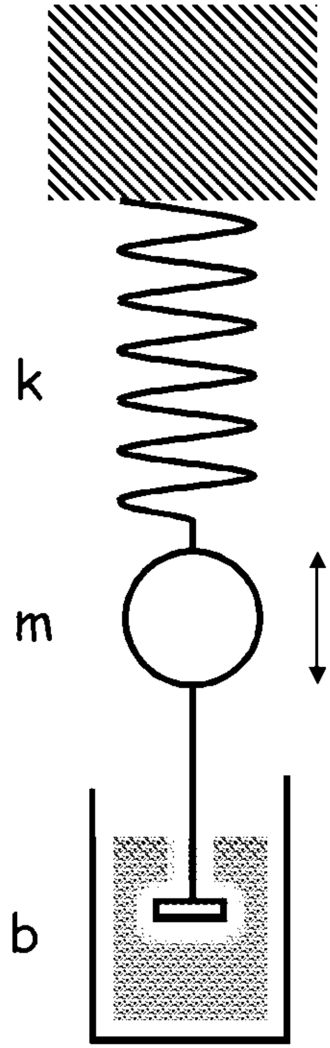
Driven SHM



$$\omega_o = \sqrt{k/m}$$

$$\gamma = b/m$$

$$F_0 \cos(\omega t)$$

$$\sum F = m\ddot{x}$$

$$-kx - b\dot{x} + F_0 \cos(\omega t) = m\ddot{x}$$

$$\boxed{\ddot{x} + \gamma\,\dot{x} + \omega_o{}^2 x = \frac{F_0}{m}\cos(\omega t)}$$ Driven SHM

steady state: $z_{ss}(t) = A_{ss} e^{j(\omega t - \partial_{ss})}$ *(Resistance is futile!!!)*

ω = steady state freq. = *the drive frequency*

A_{ss} = steady state amplitude

δ_{ss} = steady state phase lag between drive and motion

$$j^2\omega^2 A_{ss} e^{j(\omega t-\partial_{ss})} + \gamma j\omega A_{ss} e^{j(\omega t-\partial_{ss})} + \omega_o{}^2 A_{ss} e^{j(\omega t-\partial_{ss})} = \frac{F_0}{m} e^{j\omega t}$$

$$-\omega^2 A_{ss} + \gamma j\omega A_{ss} + \omega_o{}^2 A_{ss} = \frac{F_0}{m} e^{j\delta_{ss}}$$

Real

$$-\omega^2 A_{ss} + \omega_o{}^2 A_{ss} = \frac{F_0}{m}\cos(\delta_{ss})$$

Imaginary

$$\gamma j\omega A_{ss} = j\frac{F_0}{m}\sin(\delta_{ss})$$

ratio:

$$\frac{\sin(\delta_{ss})}{\cos(\delta_{ss})} = \frac{\gamma\omega A_{ss}}{-\omega^2 A + \omega_o{}^2 A_{ss}}$$

$$\boxed{\tan(\delta_{ss}) = \frac{\gamma\omega}{\omega_0{}^2 - \omega^2}}$$

(steady state)

$$\sin^2(\delta_{ss}) + \cos^2(\delta_{ss}) = 1$$

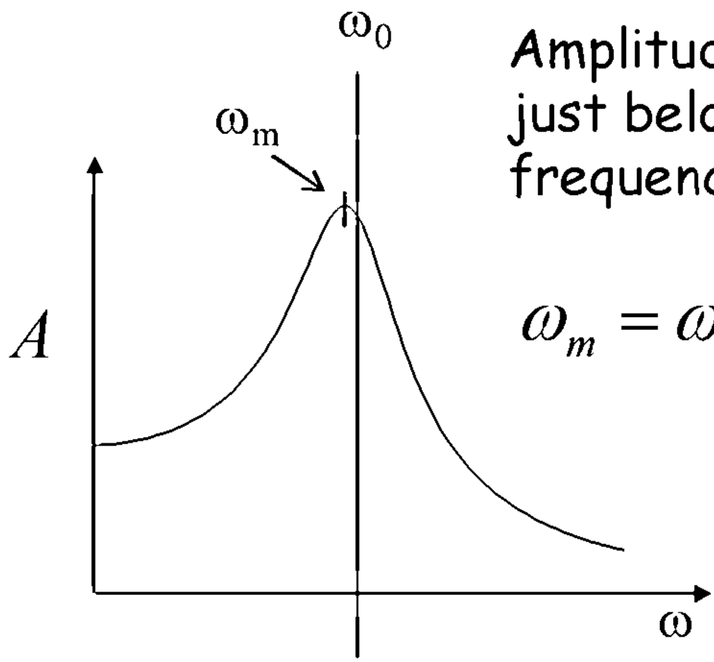
$$\left(\frac{m\gamma\omega A_{ss}}{F_0}\right)^2 + \left(\frac{m}{F_0}\right)^2 \left(A_{ss}\omega_0{}^2 - A_{ss}\omega^2\right)^2 = 1$$

$$\left(\frac{A_{ss} m}{F_0}\right)^2 \left(\left(\omega_0{}^2 - \omega^2\right)^2 + (\gamma\omega)^2\right) = 1$$

$$\boxed{A_{ss} = \frac{F_0/m}{\sqrt{\left(\omega_0{}^2 - \omega^2\right)^2 + (\gamma\omega)^2}}}$$ (steady state)

steady state solution:

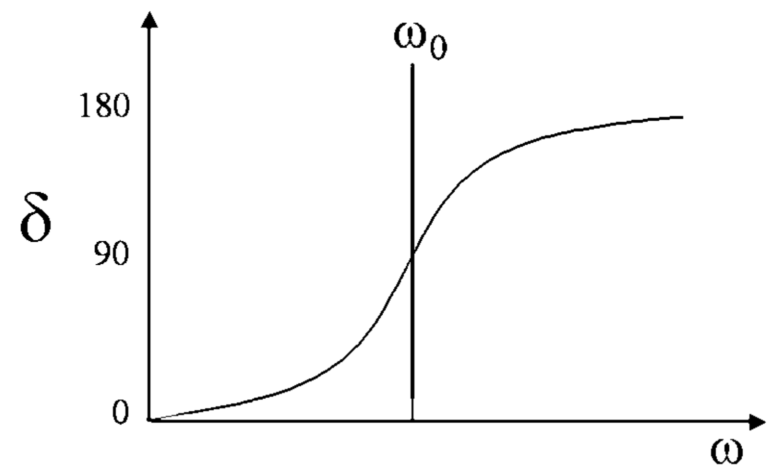
$$x_{ss}(t) = \frac{F_0/m}{\sqrt{\left(\omega_0^{\ 2} - \omega^2\right)^2 + (\gamma\omega)^2}} \cos\left(\omega t - \tan^{-1}\left(\frac{\gamma\omega}{\omega_o^{\ 2} - \omega^2}\right)\right)$$



Amplitude peaks just below natural frequency:

$$\omega_m = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$$

Phase lag through 90 deg exactly at natural frequency.



<u>Steady state</u>: A_{ss} and δ_{ss} are not adjustable parameters, they are set by the properties of the system just like ω_o!!!

For the missing part, add our result for the damped oscillator:

$$\ddot{x} + \gamma\,\dot{x} + \omega_o^{\,2} x = 0$$

$$\boxed{x_{tr}(t) = A e^{-\gamma t/2} \cos\left(\left(\omega_o^{\,2} - \gamma^2/4\right)^{1/2} t + \phi\right)}$$ (last lecture)

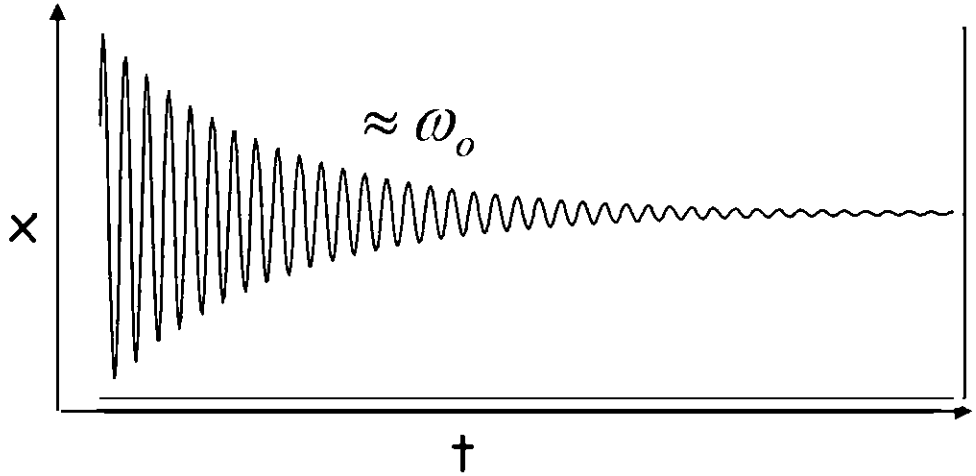
$$\ddot{x}_{ss} + \gamma\,\dot{x}_{ss} + \omega_o^{\,2} x_{ss} = \tfrac{F_0}{m}\cos(\omega t - \delta)$$

$$+ \quad \ddot{x}_{tr} + \gamma\,\dot{x}_{tr} + \omega_o^{\,2} x_{tr} = 0$$

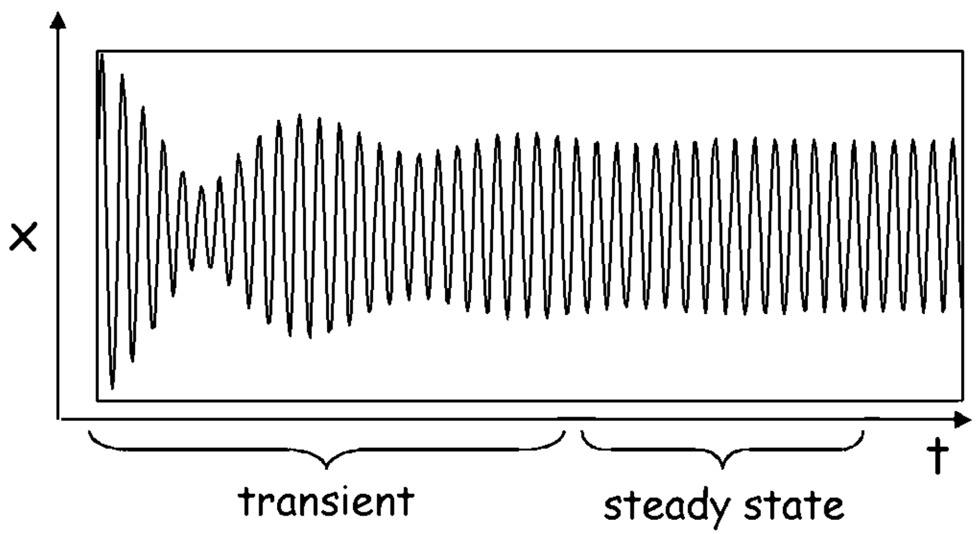
$$\overset{\bullet\bullet}{(x_{ss} + x_{tr})} + \gamma\,\overset{\bullet}{(x_{ss} + x_{tr})} + \omega_o^{\,2}(x_{ss} + x_{tr}) = \tfrac{F_0}{m}\cos(\omega t - \delta)$$

$$\boxed{\boxed{x(t) = x_{tr}(t) + x_{ss}(t)}}$$

Damped

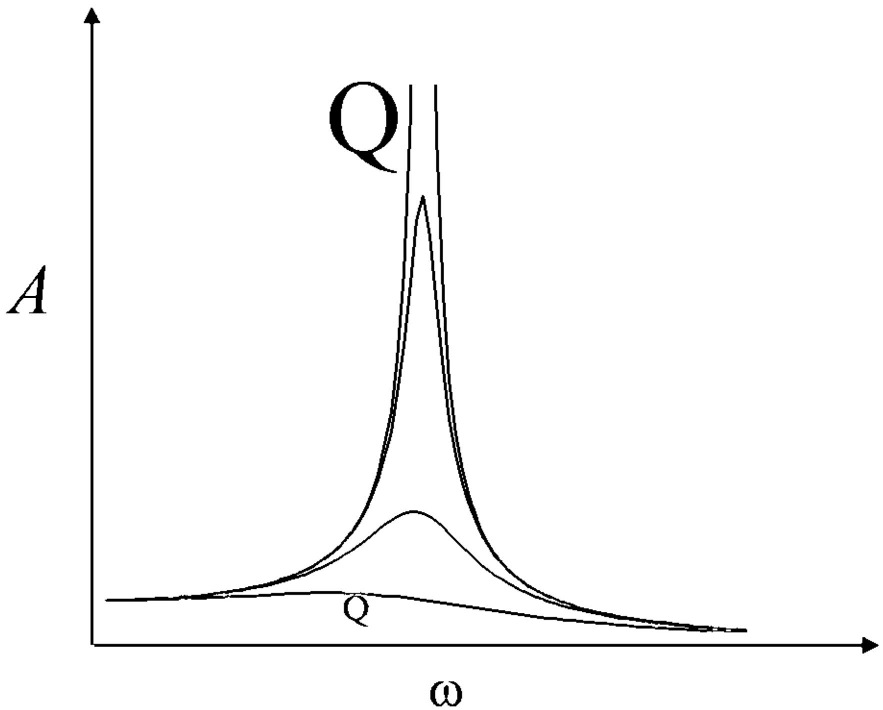


Driven & Damped



$\sqrt{\omega_o^{\,2} - \gamma^2/4}$ and ω $\qquad$ ω

High Q = "sharp" resonance



$$Q \equiv \frac{\omega_0}{\gamma}$$

$$Q \approx \frac{\omega_0}{\Delta\omega_{FWHM}}$$

(for high Q only)

A simple harmonic oscillator driven by a sinusoidal force oscillates at the frequency of the force once the transient motion at the natural frequency decays. The steady state amplitude goes through a resonance near the natural frequency.