CAAM 453: NUMERICAL ANALYSIS I

Instructor:	Maarten V. de Hoop	**Class Time:**	MWF 9:00-9:50 am
Email:	`mdehoop@rice.edu`	**Classroom:**	Duncan Hall 1046
Office:	Duncan Hall 2036	**Office Hours:**	MW 1:00-2:00 pm
Course Website:	`TBA`		and by appointment

Course Objectives and Learning Outcomes

This course is an introduction to a broad range of numerical methods for solving mathematical problems that arise in Science and Engineering. The goal is to provide a basic understanding of the derivation, analysis and use of these numerical methods, including aspects of numerical linear algebra, along with a rudimentary understanding of finite precision arithmetic. This will help you choose, develop and apply the appropriate numerical techniques for your problem, interpret the results and assess accuracy.

On completion of this course, the student should be able to:

- Numerically solve a scalar nonlinear equation.
- Understand the basics of finite precision arithmetic, conditioning of problems and stability of numerical algorithms.
- Solve dense systems of linear algebraic equations and least squares problems, and have a working knowledge of LU and QR factorizations for these problems.
- Use condition number and norms to assess accuracy of solutions to linear algebraic equations and least squares problems.
- Numerically approximate functions with polynomials. Understand and apply appropriate techniques for numerical differentiation and integration.
- Solve initial value problems for ordinary differential equations with explicit or implicit methods as appropriate.
- Numerically solve basic eigenvalue problems and have a working knowledge of the singular value decomposition (SVD).
- Use the method of lines to solve basic parabolic partial differential equations.

Prerequisites. Students must have a sound background in Calculus (I,II,III), a working knowledge of Linear Algebra, and must be proficient in Matlab programming. Required courses: MATH 212 or MATH 222 and CAAM 335 or equivalents, CAAM 210 or equivalent.

Required Texts and Materials

We will use the CAAM 453 Lecture Notes by Professor Mark Embree. This e-book may be downloaded at no cost. The material covered in this book will be supplemented with additional topics covered in class and provided by additional notes, made available on the instructor's website. Hence, regular attendance of the lectures is required.

The material of this course is covered in many books on Numerical Analysis (call number QA 297) or on Scientific Computing (call number QA 183) and you are encouraged to browse the library. In particular,

L.N. Trefethen and D. Bau III, *Numerical Linear Algebra*, SIAM, Philadelphia, 1997 is recommended. We shall use Matlab for programming exercises.

Exams and Homework

There will be 8-9 homework assignments and no exams.

Grade Policies

- Homework will typically consist of 3 regular problems and one pledged problem. Unless noted otherwise, you may discuss the regular problems with fellow classmates and this is encouraged. However, you are expected to individually write up your solutions. You may not consult solution sheets from past CAAM courses. You are not allowed to discuss pledged problems with anyone but your instructor or TA.
- The grade will be determined from your scores on the homework assignments. There will be no exams.
- NO LATE HOMEWORK will be accepted. We will automatically drop your worst two regular assignment scores. Exceptions will only be made for documented illnesses or emergencies.
- Homework frequently involves Matlab computer projects. Program source code must be turned in. On regular assignments you may discuss the coding projects with your fellow students. In no case are you allowed to share codes. Each student is responsible for writing her/his own codes.
- Look over the graded homework as soon as it is returned. If you detect mistakes in the grading, notify your instructor immediately. Homework scores will only be changed during the first two weeks after they have been returned.

Absence Policies

Students are strongly encouraged to contribute to our class community by attending and participating in the lectures.

Rice Honor Code

In this course, all students will be held to the standards of the Rice Honor Code, a code that you pledged to honor when you matriculated at this institution. If you are unfamiliar with the details of this code and how it is administered, you should consult the Honor System Handbook at `http://honor.rice.edu/honor-system-handbook/`. This handbook outlines the University's expectations for the integrity of your academic work, the procedures for resolving alleged violations of those expectations, and the rights and responsibilities of students and faculty members throughout the process.

Disability Support Services

If you have a documented disability or other condition that may affect academic performance you should: 1) make sure this documentation is on file with Disability Support Services (Allen Center, Room 111 / `adarice@rice.edu` / x5841) to determine the accommodations you need; and 2) talk with me to discuss your accommodation needs.

Syllabus Change Policy

This syllabus is only a guide for the course and is subject to change with advance notice.

Course Schedule

Week 1	Solving Nonlinear Scalar Equations: *Lectures 38-40*
08/21-08/25	Bisection, Newton's Method, Secant Method
	Convergence of the Secant Method
Week 2	Numerical Linear Algebra: *Lectures 2-4*
08/28-09/01	The Concept of Matrix Decomposition
	Matrix Analysis and Norms, Projectors and Reflectors, the QR Factorization
Week 3	Solving Systems of Linear Algebraic Equations: *Lectures 5-7*
09/06-09/08	QR via Gram-Schmidt, QR via Householder method
	Solving $Ax = b$ via QR factorization
	Conditioning and Stability of Linear Systems
	Backward Error Analysis: Assessing Accuracy from Residual and Condition Number
Week 4	Polynomial Interpolation: *Lectures 9-12*
09/11-09/15	Constructing Polynomial Interpolants
	Selection of Interpolation Bases
	Error Bounds for Polynomial Interpolation
	Hermite Interpolation, Piecewise Polynomial Interpolation
Week 5	Polynomial Interpolation, Least Squares Approximation: *Lectures 14, 16-18*
09/18-09/22	Splines
	Least Squares Approximation
	Normal Equations Solution
	QR Solutions: Gram-Schmidt, Householder QR, Modified Gram-Schmidt
	The Singular Value Decomposition, Low-Rank Approximation
Week 6	Continuous Least Squares: *Lectures 19-20*
09/25-09/29	Continuous Least Squares Approximation
	Orthogonal Polynomials for Continuous Least Squares Problems
Week 7	Minmax Approximation: *Lectures 21-22*
10/02-10/06	Oscillation Theorem
	Minimax Approximation, Optimal Interpolation, Chebyshev Polynomials
Week 8	Finite Precision Arithmetic: *Lecture 8*
10/11-10/13	Floating Point Arithmetic
	Idealized Floating Point Number System
	Binary Representation, Floating Point Representation, IEEE Standard
Week 9	Quadrature: *Lectures 23-26*
10/16-10/20	Interpolatory Quadrature
	Richardson Extrapolation and Romberg Integration
	Gaussian Quadrature

Week 10	Quadrature, ODE Solvers: *Lectures 26-28*
10/23-10/27	Gaussian Quadrature
	Introduction to ODE Solvers
	Analysis of One Step ODE Integrators
Week 11	ODE Solvers: *Lectures 29-30*
10/30-11/03	Global Error Analysis
	Linear Multistep Methods: Truncation Error
	Von Neumann Analysis and the Diffusion Equation
Week 12	ODE Solvers: *Lectures 31-32*
11/06-11/10	Linear Multistep Methods: Zero Stability
	Absolute Stability
Week 13	ODE Solvers, LU: *Lectures 33, 35*
11/13-11/17	Stiff Problems
	Gaussian Elimination
Week 14	LU: *Lecture 35*
11/20-11/22	Gaussian Elimination
	Thanksgiving
Week 15	Cholesky, Eigenvalues: *Lectures 36-37*
11/27-12/01	Some Lagrangian mechanics
	Cholesky Factorization
	Eigenvalues Computation