workers) and the product; so that the determination of profit by the difference between total product and capital advanced, and also the determination of the ratio of this profit to the capital, is done directly between quantities of corn without any question of valuation.... It follows that if there is to be a uniform rate of profit in all trades it is the exchangeable values of the products of other trades relatively to their own capitals (i.e. relatively to corn) that must be adjusted so as to yield the same rate of profit as has been established in the growing of corn. [Sraffa (1951-73), vol. I, p. xxxi]

The 'corn model' that Sraffa thus attributes to Ricardo has been, in the intervening years, often discussed, but seldom formally laid out (it
seems rather obvious) and never applied, in extenso, to the argument of Ricardo’s *Essay on Profits*. Recently, the aptness of the model to Ricardo’s scheme of thought has been called into question by Samuel Hollander [1975, 1979] and Terry Peach [1984], each for a somewhat different set of reasons. To Sraffa’s defense have come several authors including John Eatwell [1975], Pierangelo Garegnani [1982], Giancarlo de Vivo [1985], and the present writer [1982]. The task of defending Sraffa’s interpretation is made difficult by the absence of any explicit corn model in Ricardo’s extant papers and correspondence. As antidote to this difficulty I have elsewhere [Langer (1982)] noted (i) that an explicit corn model is present in an 1819 *Edinburgh Review* article written by Ricardo’s then doctrinal ally, Robert Torrens, (ii) that Ricardo was aware of the article, and (iii) that he praised it highly. de Vivo [1985] has recently presented evidence that Torrens formulated that article (and others like it) inspired by and acknowledging the influence of Ricardo’s *Essay on Profits*. Garegnani [1982], in responding to Sraffa’s critics, has posed a closely related question: if the ‘corn model’ is not the "rational foundation" of the analysis found in the *Essay on Profits* then from whence does Ricardo derive his conclusions?

The purpose of this paper is to show that the corn model is the "rational foundation" of not merely the dictum that the "profits of the farmer regulate the profits of all other trades" but of all of the substantive theoretical propositions contained in the *Essay on Profits*. This purpose is pursued by formalizing Sraffa’s simplest corn model and addressing it directly to the questions addressed by the *Essay on Profits*. The model that is elaborated below shows that no eccentric theoretical
twists are required to deduce Ricardo's most fundamental conclusions regarding the determination of profit, the effects of technological change, the determination of rent, and the effects of accumulation and population growth from the corn model.

The paper also situates the Essay and its arguments within the historical context within which they were formulated. Not only is this intended to help the reader understand Ricardo's appeal to his contemporaries, but also to persuade the reader that Ricardo's theoretical intentions were much broader than merely rationalizing the contention that the "profits of the farmer ... regulate the profits of all other trades", or that the "interests of the landlord are opposed to the interest of every other class". Instead, it will be made apparent that his intentions were to provide a comprehensive account of the effects of capital accumulation and population growth during the Napoleonic war years (1794-1815) and to use that account as a basis for directing post-war policy. For such a purpose the sweeping generalizations of a corn model are far more apt than the intricate and ad hoc lines of reasoning attributed to Ricardo by Sraffa's critics.

It is also hoped that the model that is presented below will be of some incidental interest to theorists, for it is able to demonstrate the usual Ricardian conclusions without having to make reference to the marginal product of capital applied to land or to any curves purporting to show such a relationship. This is accomplished without any sacrifice of clarity or analytical rigor.
II. Historical Background to the *Essay on Profits*.

In the decade following Napoleon's defeat in 1815, there was no matter of national economic policy in Britain that aroused more attention, controversy, and hostility than that surrounding the regulation of the 'corn trade'. Ricardo's *Essay on Profits*, published on 24 February 1815, was intended as a contribution to the debates on the 'Corn Law' expected to take place in the House of Commons in March.

The political struggle surrounding the question of whether and how the corn trade ought to be regulated was generally perceived as a contest of power between the landed classes, who controlled the Parliament, and the middle and working classes. The war, lasting from roughly 1793 to 1815, had enhanced the fortunes of both landlords and farmers, and had sparked a boom in land improvements such as clearing, draining, manuring, and irrigating. Because European supplies of 'corn' (a generic term used to refer to grains in general—most particularly wheat, barley, and oats) were either cut-off entirely or unreliable throughout most of the war, and because population growth was especially rapid, an historically unprecedented stimulus was given to extending domestic production. It was a source of both economic and patriotic pride to the landlords that they had successfully responded to the wartime emergency and brought hundreds of thousands of acres of previously unproductive land into tillage. As the cessation of hostilities between England and the continent approached, it was widely believed that if English ports were reopened to imported grains, corn prices would plummet and the vaunted wartime land improvements would be bankrupted. At the impetus of landlords, Parliamentary committees were formed in 1813 and 1814 to search for remedial measures. In 1815, a bill
was pushed through Parliament which virtually prohibited the importation of foreign grains. In the meantime, the middle and working classes organized, petitioned, rallied, and rioted. The battles over the Corn Law in 1815 and those continuing in subsequent years polarized English society and symbolized the struggle of the following several decades for both economic and political power between the landed and the middle classes.

Ricardo's *Essay on Profits* was as much polemic as it was scientific. It was by no means the sort of arcane scientific piece than so many now, mistakenly, attribute to the Classical economists. Its objective was to argue upon the grounds of the 'principles of political economy' for "leaving the importation of corn unrestricted by law". Its most infamous suggestion was "that the interest of the landlord is always opposed to the interest of every other class in the community". [Ricardo, p.9 and p. 21]

The *Essay on Profits* followed closely on the heels of pamphlets by Malthus, Torrens, and Edward West which detailed a theory which has since been called the "Ricardian theory of rent". Although much the same theory is contained in the *Essay*, Ricardo claimed no originality for it. His contribution was to integrate that theory along with the Malthusian theory of wages and his own already formulated views on profits into a coherent and complete explanation of accumulation and the distribution of income. Needless to say, its simplicity and elegance, as will be seen below, have rendered it one of the most admired creations of modern economics.

III. The Model.

Ricardo begins the *Essay* by asking the reader to consider "the first settling of a country rich in fertile land, and which may be had by anyone who chooses to take it". [p. 10] He follows with a discussion of the
factors which, in such circumstances, will regulate profits and the rate of profit.

We propose to formalize his argument, using the method of presentation of Sraffa's *Production of Commodities by Means of Commodities* [1960], in the following way:

Consider an economy in which corn is produced by means of labor (productive activity) and itself. We thus incorporate Sraffa's interpretation at the outset. Assume, furthermore, that production takes exactly one year, that there are constant returns to scale, and that land of a uniform quality exists in abundance. The annual production of such a society may be tabulated in the following way:

\[
\text{corn + labor} \rightarrow \text{corn},
\]

where the arrow means "produces in one year" and the units in which labor is measured are "man-years". The 'quantity relations' between corn and labor as inputs and corn as output may be expressed more explicitly as

\[
\frac{A_A}{A} + \frac{1}{A} \rightarrow A,
\]

where \(A_A\) is the quantity of corn used to produce \(A\) units of corn and \(\frac{1}{A}\) is the quantity of direct labor necessary to produce \(A\) units of corn. It is convenient to redefine the units in which corn is measured such that the gross output of corn, \(A\), becomes the unit of measurement. Since constant returns to scale are being assumed nothing is is lost by so doing, and the quantity relations may be rewritten as the simpler

\[
a + 1 \rightarrow 1 \text{ unit of corn}.
\]

Clearly, \(a = \frac{A_A}{A}\), the quantity of corn necessary to produce a unit of corn, and 1 is the quantity of labor required to produce a unit of corn.
The convenience of this respecification is that it makes it easier to discuss outputs of corn other than A and the concomittant input requirements which, accordingly, will be other than a and l. Thus, if x units of corn are being produced, xa units of corn and x l units of labor are needed to produce it. Thus also, the net output of the system will be \(x - xa\). The matrix equivalent of this specification is now very conventional in most presentations of multiproduct linear models.

Finally, assume that the laborers' wages are paid entirely in corn and let \(w\) denote the corn wage per unit of labor. The total wage bill (in corn) thus becomes \(xlw\) and total profits (also in corn) become \(x - xa - xlw\). Since the capital advanced by capitalists consists of the corn that they advance as seed, \(xa\), and the corn that they advance as wages, \(xlw\), the rate of profit, which will be denoted \(R\), may be written

\[
R = \frac{(x - xa - xlw)}{(xa + xlw)}
= \frac{(1 - a - lw)}{(a + lw)}
\]  

(1)

\(R\) is clearly a pure number, since it is a ratio of physically homogeneous stuff (corn), and independent of valuation. The latter observation implies that even if other commodities were being produced in addition to corn, if competition enforces uniformity in the rate of profit in all branches of production, then that uniform rate of profit must be the one prevailing in the production of corn. Hence we may make sense of Ricardo's proposition in the *Essay on Profits* that "when the profits on agricultural stock, by supposition, are fifty per cent, the profits on all other capital, employed either in ... rude manufactures ... or in foreign commerce ... will be also, fifty per cent." [p. 12] It is upon this proposition that the 'Ricardo controversy' has focused. But as we shall see in what follows,
the corn model is the 'rational foundation' of much more. Indeed, it is the basis of the entirety of Ricardo's theoretical discussion in the Essay on Profit.

He begins his discussion by noting that in this setting, or, to use Ricardo's preferred expression, in this "state of society", profits might change owing to two changes in circumstances. "Profits might increase," he writes, "because the population increasing at a more rapid rate than capital, wages might fall," and "profits might also increase, because improvements might take place in agriculture, or in the implements of husbandry, which would augment the produce with the same cost of production." [p. 11] Simple inspection of Equation 1 reveals that the rate of profit is a decreasing function of \( a, l, \) and \( w. \) Hence, a decrease in wages, \( w, \) will increase \( R, \) as would any decrease in \( a \) or in \( l \) that might result from an 'improvement' in agricultural technique. If it were supposed that a part of the labor employed to produce corn is engaged in fashioning and maintaining the 'implements of husbandry', such as plows, hoes, shovels, etc., an improvement in these implements "which would augment the produce with the same cost of production" would imply a decrease in \( a, \) which would increase profits and the rate of profit.

Ricardo then takes up the much more important circumstances which would arise "after all the fertile land in the immediate neighborhood of the first settlers were cultivated". In such a case, if population increased and more food were required, than it might only be obtainable from land which was "less fertile" or "not so advantageously situated". [p. 13] The annual production of such a 'state of society' would be tabulated as
corn + labor + first quality land $\rightarrow$ corn

corn + labor + second quality land $\rightarrow$ corn.

Explicitly, the quantity relations might be written

\[ a_1 + l_1 + S_1 \rightarrow x_1 \]
\[ a_2 + l_2 + S_2 \rightarrow x_2. \]

Here, the symbols are given the following meaning: \( a_i \) is the quantity of corn used to produce \( x_i \) units of corn on land of the \( i \)th quality, \( S_i \), and \( l_i \) is the quantity of labor required to produce it.

The meaning to be given to the statement that land of the second quality is "less fertile" or "less advantageously situated" than land of the first quality is therefore that \( (x_2/(a_2 + l_2 W)) \) is less than \( (x_1/(a_1 + l_1 W)) \), the terms of which are the output/capital ratios of capital employed on the two lands.

It may be noted, in passing, that unless the proportions of labor to means of production on the different lands under cultivation are equal, \( i.e. \) unless \( (l_1/a_1) = (l_2/a_2) \), it might not be possible to unambiguously rank the different soils in order of 'fertility' independently of a knowledge of the wage rate. Soils requiring a high proportion of labor to means of production for their cultivation will, relative to soils requiring lower proportions, appear increasingly more 'fertile' as the wage falls, and there may, at some point, occur a switch in the 'order of fertility'. Ricardo does not appear to have noticed this in the Essay on Profits, but in as much as he assumes either that "capital and labor advance in the proper proportion" or that "the real wages of labour continue uniformly the same", he either obviates the difficulty, in the latter case, or implicitly assumes equality in such proportions, in the former.
We now come to the most crucial part of the argument and will soon be confronted with Ricardo's most startling conclusion.

Since competition among capitalist farmers may be assumed to equalize the rate of profit obtained on lands of every quality, a 'rent' will arise on the better land. This rent will be equal to the difference between the gross output obtained on the better land and the capital advanced on such land plus the profit thus obtained at the uniform rate. If there is unutilized land of the worst quality, competition for tenants amongst the owners of such land may be expected to reduce the rent obtainable from it to zero, or the least necessary to recompense the landlord for the trouble of attending to the lease of his land. The general and uniform rate of profit will therefore be determined by the ratio of the corn profit to the expenses of production (in corn) on the worst land under cultivation. In Ricardo's words, "the general profits of stock" are "regulated by the profits made on the least profitable employment of capital in agriculture." [p. 13]

Formally, the 'competitive requirements' of uniformity of the rate of profit and zero rent on the worst land in cultivation may be written

\[(a_1 + \alpha_1 \omega)(1 + R) + N_1 = x_1\]  
\[(a_2 + \alpha_2 \omega)(1 + R) = x_2,\]  

where \(N_1\) is the rent of the land of the first quality. These expressions, familiar to those acquainted with the linear production models of Sraffa, Leontief, and Von Neuman, simply say that the gross receipts of capitalists (the right-hand sides of the equalities) must equal the expenses of production plus profit plus rent (the left-hand sides of the equalities).
Clearly, the rate of profit is determined by Equation 3, from which it can be seen that

\((1 + R) = x_2/(a_2 + \frac{1}{2}w)\),

which implies that

\[ R = \left[ x_2/(a_2 + \frac{1}{2}w) \right] - 1 \]
\[ = (x_2 - a_2 - \frac{1}{2}w)/(a_2 + \frac{1}{2}w). \]  \(\text{(4)}\)

Thus, the rate of profit is the ratio of the profit obtainable on the worst land, \(x_2 - a_2 - \frac{1}{2}w\), to the capital required for its production, \(a_2 + \frac{1}{2}w\).

It is also evident that the rent of the land of the first quality can be calculated from Equation 2 as

\[ N_1 = x_1 - (a_1 + \frac{1}{2}w)(1 + R) \]
\[ = x_1 - (a_1 + \frac{1}{2}w)[x_2/(a_2 + \frac{1}{2}w)]. \]  \(\text{(5)}\)

If it is now supposed that population continues to increase, and if it is necessary to bring still worse land into cultivation, we come to the most startling implication of the model. In Ricardo’s words, “by bringing successively land of a worse quality, or less favorably situated into cultivation, rent would rise on the land previously cultivated, and in precisely the same degree would profits fall; and if the smallness of profits do not check accumulation, there are hardly any limits to the rise of rent and the fall of profit”. [p. 14] Indeed, as the model will clearly show, with the increase of population and the extension of cultivation to progressively less fertile land, profits will eventually approach zero and rents will engross all of net revenue. This may be seen most simply by letting the capital advanced on land of the \(i^{th}\) quality, \(a_i + \frac{1}{2}w\), equal \(a_i^x\). Thus the output/capital ratio, \(x_i/a_i^x\), may be taken as an ‘index of
fertility'. Equations 4 and 5 may therefore be written more simply as

$$R = \left( \frac{x_k}{a_k^x} \right) - 1$$  \hspace{1cm} (4')

and

$$N_i = x_i - a_i^x \left( \frac{x_k}{a_k^x} \right).$$  \hspace{1cm} (5')

where the 'k^{th}' land in use is here taken as meaning the 'worst land' in use and $N_i$ is the rent of land of the $i^{th}$ quality.

The long run effects of accumulation and population growth alluded to by Ricardo can now be clearly seen by examining Equations 4' and 5'. As less and less fertile land is brought into cultivation, $x_k/a_k^x$, which is the 'index of fertility' of the 'worst land' in use, becomes successively smaller. Hence, $R$ will become successively smaller. If a point is reached where it is necessary to bring land into cultivation that yields no surplus, i.e., $x_k = a_k^x$,

then, from Equation 4', we can see that

$$R = \left( \frac{x_k}{a_k^x} \right) - 1 = (1/1) - 1 = 0.$$  

Simultaneously, we can see from Equation 5' that as $x_k/a_k^x$ becomes successively smaller, $N_i$, the rent of land of better quality, becomes successively larger, until, at last, when $x_k/a_k^x = 1$,

$$N_i = x_i - a_i^x \left( \frac{x_k}{a_k^x} \right) = x_i - a_i^x,$$

which is to say that the surplus from land of any quality, $x_i - a_i^x$, is paid entirely to landlords as rent. In one of the classic understatements in the history of economic thought, Ricardo remarks, "This is a view of the effects of accumulation which is exceedingly curious, and has, I believe, never before been noticed." [p. 16]

It should not be hard for the reader to see the connection of this argument to the Corn Law. Indeed, this was Ricardo's point and purpose.
In England, which was already a densely populated country, tillage had been carried to quite unproductive lands, especially during the war. As a result, rents had risen—hence the wartime prosperity in agriculture (indeed, during Parliamentary debates of 1813-15 it was claimed that rents had tripled). It follows that if imports of grain were prohibited and population continued to increase, it would be necessary to extend cultivation to even less fertile lands, leading to higher rents and lower profits. Assuming, as the English middle classes generally did, that profits are accumulated and rents are spent in extravagant living, capital would increase progressively more slowly relative to population; wages would consequently also fall. In the process, rents would be continuously increasing, and hence we are led to Ricardo's conclusion that "the interest of the landlord is always opposed to the interest of every other class in the community". On the other hand, landlords' fears of the consequences of free importation are also vindicated. For if grains may be imported at a lower price than they may be produced on the least fertile lands in use, capital will leave agriculture, profits will rise, and rents will fall.

The deductive severity of Ricardo's analysis has made it hard for some modern readers to understand his appeal to his contemporaries. But as the foregoing remarks have indicated, there was a near perfect correspondence between the events that transpired during the war (namely, the extension of cultivation to formerly barren lands, rising corn prices, and rising rents) and the course of events that is predicted by Ricardo's theory. If the corn model interpretation of Ricardo is correct, hardly ever in the history of economic thought has a theory been at once so comprehensive, elegant, and politically relevant.
IV. Conclusion.

In the preceding sections of this paper we have applied the corn model interpretation in *extenso* to the arguments contained in Ricardo's *Essay on Profits* and have seen that all of Ricardo's most substantive theoretical assertions are fully vindicated. A table that Ricardo presents in the text of the Essay may also be so vindicated. In the Appendix to this paper it is shown that there is a *perfect* correspondence between the headings and entries of that table and the terms in which the 'corn model' has been expressed above.

At best this paper may be regarded as an 'essay in persuasion'. Its object of persuasion has been the 'corn model' interpretation of Ricardo. It has not attempted to marshall all of the evidence that may be put in favor of Sraffa's interpretation but only a small, but neglected, portion of it; namely, the very easy confluence between, on the one hand, the leading themes of Ricardo's *Essay on Profit*, the table used by Ricardo to illustrate his arguments, and the historical context of the essay, and, on the other hand, a very formal and explicit Sraffian model of those arguments. As such it is hoped that the paper will complement and enhance the persuasiveness of the earlier papers of Eatwell, Garegnani, de Vivo, et al.
APPENDIX

Ricardo's Table from the Essay on Profits

**TABLE**, showing the Progress of Rent and Profit under an assumed Augmentation of Capital.

<table>
<thead>
<tr>
<th>Capital estimated in quarters of wheat</th>
<th>Profit per cent.</th>
<th>Rent produce in quarters of wheat after allowing the cost of production on each capital</th>
<th>Rent of 3rd portion of land in quarters of wheat</th>
<th>Profit of 2nd portion of land in quarters of wheat</th>
<th>Rent of 4th portion of land in quarters of wheat</th>
<th>Profit of 1st portion of land in quarters of wheat</th>
<th>Rent of 5th portion of land in quarters of wheat</th>
<th>Profit of 6th portion of land in quarters of wheat</th>
<th>Rent of 7th portion of land in quarters of wheat</th>
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Explanation: The first three columns of each row present what have been termed in this essay as the "quantity relations" between corn as input and corn as output on portions of land which are assumed to be successively brought into cultivation. The first column gives $a_1 + l_1w$, or, in simplified form, $a^*_1$. The third column is simply $x_1 - a^*_1$. The second column is $R$, i.e. the third column divided by the first. The remaining fifteen columns are to be considered in pairs. The first pair are, respectively, $Ra^*_1$ and $N_1$. The second pair are $Ra^*_2$ and $N_2$. The "1st" pair are $Ra^*_1$ and $N_1$. 
NOTES

1 Formal treatments of the 'corn model', such as that in Marglin [1984], apply it to questions other than those of Ricardo in the Essay on Profits.

2 A careful and scholarly account of post-war controversies over economic policy is contained in Hilton [1977], to which the reader is referred for a more detailed treatment of the politics of the Corn Law.

3 A technical discussion of problems involved in ordering lands according to 'fertility' is elegantly provided by Quadrio-Curzio [1980].
REFERENCES


