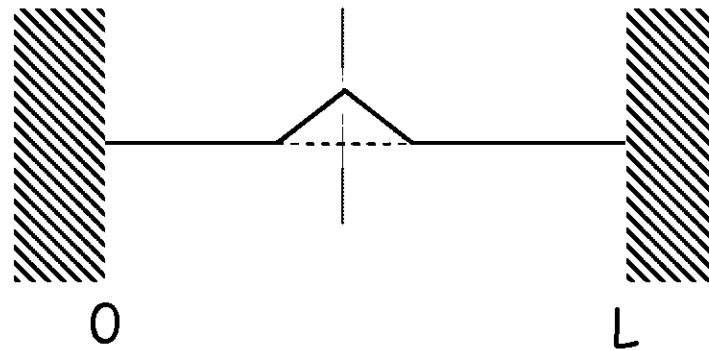


10. Wave Motion

A photograph showing a hand holding a string that is connected to a speaker. The string is vibrating, creating a wave pattern. A speech bubble is overlaid on the image, containing the text "How do we describe this?".

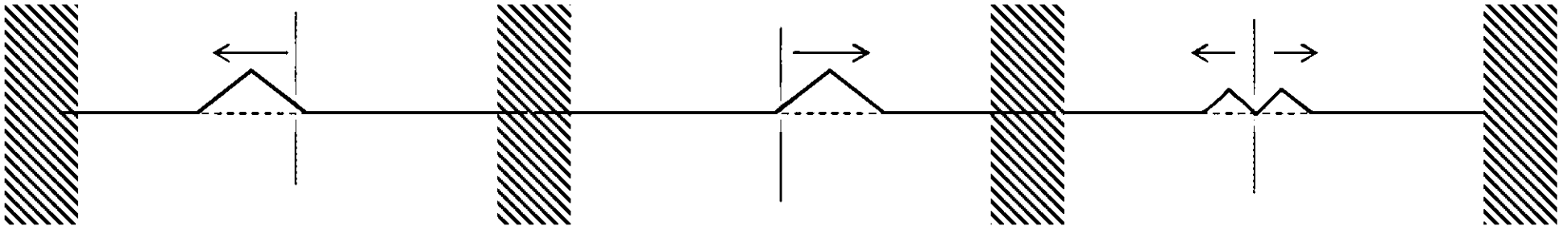
How do we describe this?

An initial shape...



...so can we just let the normal modes oscillate at ω_n ?

No! Because that initial shape can do many things:



To get the answer right, we have to get *all* the boundary conditions right!

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

Wave equation needs two spatial and two temporal boundary conditions.


Two spatial boundary conditions

Clamped string: $y(0,t) = 0$

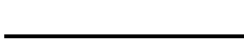
$$y(L,t) = 0$$

"a certain point in space for all time"

Two temporal boundary conditions

Initial shape: $y(x,0) =$  "a certain point in time for all space"

rather than an analogous second point in time (which could work), try this:

Initial velocity: $\dot{y}(x,0) =$ 

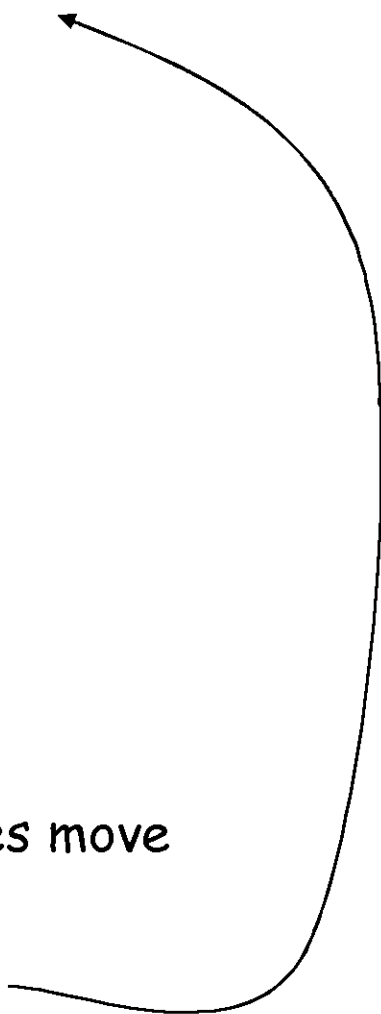
$$y(x,t) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{\omega_n}{v}x + \phi_n\right) \cos(\omega_n t + \delta_n)$$

$$y(0,t) = 0 \quad \longrightarrow \quad \phi_n = 0$$

$$y(L,t) = 0 \quad \longrightarrow \quad \omega_n = \frac{n\pi v}{L}$$

$$y(x,0) = \text{shape} \quad \longrightarrow \quad A_n \text{'s}$$

The $\dot{y}(x,0) = \text{velocities}$ tell you how the normal modes move with different phase offset. But don't do it this way!!!



$$y(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{\omega_n}{v} x\right) [B_n \cos(\omega_n t) + A_n \sin(\omega_n t)]$$

(spatial boundary conditions) 

$$y(x,0) = \sum_{n=1}^{\infty} \sin\left(\frac{\omega_n}{v} x\right) B_n$$

$$= \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{L} x\right) B_n \quad \text{French's Fourier series!}$$

$$\dot{y}(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{\omega_n}{v} x\right) [-B_n \omega_n \sin(\omega_n t) + A_n \omega_n \cos(\omega_n t)]$$

$$\dot{y}(x,0) = \sum_{n=1}^{\infty} \sin\left(\frac{\omega_n}{v} x\right) [A_n \omega_n] = 0 \quad \begin{array}{l} \text{Initially at rest?} \\ \text{Then } A_n = 0 \end{array}$$

