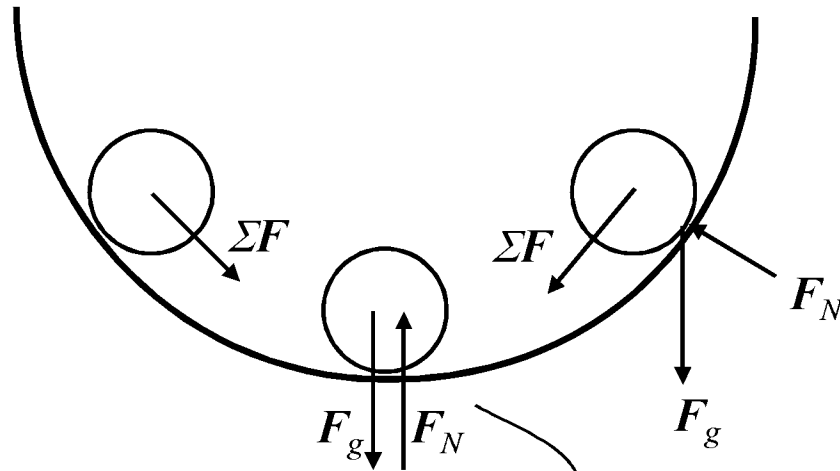


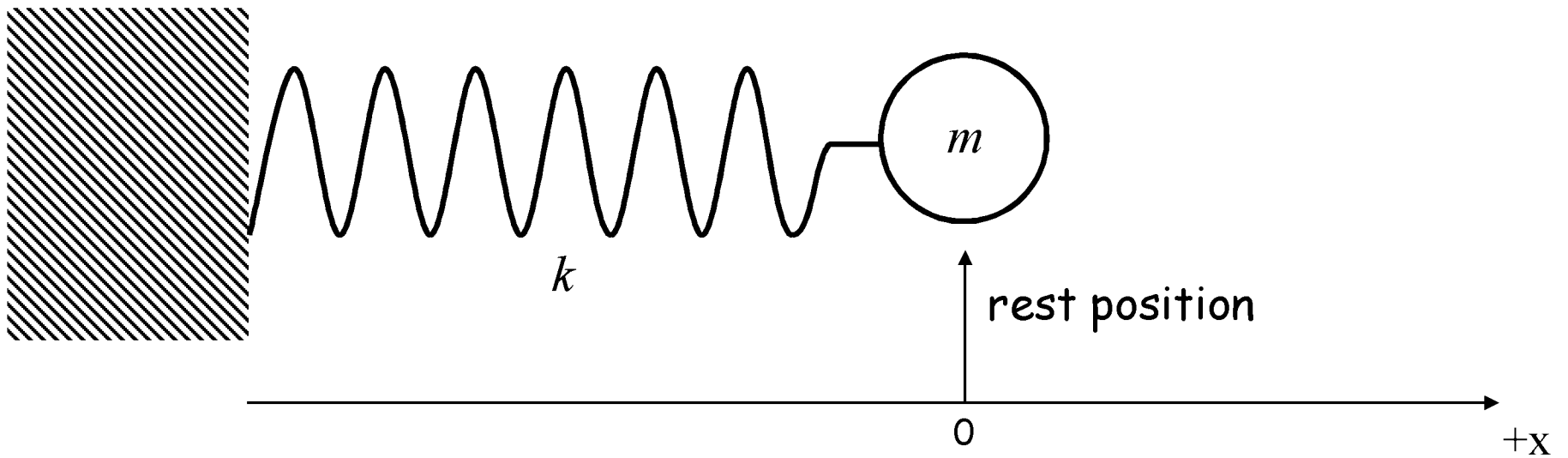
Simple Harmonic Motion (SHM)

Ball in a Bowl:



$$\Sigma \vec{F} = 0$$

Stable Equilibrium
(*restoring* force, not constant force)



$$\sum \vec{F} = m\vec{a}$$

$$-kx = m\ddot{x}$$

(equation of motion)

Hooke's Law

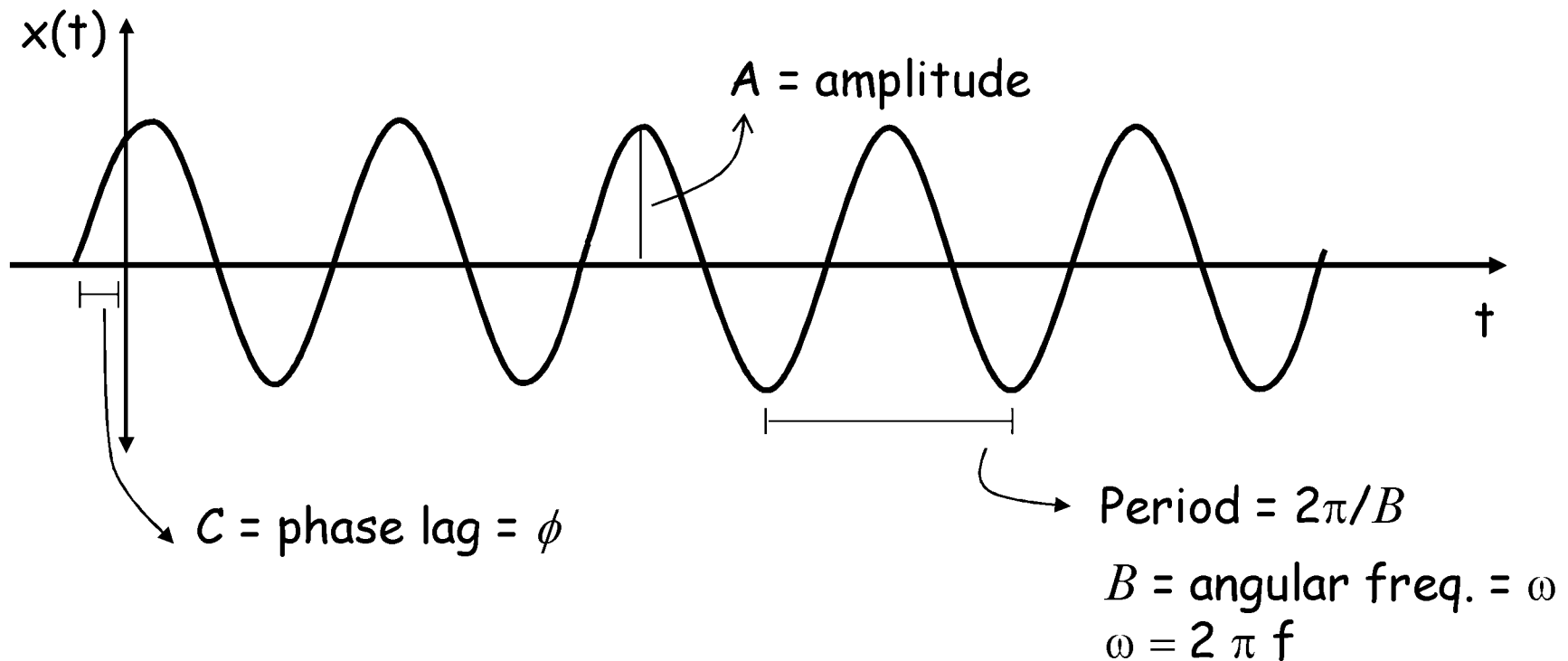
"Guess" a solution: $x(t) = A \sin(Bt + C)$

Does it work?

$$-kA \sin(Bt + C) = -mAB^2 \sin(Bt + C)$$

Yes, if:

$$k = mB^2 \quad \dots A \text{ and } C \text{ can take any value}$$



So usually written... $x(t) = A \sin(\omega_o t + \phi)$

$$\omega_o = \sqrt{\frac{k}{m}} \quad A \text{ and } \phi \text{ from initial conditions.}$$

Example: Mass (m) on spring (k) displaced $x = 1 \text{ cm}$ at $t=0$:

$$x(t) = A \sin(\omega_o t + \phi) \quad \dot{x}(t) = A \omega_o \cos(\omega_o t + \phi)$$

$$x(0) = A \sin(\phi) = 1 \text{ cm} \quad \dot{x}(0) = A \omega_o \cos(\phi) = 0$$

$$A \sin\left(\frac{\pi}{2}\right) = 1 \text{ cm} \longleftarrow \phi = \frac{\pi}{2}$$

$$A = 1 \text{ cm}$$

$$x(t) = (1 \text{ cm}) \sin\left(\sqrt{\frac{k}{m}} t + \frac{\pi}{2}\right)$$

When a mechanical system is at a point of *stable equilibrium*, a perturbation will result in a restoring force that drives it back to equilibrium. The resulting equation of motion has the following form:

$$\ddot{x} + \omega_o^2 x = 0$$

The solution is an oscillating trajectory at frequency ω_o known as *Simple Harmonic Motion*.