Comparing Classical and Quantum Finite Automata

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Quantum computers offer the advantage of running more efficient algorithms because they are composed of qubits rather than conventional bits that are used in classical computers. This motivates the exploration of quantum algorithms and models with the expectation that they can solve many more complex problems. Here, we are exploring one of the simplest models of computation by comparing Deterministic Finite Automata and Quantum Finite Automata (Ambainis and Freivalds, 1998). This research focuses on applying quantum principles to the following problem: Consider a string $a^i$ with $i$ letters. We want to determine whether the string is in the language $L$ where $L = \{a^i \mid i \text{ is divisible by } p\}$ and $p$ is a given prime number. If $i$ is divisible by $p$, we accept the string into the language, and if not, we reject it. $|0\rangle$ and $|1\rangle$ qubit states serve as the accept and reject states. Classically, using the highest known prime integer, this algorithm requires a minimum of 77,232,917 bits, whereas the quantum finite automata only requires 27 qubits. Using Python’s Quantum Information Software Kit (Qiskit), I have implemented a program [1] that can determine if the length of a string is divisible by a large prime number with exponentially fewer qubits, thus further demonstrating the potential of quantum models.

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Classical Finite Automata

- Flip switch up
- Flip switch down

L = {\alpha^i | i is odd}

Goal: To accept or reject an input from some character set based on the pattern defined by the machine

Applications
- Traffic Lights
- Elevators
- Turnstiles

Quantum Finite Automata

Bloch Sphere ~ Single Qubit Representation

\[ \psi = \alpha|0\rangle + \beta|1\rangle \]

\[ \alpha = \cos \theta \]

\[ \beta = e^{i \varphi} \sin \theta \]

\[ |\alpha|^2 + |\beta|^2 = 1 \]

Goal: To accept or reject an input from some character set using quantum properties such as superposition and the probability that the qubit is in a specific state

Motivation
- Interactive Proof Systems
- Demonstrate a quantum advantage in simple models
- Invention of quantum state machines (traffic lights, elevators, etc.)

Wave Superposition

The Problem: Prime Divisibility

Consider a string with \( a^i \) letters and we want to know if the string is accepted into the language \( L \), where \( L = \{a^i | i \text{ is divisible by } p \} \) and \( p \) is a prime number\(^1\) Will qubits offer an advantage to this finite automata problem?

The Solution: Classical vs. Quantum

Example: String = “aaa”, Prime = 3

Classical

Start

\[ \begin{align*}
\alpha &: 10 \\
\beta &: 01 \\
\end{align*} \]

Target Qubit Control Qubit

\[ |0\rangle \text{ Heads} \]

\[ |1\rangle \text{ Tails} \]

Quantum

\[ \text{Measure: } \Theta = \Phi_1 + \Phi_2, \Phi_1 = 2\pi ik/p, \Phi_2 = 2\pi i/p \]

\[ \text{Demonstrates that we can run multiple QFAs at the same time} \]

\[ \text{Therefore, we can use exponentially fewer qubits} \]

QISKit Implementation in Python

Local Simulator

Real Device

References


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Future Work

- Test for noise and use error correction techniques to improve the solution on the 5-qubit computer
- Test the solution with larger qubit systems such as IBM Q’s 20-qubit computer
- Apply to real world finite automata

Conclusion

Classical

- Requires \( \log(p) \) bits and \( p \) states\(^1\)
- Dividing by the largest known prime integer requires a minimum of 77,232,917 bits to solve the problem

Quantum

- Requires \( \log(\log(p)) \) qubits and \( \log(p) \) states\(^1\)
- Dividing by the largest known prime integer requires a minimum of 27 qubits to solve the problem
- Requires exponentially less memory