Electrical & Computer Engineering 241
Problem Set VIII

Problem 5.11: Discrete-Time Filtering

We can find the input-output relation for a discrete-time filter much more easily than for analog filters. The key idea is that a sequence can be written as a weighted linear combination of unit samples.

(a) Show that $x(n) = \sum_i x(i)\,\delta(n-i)$ where $\delta(n)$ is the unit-sample.

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases}$$

(b) If $h(n)$ denotes the **unit-sample response**—the output of a discrete-time linear, shift-invariant filter to a unit-sample input—find an expression for the output.

(c) In particular, assume our filter is FIR, with the unit-sample response having duration $q+1$. If the input has duration N, what is the duration of the filter's output to this signal?

(d) Let the filter be a boxcar averager: $h(n) = \dfrac{1}{q+1}$ for $n = \{0, \dots, q\}$ and zero otherwise. Let the input be a pulse of unit height and duration N. Find the filter's output when $N = \dfrac{q+1}{2}$, q an odd integer.

Problem 5.17: DSP Tricks

Sammy is faced with computing **lots** of discrete Fourier transforms. He will, of course, use the FFT algorithm, but he is behind schedule and needs to get his results as quickly as possible. He gets the idea of computing **two** transforms at one time by computing the transform of $s(n) = s_1(n) + js_2(n)$, where $s_1(n)$ and $s_2(n)$ are two real-valued signals of which he needs to compute the spectra. The issue is whether he can retrieve the individual DFTs from the result or not.

(a) What will be the DFT $S(k)$ of this complex-valued signal in terms of $S_1(k)$ and $S_2(k)$, the DFTs of the original signals?

(b) Sammy's friend, an Aggie who knows some signal processing, says that retrieving the wanted DFTs is easy: "Just find the real and imaginary parts of $S(k)$." Show that this approach is too simplistic.

(c) While his friend's idea is not correct, it does give him an idea. What approach will work? **Hint**: Use the symmetry properties of the DFT.

(d) How does the number of computations change with this approach? Will Sammy's idea ultimately lead to a faster computation of the required DFTs?

Problem 5.21: Yet Another Digital Filter

A filter has an input-output relationship given by the following difference equation.

$$y(n) = \frac{1}{4}x(n) + \frac{1}{2}x(n-1) + \frac{1}{4}x(n-2)$$

(a) What is the filter's transfer function? How would you characterize it?

(b) What is the filter's output when the input equals $\cos\left(\frac{\pi n}{2}\right)$?

(c) What is the filter's output when the input is the depicted discrete-time square wave (Figure 5.32)?

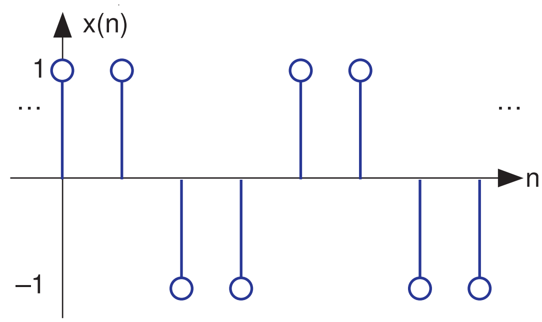

Figure 5.32

Problem 5.23: Digital Filters
A discrete-time system is governed by the difference equation

$$y(n) = y(n-1) + \frac{x(n) + x(n-1)}{2}$$

(a) Find the transfer function for this system.
(b) What is this system's output when the input is $\sin\left(\frac{\pi n}{2}\right)$?
(c) If the output is observed to be $y(n) = \delta(n) + \delta(n-1)$, then what is the input?

Problem 5.33: Digital Filtering of Analog Signals
RU Electronics wants to develop a filter that would be used in analog applications, but that is implemented digitally. The filter is to operate on signals that have a 10 kHz bandwidth, and will serve as a lowpass filter.

(a) What is the block diagram for your filter implementation? Explicitly denote which components are analog, which are digital (a computer performs the task), and which interface between analog and digital worlds.
(b) What sampling rate must be used and how many bits must be used in the A/D converter for the acquired signal's signal-to-noise ratio to be at least 60 dB? For this calculation, assume the signal is a sinusoid.
(c) If the filter is a length-128 FIR filter (the duration of the filter's unit-sample response equals 128), should it be implemented in the time or frequency domain?
(d) Assuming $H\left(e^{j2\pi f}\right)$ is the transfer function of the digital filter, what is the transfer function of your system?