Topic 9:
Product Differentiation

EC 3322

Semester I – 2008/2009

Introduction

- Firms produce similar but not identical products (differentiated) → in many different ways.
  - **Horizontally**
    - Goods of **similar quality** targeted at **consumers of different types/ preference/ taste/ location**
      - How is variety determined?
      - How does competition influence the equilibrium variety.
  - **Vertically**
    - Consumers agree that there are **quality differences.**
    - They **differ** in **willingness to pay for quality.**
      - What determine the quality of goods?

Introduction …

- **Modeling** horizontal product differentiation:
  - **Representative Consumer Model**
    - Firms producing differentiated goods **compete equally for all consumers.**
    - Demand is continuous → the usual (inverse) demand function → a small change in any one firm's quantity (or price) → a small change in demand.
  - **Spatial/ Location/ Address Model**
    - Consumers may **prefer products with certain characteristics** (taste, location, sugar contents, etc) → are **willing to pay premium** for the preferred products.
    - Demand maybe **independent** (not close substitutes) or **highly dependent** (close substitutes)

Representative Consumer Model

- Recall that with **homogenous goods** → demand:

$$P_i = P = D\left(q_1, q_2 ..., q_N\right)$$

$$P_i = P = D\left(q_1 + q_2 + ... + q_N\right)$$

- With 2 firms $P = A - B\left(q_1 + q_2\right)$ with $B_1 = B_2$
- With differentiated products: $P_i = A - B_i q_i - B_j q_j$ with $j \neq i$ where $A > 0$ and $\left|B_i\right| > \left|B_j\right|$
- With more than two firms, we can write:

$$P_i = A - B_i q_i - B_j \sum_{j=1}^{N} q_j \text{ with } j \neq i$$

$\sum_{j=1}^{N} q_j$ = the sum of the quantity of all firms except firm *i*.

Representative Consumer Model…

Coke and Pepsi are similar but not identical. As a result, the lower priced product does not win the entire market.

Suppose that econometric estimation gives:

$$Q_C = 63.42 - 3.98P_C + 2.25P_P$$

$$MC_C = \$4.96$$

$$Q_P = 49.52 - 5.48P_P + 1.40P_C$$

$$MC_P = \$3.96$$

There are at least two methods for solving this for P_C and P_P. Assume that we have **Bertrand competition**.

Representative Consumer Model…

Method 1: Calculus

Profit of Coke: $\Pi_C = (P_C - 4.96)(63.42 - 3.98P_C + 2.25P_P)$

Profit of Pepsi: $\Pi_P = (P_P - 3.96)(49.52 - 5.48P_P + 1.40P_C)$

Differentiate with respect to P_C and P_P respectively → first order conditions → optimal P_c and P_p.

Method 2: MR = MC

Reorganize the demand functions

$P_C = (15.93 + 0.57P_P) - 0.25Q_C$ ------> $P_c(Q_c, Q_p)$

$P_P = (9.04 + 0.26P_C) - 0.18Q_P$ ------> $P_p(Q_c, Q_p)$

Calculate marginal revenue, equate to marginal cost, solve for Q_C and Q_P and substitute in the demand functions

Representative Consumer Model…

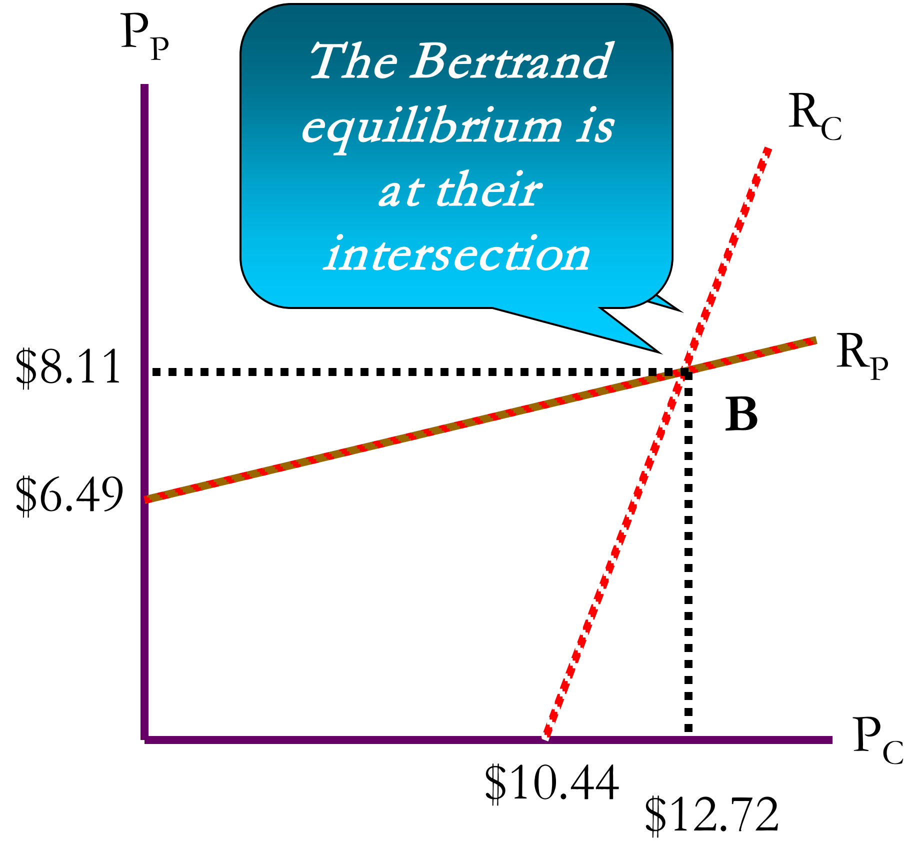
Both methods give the best response functions:

$P_C = 10.44 + 0.2826P_P$

$P_P = 6.49 + 0.1277P_C$

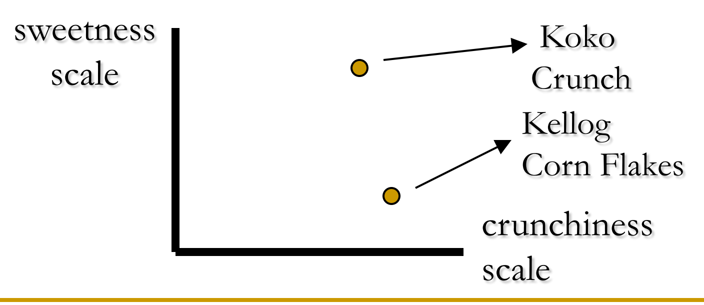
These can be solved for the equilibrium prices as indicated

The **equilibrium prices are each greater than marginal cost**



Location Model

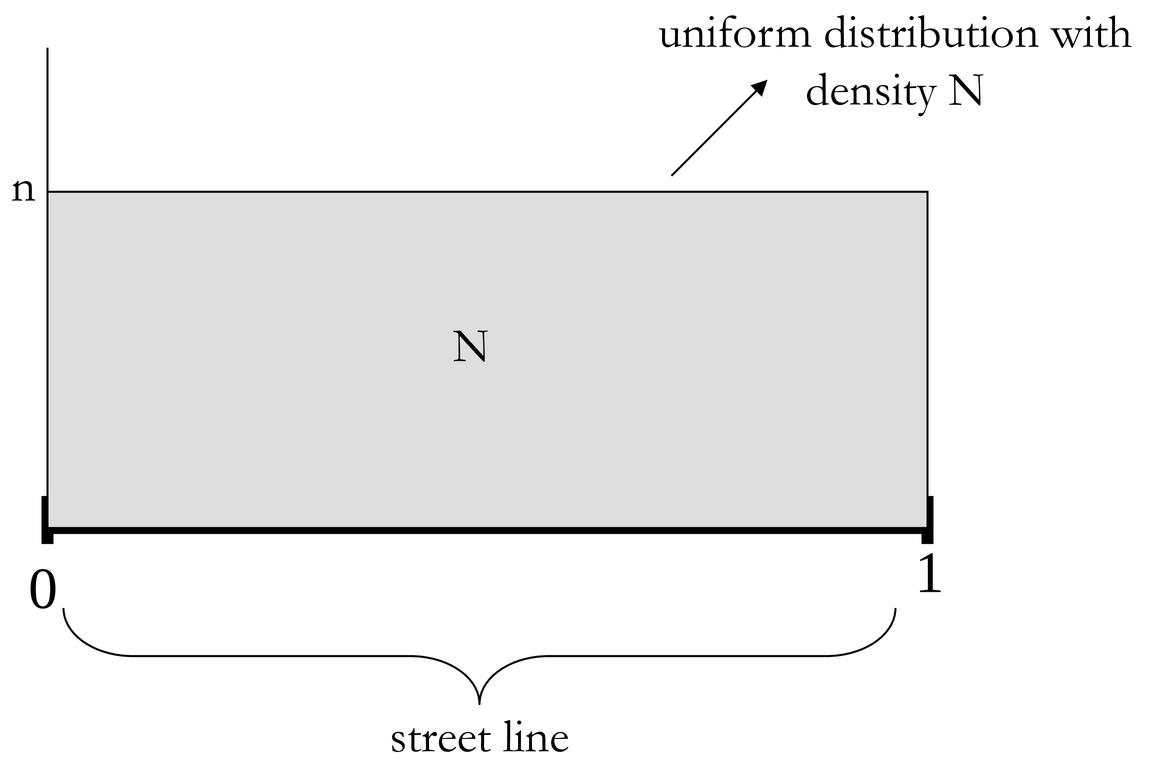
- Typically, brands (products) compete vigorously with those that consumers view as **close substitutes**.
- Close substitutes-ness → could either depends on the **perception of consumers** or **physical** or **product attributes**.
- **"Location"** based model → tries to capture the notion of close substitutes → location can be interpreted as:
  - **Geographic** location → e.g. the location of the outlet (store).
  - **Time** → e.g. departure time, showing time.
  - **Product characteristics** → design and variety → e.g. diet coke vs regular coke, sweetness and crunchiness of cereals.



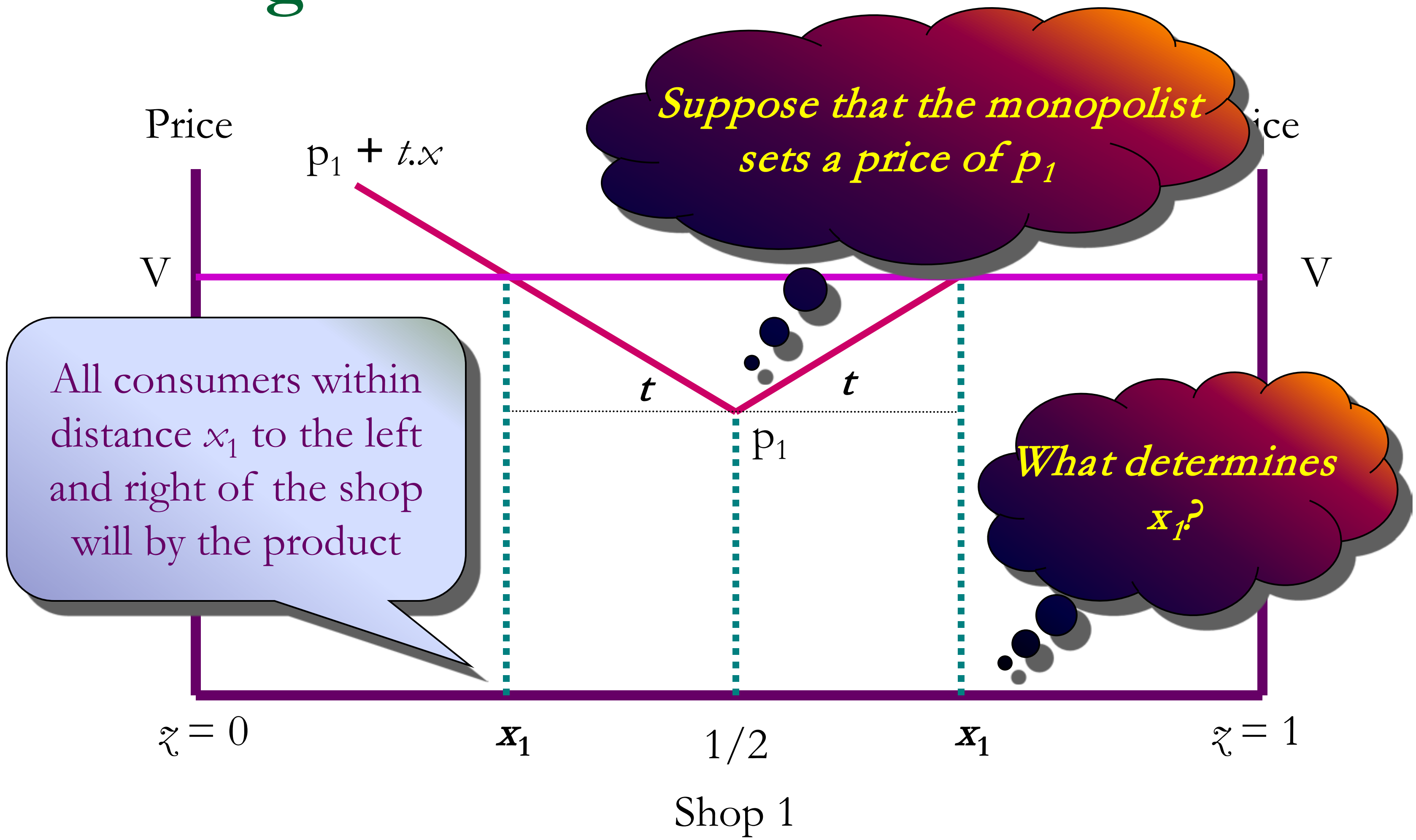
Location Model…

- Based on Hotelling (1929) → Hotelling's Linear Street Model.
- Imagine → e.g. a long stretch of beach with ice cream shops (sellers) along it.
- The model discusses the "**location**" and "**pricing behavior**" of firms.
- **Basic Setup**:
  - N-consumers are **uniformly distributed** along this linear street → thus in any block of the street there are an equal number of consumers.
  - Consumers are **identical except for location** and each of them are considering **buying exactly one unit** of product as long as the **price paid** + **other costs** are **lower** than the value derived from consuming the product (**V**).
  - As a **benchmark**, consider for the time being the case of a **monopoly seller** → operates only 1 store → it is reasonable to expect that it is located in the middle.
  - Consumers incur "***transportation***" **costs** per unit of distance (e.g. mile) traveled, ***t***.

Location Model…

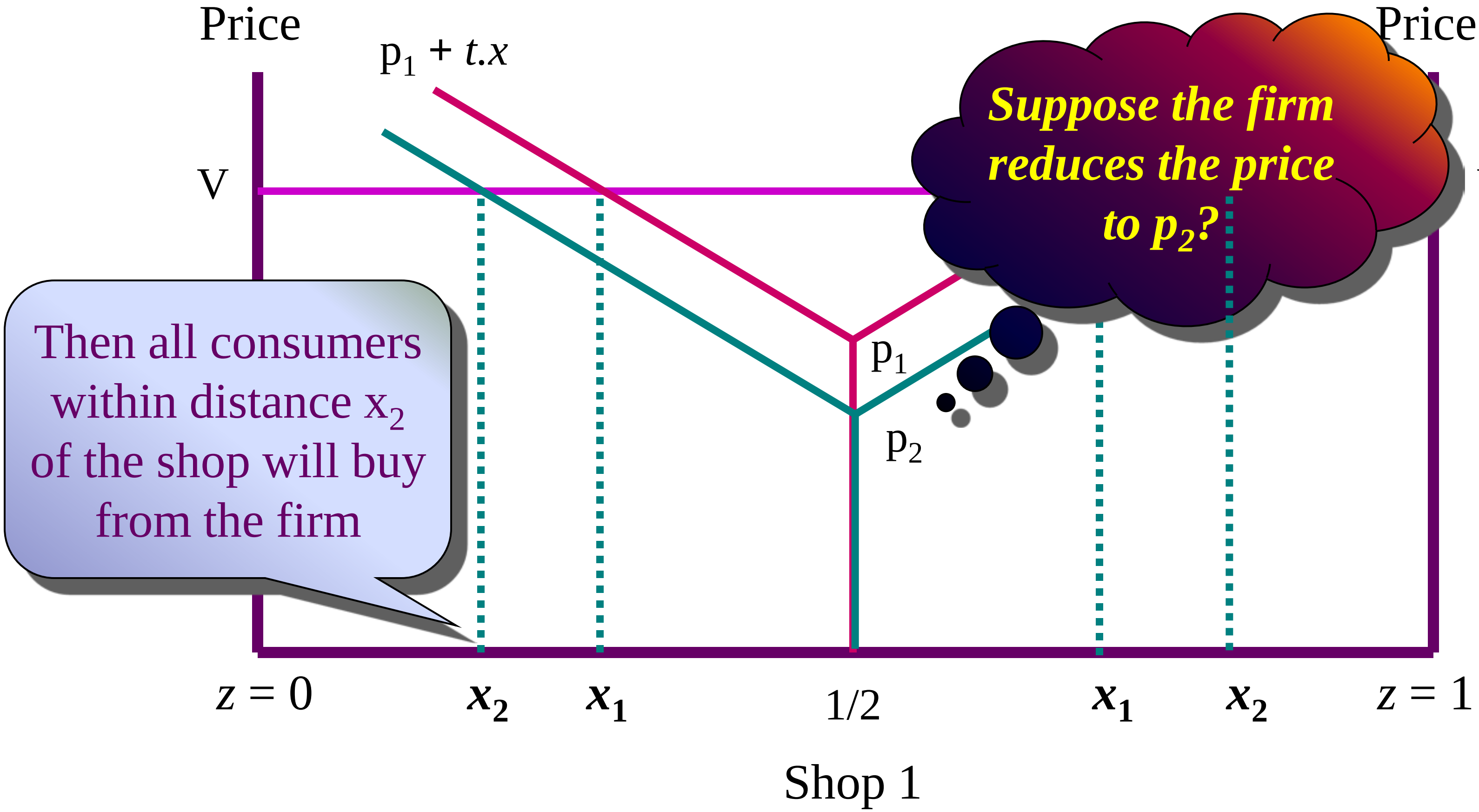


Hotelling Model…



$p_1 + t.x_1 = V$, so $x_1 = (V - p_1)/t$

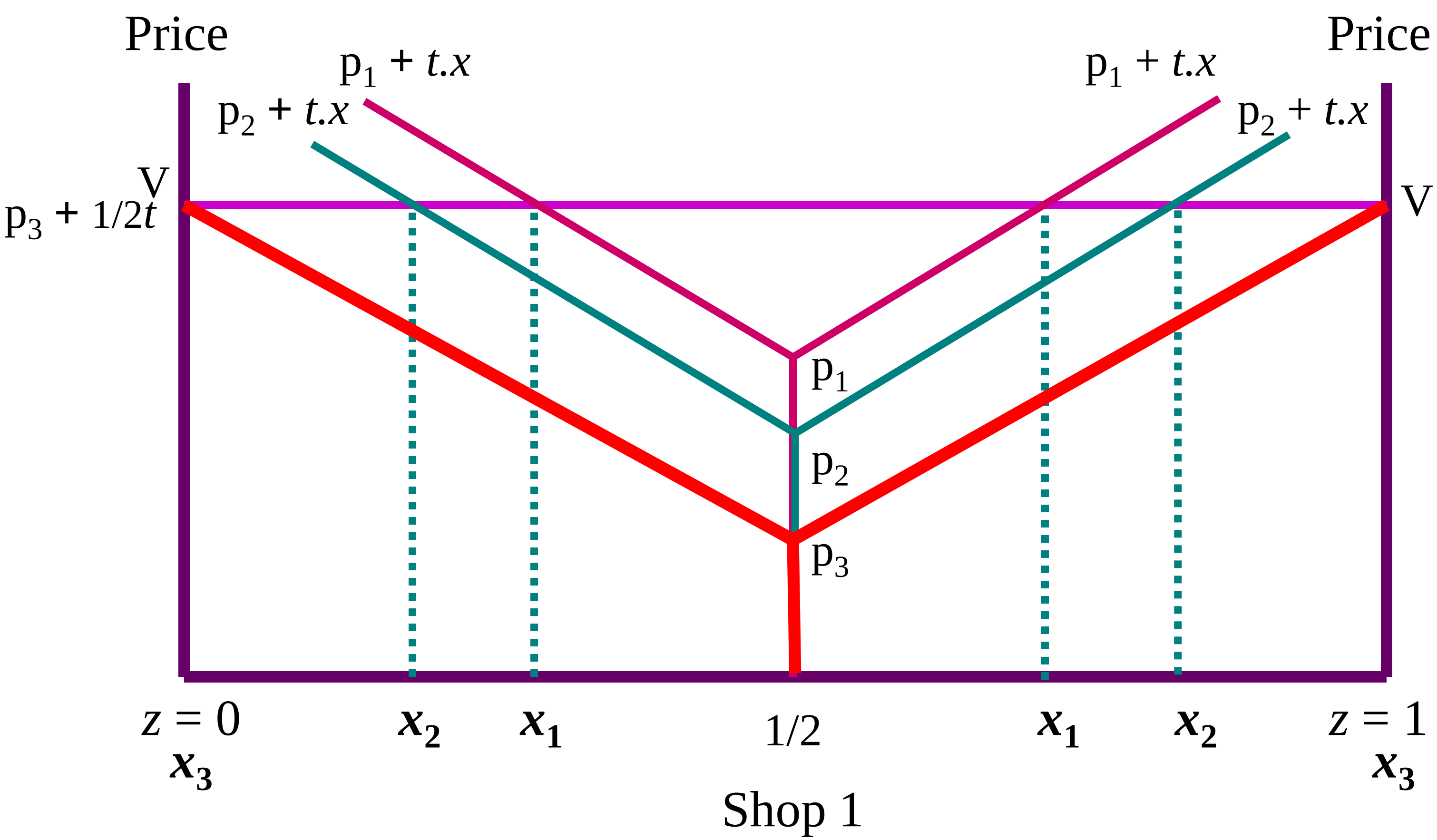
Hotelling Model…



Hotelling Model…

- Suppose that all consumers are to be served at price p.
  - The highest price is that charged to the consumers at the ends of the market.
  - Their transport costs are $t/2$: since they travel ½ mile to the shop
  - So they pay $p + t/2$ which must be no greater than V.
  - So $p = V - t/2$.
- Suppose that marginal costs are c per unit.
- Suppose also that a shop has set-up costs of F.
- Then profit is $\Pi = N\left(V - \frac{t}{2} - c\right) - F$

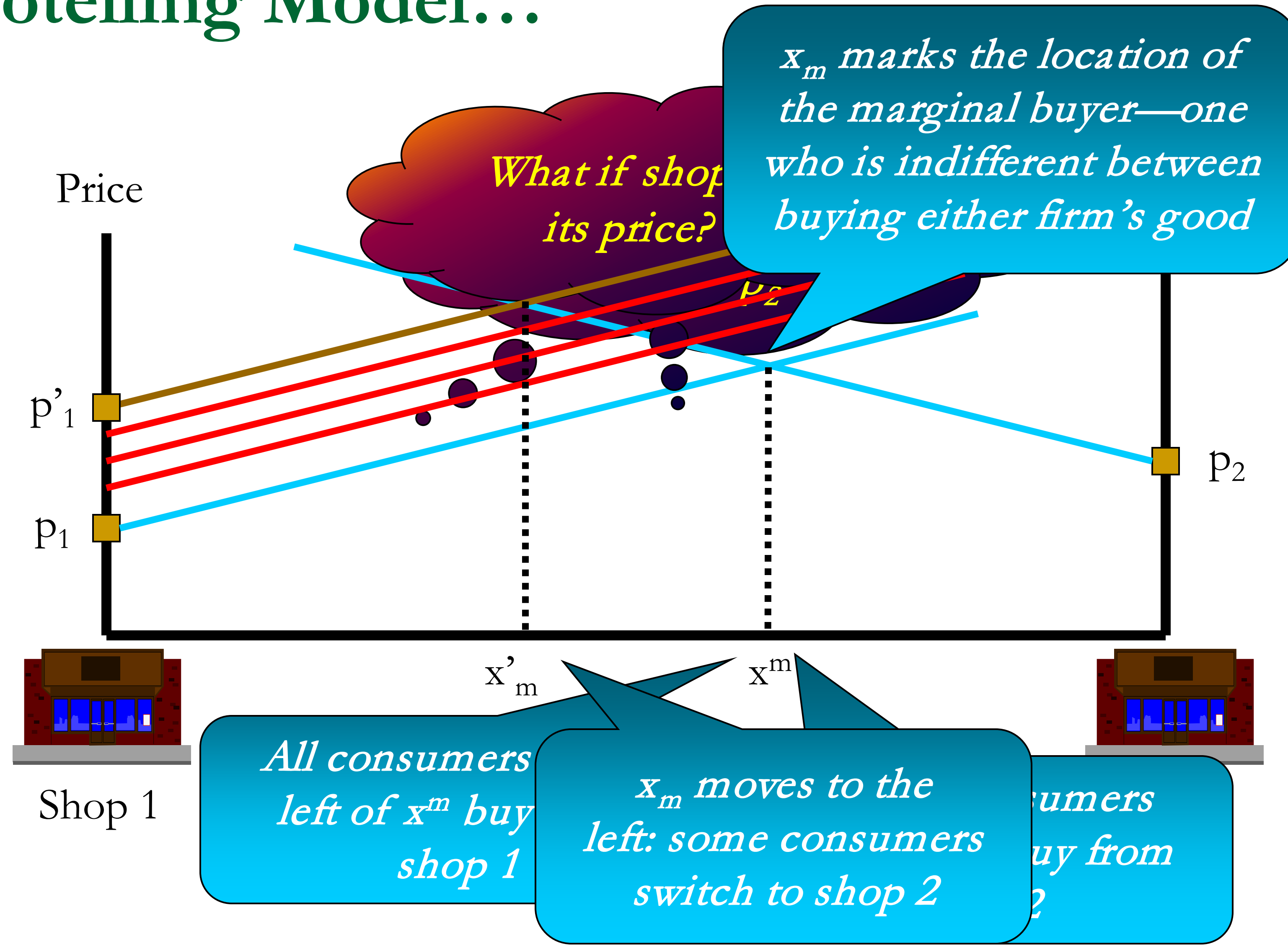
Hotelling Model…



Hotelling Model…

- What if there are two shops and these **two shops** are **competitors**?
- Consumers buy from the shop who can offer the **lower full price** (product price + transportation cost).
- Suppose that **location of these two shops are fixed** at **both ends** of the street, and they **compete only in price**.
- How large is the demand obtained by each firm and what prices are they going to charge?

Hotelling Model…



Hotelling Model…

$p_1 + tx^m = p_2 + t(1 - x^m) \quad \therefore 2tx^m = p_2 - p_1 + t$

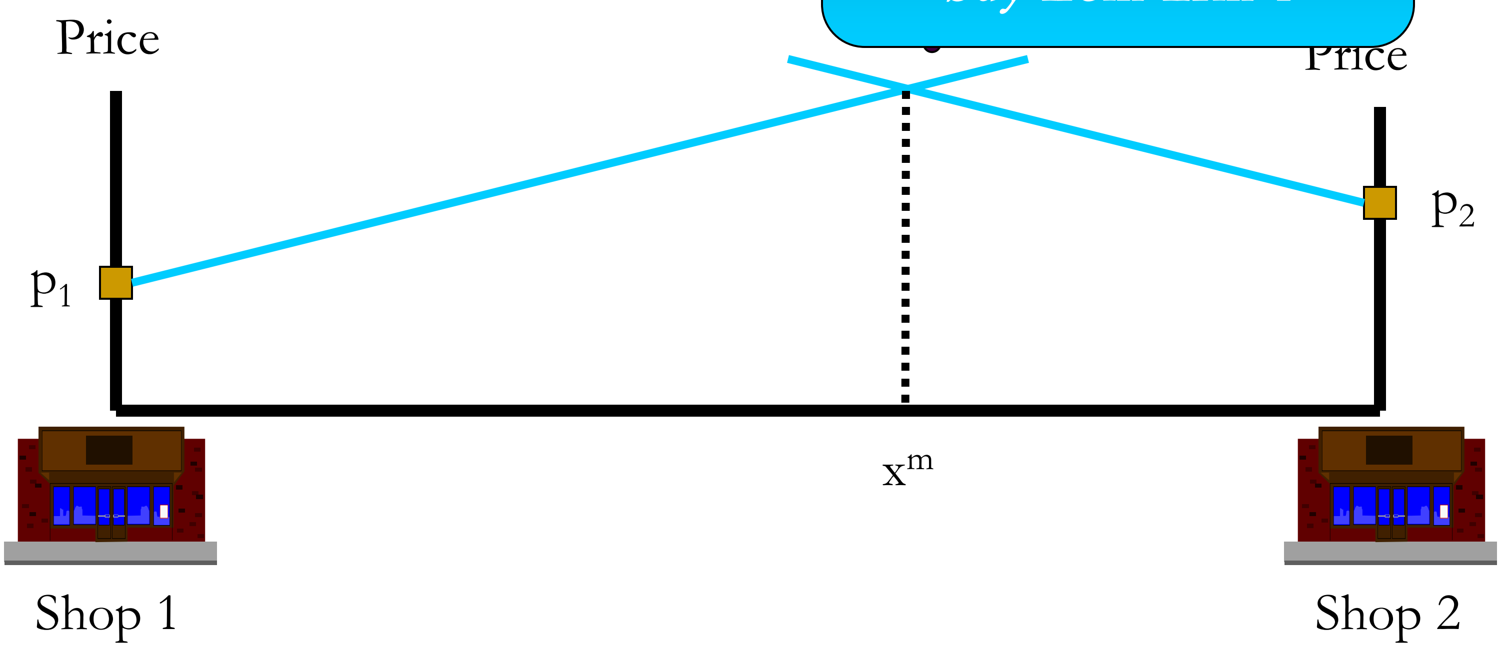
$\therefore x^m(p_1, p_2) = (p_2 - p_1 + t)/2t$

How is x^m

This is the fraction of consumers who buy from firm 1

There are N consumers in total

So demand to firm 1 is $D^1 = N(p_2 - p_1 + t)/2t$



Hotelling Model…

Profit to firm 1 is $\pi_1 = (p_1 - c)D^1 = N(p_1 - c)(p_2 - p_1 + t)/2t$

$\pi_1 = N(p_2p_1 - p_1^2 + tp_1 + cp_1 - cp_2 - ct)/2t$

Solve this for p_1

Differentiate with re

$\partial\pi_1/ \partial p_1 = \frac{N}{2t}$ (p

This is the best response function for firm 1

$p^*_1 = (p_2 + t + c)/2$

What about firm 2?

similar best respons on.

This is the best response function for firm 2

$p^*_2 = (p_1 + t + c)/2$

Hotelling Model…

Finding the Bertrand-Nash Eq.:

$p^*_1 = (p_2 + t + c)/2$

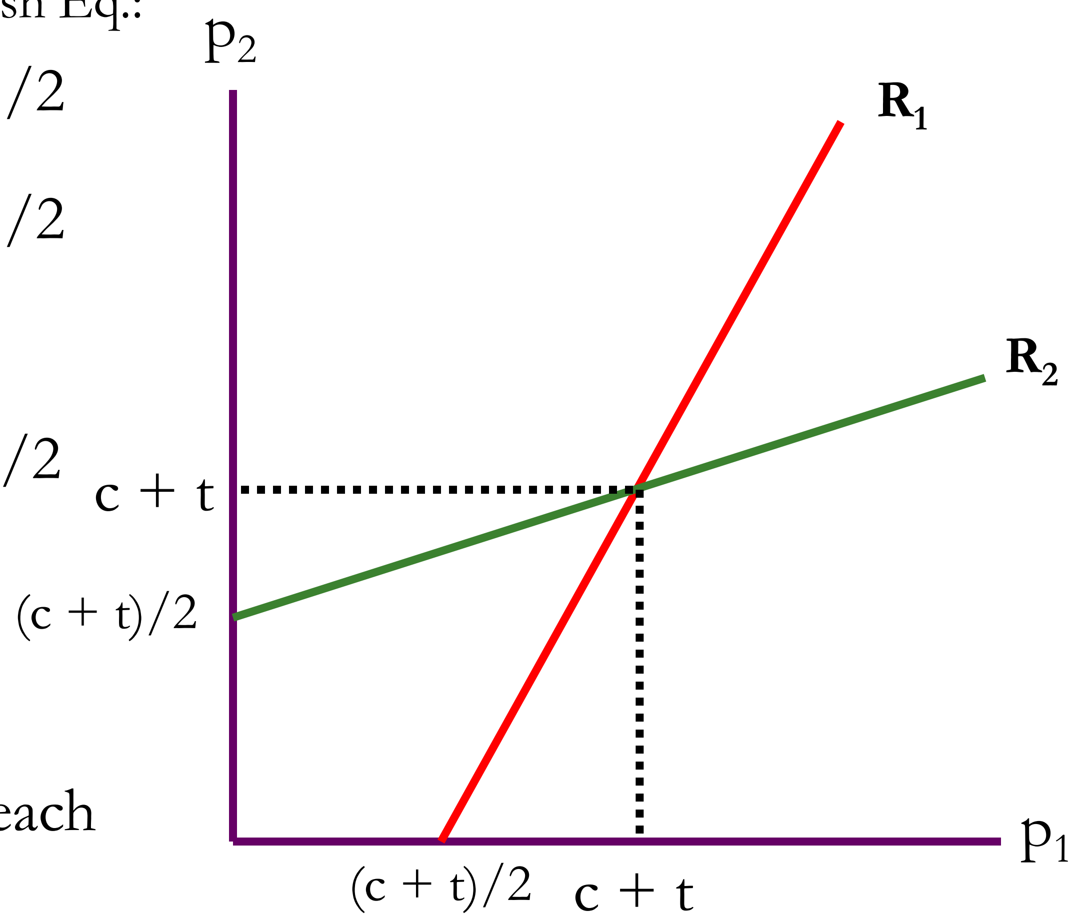
$p^*_2 = (p_1 + t + c)/2$

$2p^*_2 = p_1 + t + c$

$= p_2/2 + 3(t + c)/2$

$\therefore\ p^*_2 = t + c$

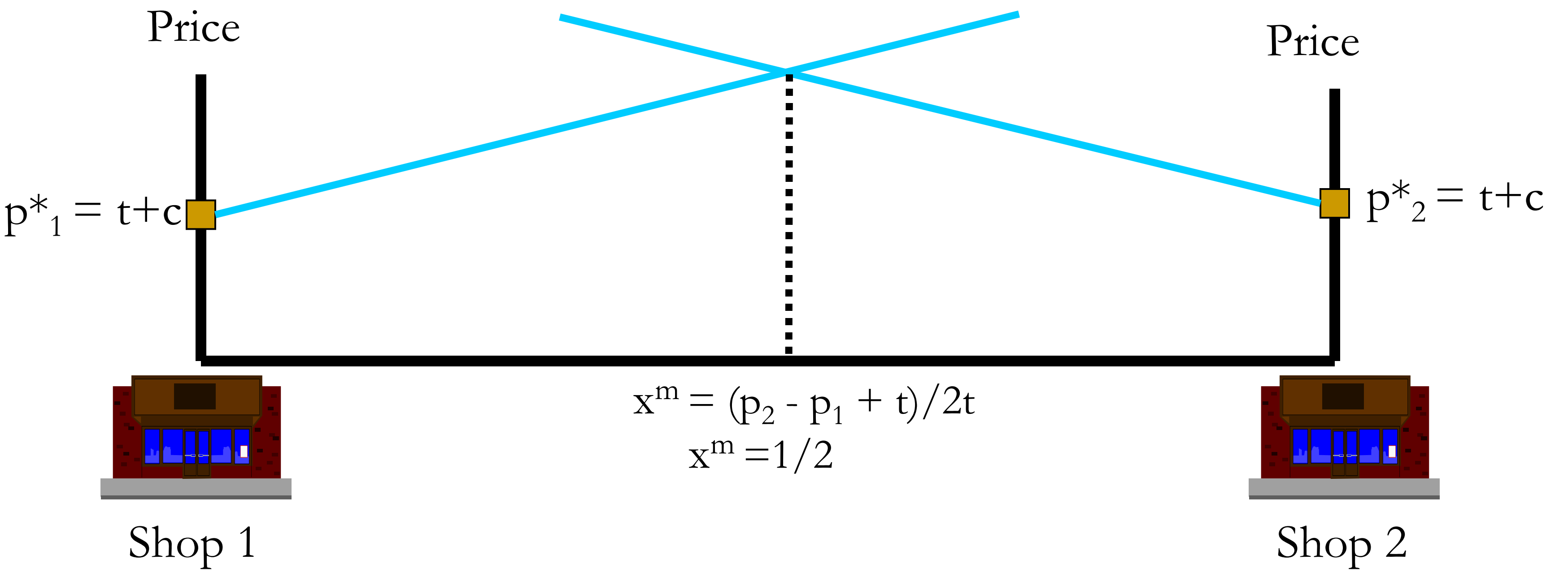
$\therefore\ p^*_1 = t + c$



Profit per unit to each firm is t

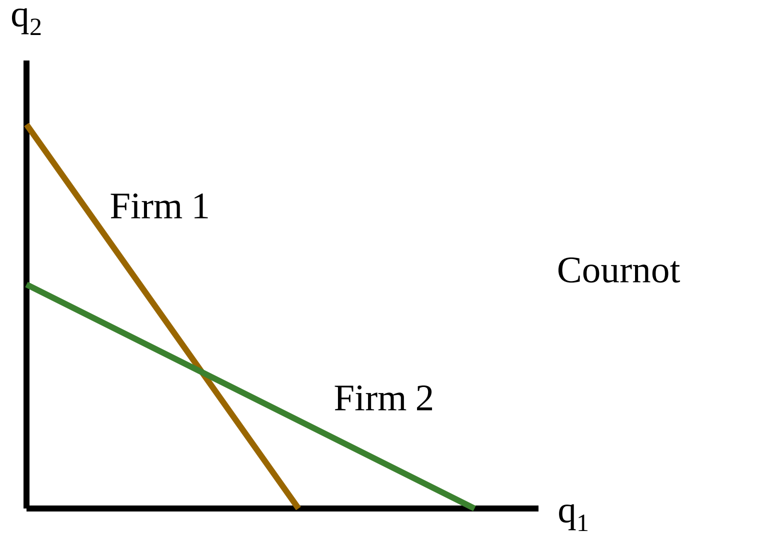
Aggregate profit to each firm is Nt/2

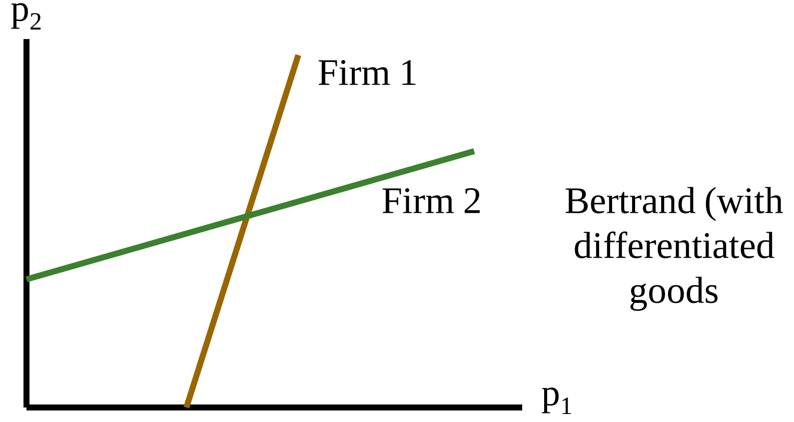
Hotelling Model…



Strategic Complements and Substitutes

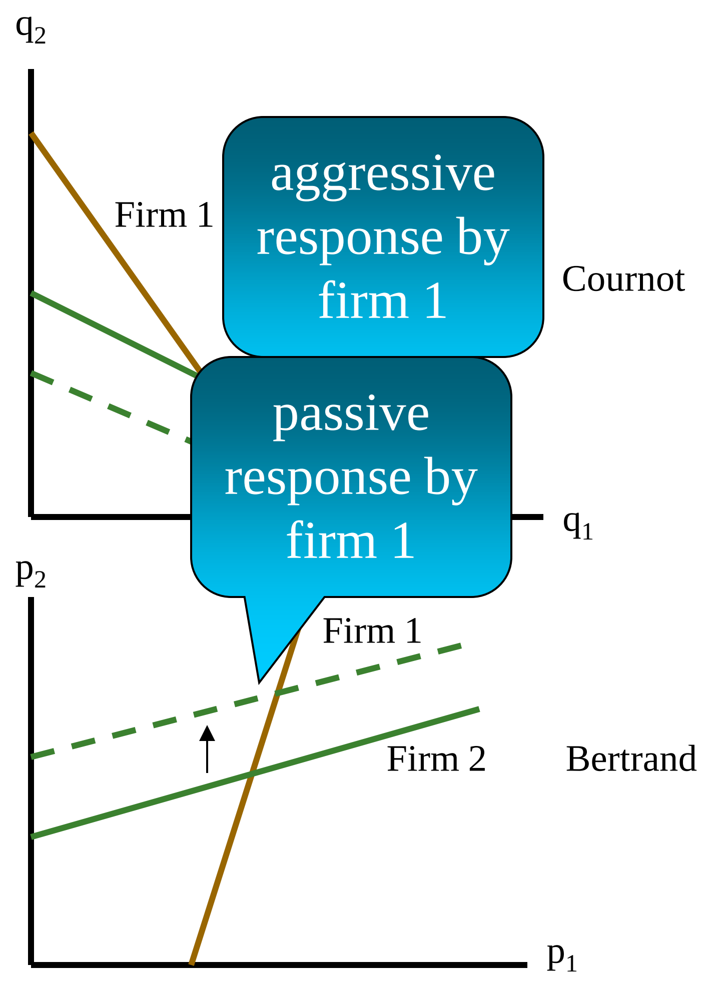
- Best response functions are very different between Cournot and Bertrand
  - they have opposite slopes
  - reflects very different forms of competition
  - firms react differently e.g. to an increase in costs



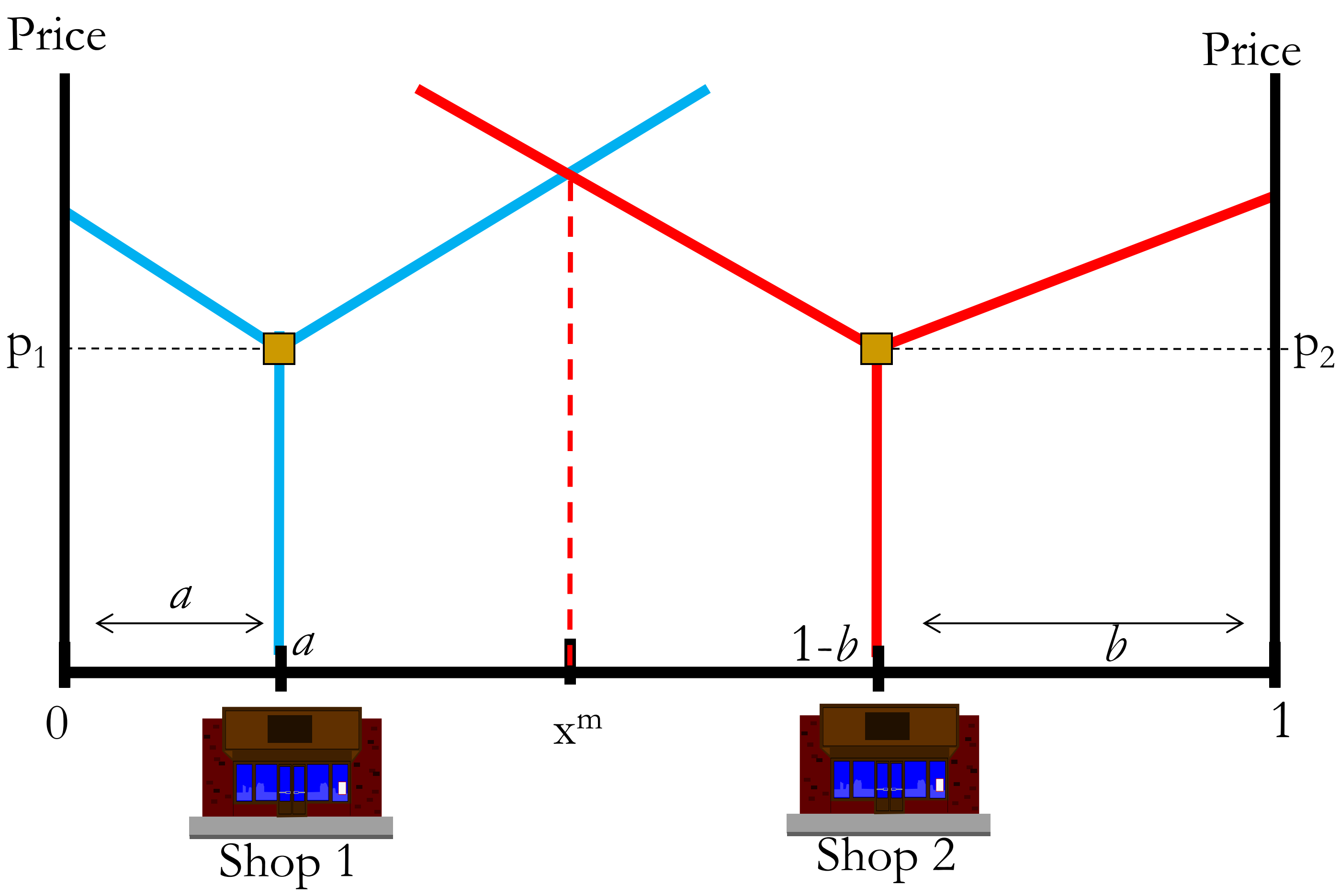


Strategic Complements and Substitutes…

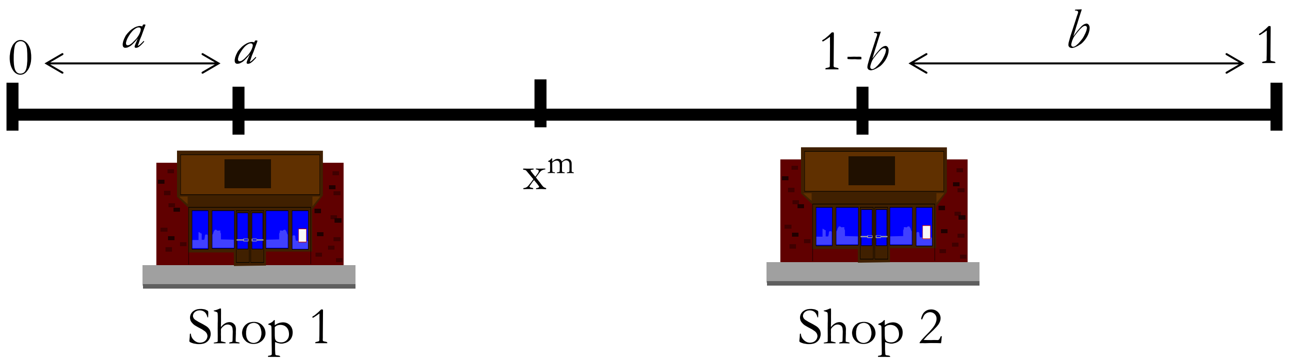
- Suppose firm 2's costs increase
- This causes Firm 2's Cournot best response function to fall
  - at any output for firm 1 firm 2 now wants to produce less
- Firm 1's output increases and firm 2's falls
- Firm 2's Bertrand best response function rises
  - at any price for firm 1 firm 2 now wants to raise its price
- firm 1's price increases as does firm 2's



Hotelling Model (continued)



Hotelling Model (continued)



$$U_x = \begin{cases} V - p_1 - t\left(x^m - a\right) & \text{if buys from shop 1} \\ V - p_2 - t\left(\left(1-b\right) - x^m\right) & \text{if buys from shop 2} \end{cases}$$

$$V - p_1 - t\left(x^m - a\right) = V - p_2 - t\left(\left(1-b\right) - x^m\right)$$

$$D^1 = Nx^m = N\left(\frac{p_2 - p_1}{2t} + \frac{1-b+a}{2}\right) \quad \text{Demand for Shop 1}$$

$$D^2 = N\left(1 - x^m\right) = N\left(\frac{p_1 - p_2}{2t} + \frac{1+b-a}{2}\right) \quad \text{Demand for Shop 2}$$

Hotelling Model…

Finding the Bertrand-Nash Eq.:

For simplicity assume N=1 and c=0

$$\pi_1 = D^1(p_1 - c) = N\left(\frac{p_2 - p_1}{2t} + \frac{1-b+a}{2}\right)(p_1 - c)$$

$$\pi_1 = D^1 p_1 = \left(\frac{p_2 - p_1}{2t} + \frac{1-b+a}{2}\right)p_1$$

$$\frac{\partial \pi_1}{\partial p_1} = \left(\frac{p_2 - 2p_1}{2t} + \frac{1-b+a}{2}\right) = 0$$

Similarly for firm 2, the first order condition for max can be derived as,

$$\pi_2 = D^2 p_2 = \left(\frac{p_1 - p_2}{2t} + \frac{1+b-a}{2}\right)p_2$$

$$\frac{\partial \pi_2}{\partial p_2} = \left(\frac{p_1 - 2p_2}{2t} + \frac{1+b-a}{2}\right) = 0$$

Hotelling Model…

Best response functions can be derived:

$$p^*_1 = \frac{t(1-b+a)}{2} + \frac{1}{2}p_2$$

$$p^*_2 = \frac{t(1+b-a)}{2} + \frac{1}{2}p_1$$

Bertrand Nash Equilibrium:

$$p^*_1 = \frac{t(3-b+a)}{3} \qquad D^1 = x^m = \frac{3-b+a}{6} \qquad \pi_1 = \frac{t(3-b+a)^2}{18}$$

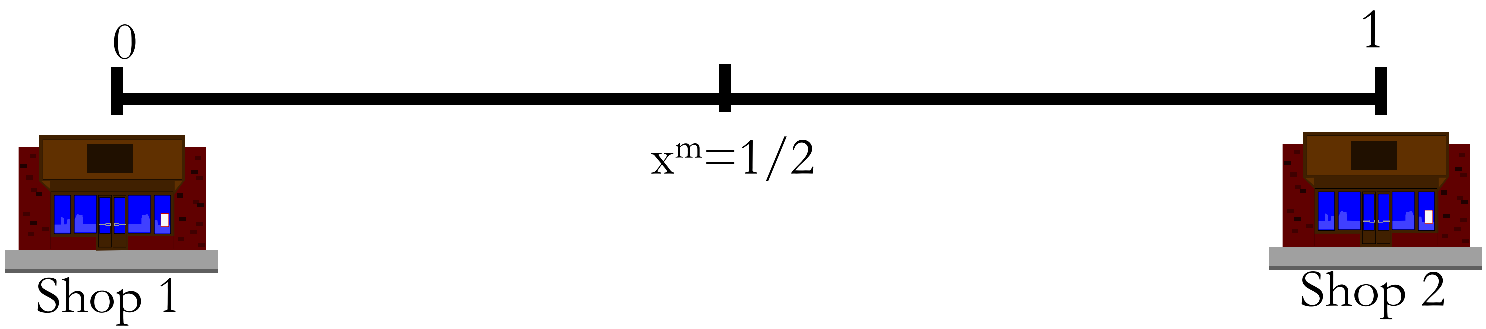
$$p^*_2 = \frac{t(3+b-a)}{3} \qquad D^2 = (1-x^m) = \frac{3+b-a}{6} \qquad \pi_2 = \frac{t(3+b-a)^2}{18}$$

Prices and profits increase with the transportation cost (t) → some degree of monopoly power.

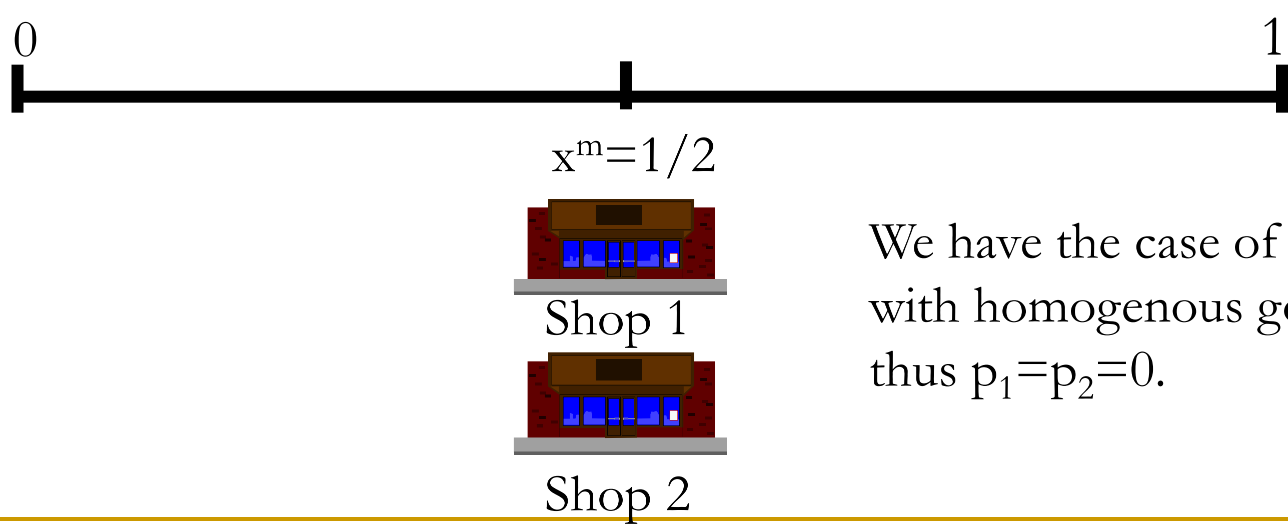
Prices and profits increase with the distance between firms (1-(*a*+b)).

Hotelling Model…

When firms are located at the **extreme ends (a=0 and b=0)**, **prices are highest** → our previous results.



When firms are located at the same location (a=1/2 and b=1/2)



We have the case of Bertrand with homogenous good, and thus $p_1=p_2=0$.

Hotelling Model…

- Two final points on this analysis
- t is a measure of transport costs
  - it is also a measure of the value consumers place on getting their most preferred variety
  - when t is large competition is softened
    - *and profit is increased*
  - when t is small competition is tougher
    - *and profit is decreased*
- **Locations have been taken as fixed → what happen when firms also choose <u>locations</u> in addition to <u>prices</u>?**

Hotelling Model…

- **If firms choose location first and then compete in prices.**

- Given the price and location of its opponent, firm 2, would firm 1 want to relocate?

$$\pi_1 = \frac{t(3-b+a)^2}{18} \quad \rightarrow \quad \frac{\partial \pi_1}{\partial a} = \frac{t(3-b+a)}{9} > 0$$

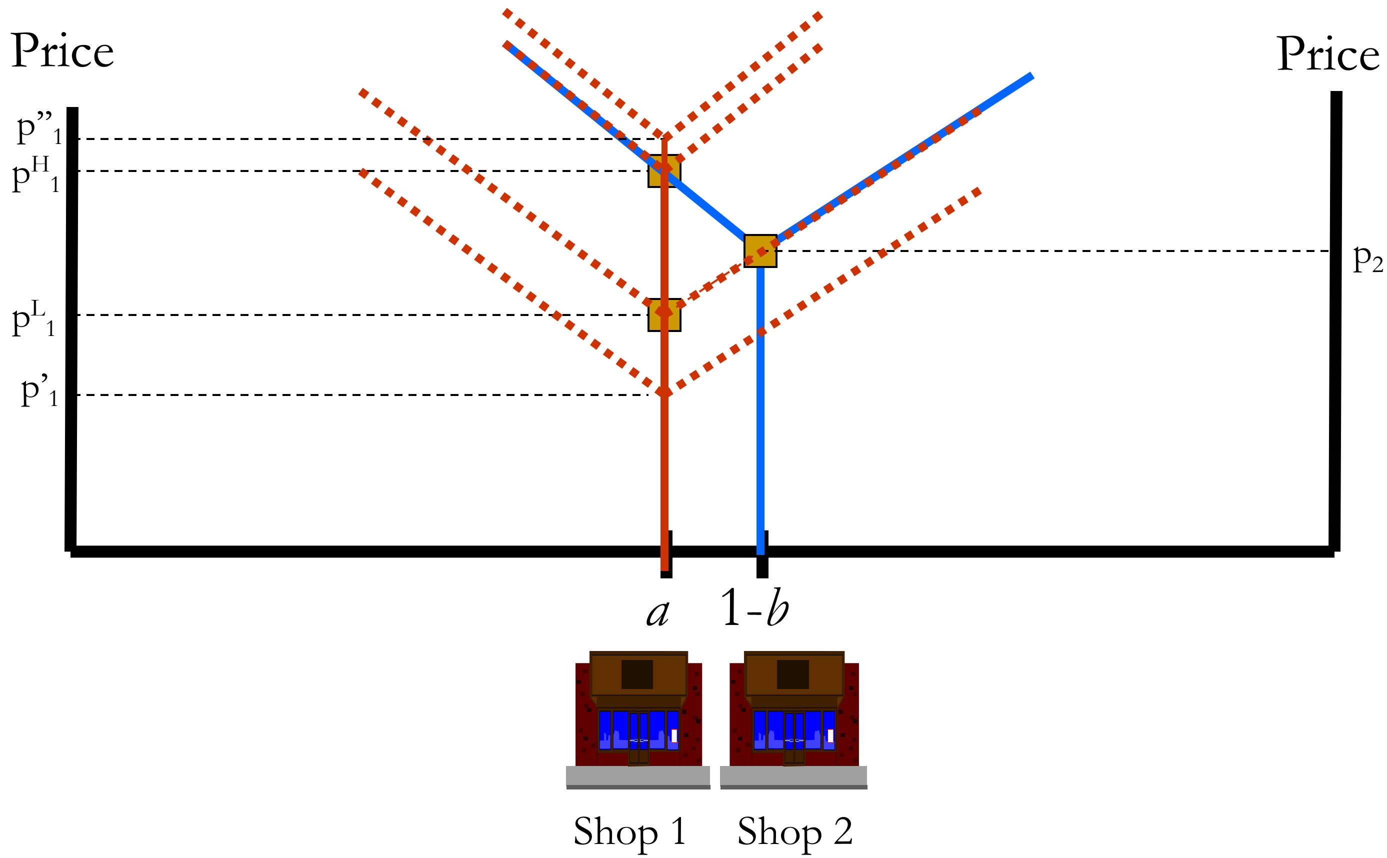
$$\pi_2 = \frac{t(3+b-a)^2}{18} \quad \rightarrow \quad \frac{\partial \pi_2}{\partial b} = \frac{t(3+b-a)}{9} > 0$$

- For any location b, firm 1 could increase its profit by moving closer to firm 2 (towards center) → similarly firm 2 will have the same intention.

- However, when they get **too close** to each others they become **less differentiated** → moving closer to Bertrand paradox → profits =0, so they want to avoid this → **better off to move back**.

- Thus, when firms choose both prices and locations → **non-existence of equilibrium** → the drawback of Hotelling's model.

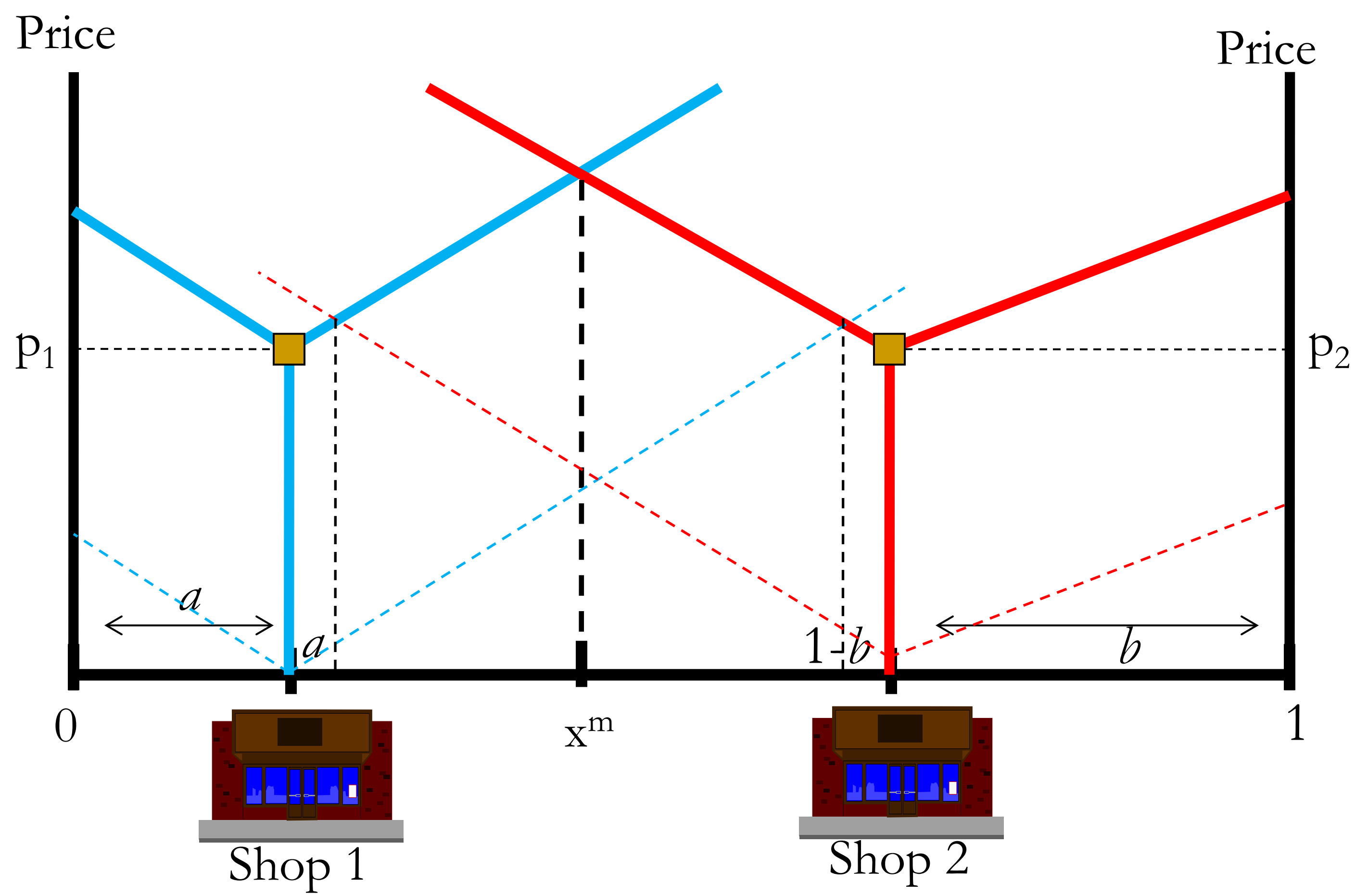
Hotelling Model…



Demand and profit functions are discontinuous → discontinuity in the best response fu. → no intersection → no pure strategy NE.

Hotelling Model…

When they are located sufficiently far from each other, the problem does not exist.

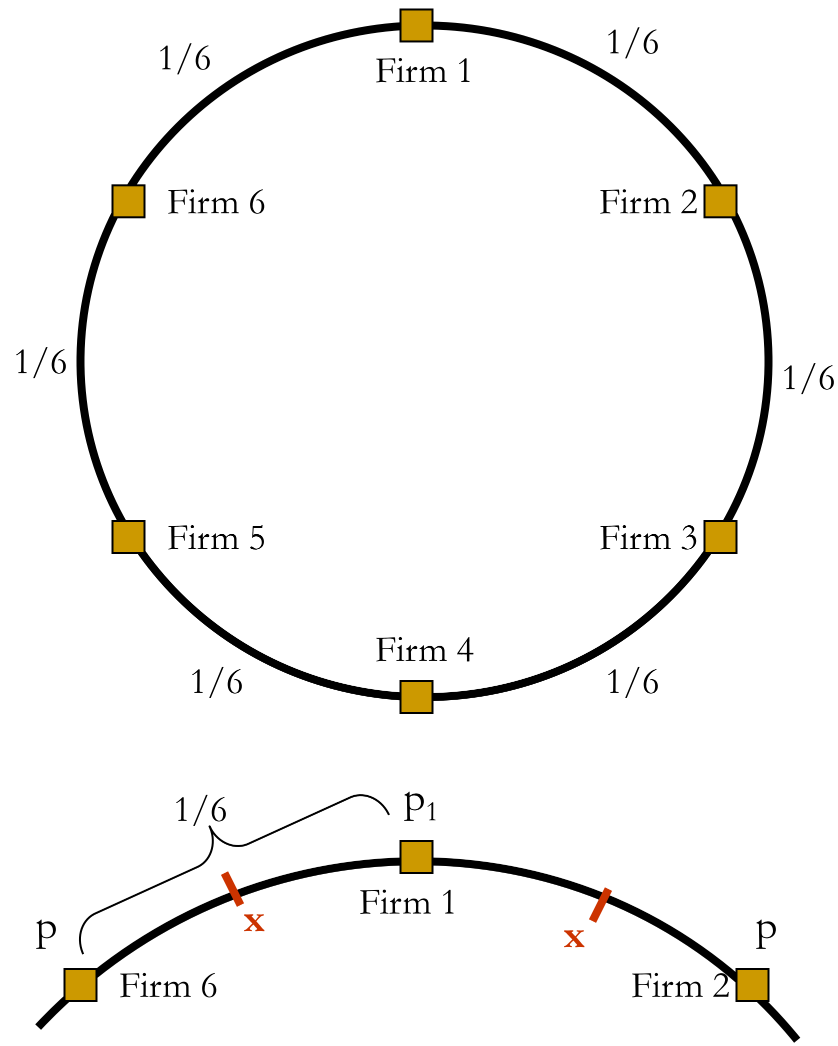


Salop's Circle Model

- To avoid the problem of non existence of equilibrium, Salop (1979) developed circle model which introduce 2 major changes to the Hotelling model.
  - Firms are located around a circle instead of a long line.
  - Consideration of an **outside (second) good**, which is **undifferentiated** and **competitively** supplied.

- **Firms**
  - Firms are located around a circle (circumference=1) with equal distance (1/N) from each other.
  - Fixed cost f, and marginal cost, c.
  - Profit: $\pi_i(q_i) = (p_i - c)q_i - f$

Salop's Circle Model (example: N=6)



Salop's Circle Model…

- **<u>Consumers</u>**
  - **Uniformly located around the circle** (e.g. round the clock airline, bus, and train services, etc).
  - A consumer's location x* represents the consumer's most preferred type of product.
  - Each consumer buys one unit
  - Transportation cost per unit of distance = t.
  - Given the price, **p**, charged by the **adjacent firms** (**left** and **right**) and $\mathbf{p_1}$ charged by **firm 1**, we can derive the location of the **indifferent consumer** located at the distance $x \in (0, 1/N)$.

$$V - p_1 - tx = V - p - t\left(\frac{1}{N} - x\right)$$

$$D^1(p_1, p) = 2x = \frac{p - p_1}{t} + \frac{1}{N}$$

Firm 1's market share (demand)

Salop's Circle Model…

Therefore:

$$\pi_1 = (p_1 - c)\left(\frac{p - p_1}{t} + \frac{1}{N}\right) - f$$

$$\frac{\partial \pi_1}{\partial p_1} = 0 \quad \rightarrow \quad p_1 = \frac{t}{2N} + \frac{p + c}{2}$$

By symmetry, we have p_1=p, and thus,

$$p = c + \frac{t}{N} \quad \rightarrow \quad (p - c) = \frac{t}{N}$$

Similar as in the Hotelling model, **price & profit margin increases with transportation cost t and decrease with N**.

Suppose that entry by new firms is possible (**free-entry**) → entry will take place until profit is fully dissipated.

$$\pi_i = (p - c)\frac{1}{N} - f = \frac{t}{N^2} - f = 0$$

$$N^c = \sqrt{\frac{t}{f}} \quad \text{and} \quad p^c = c + \sqrt{tf}$$

Firm's price is above MC, but yet it earns no profit.

Salop's Circle Model…

- Under free-entry, an increase in fixed cost (f) cause a decrease in the number of firm (N) and an increase in the profit margin (p-c).

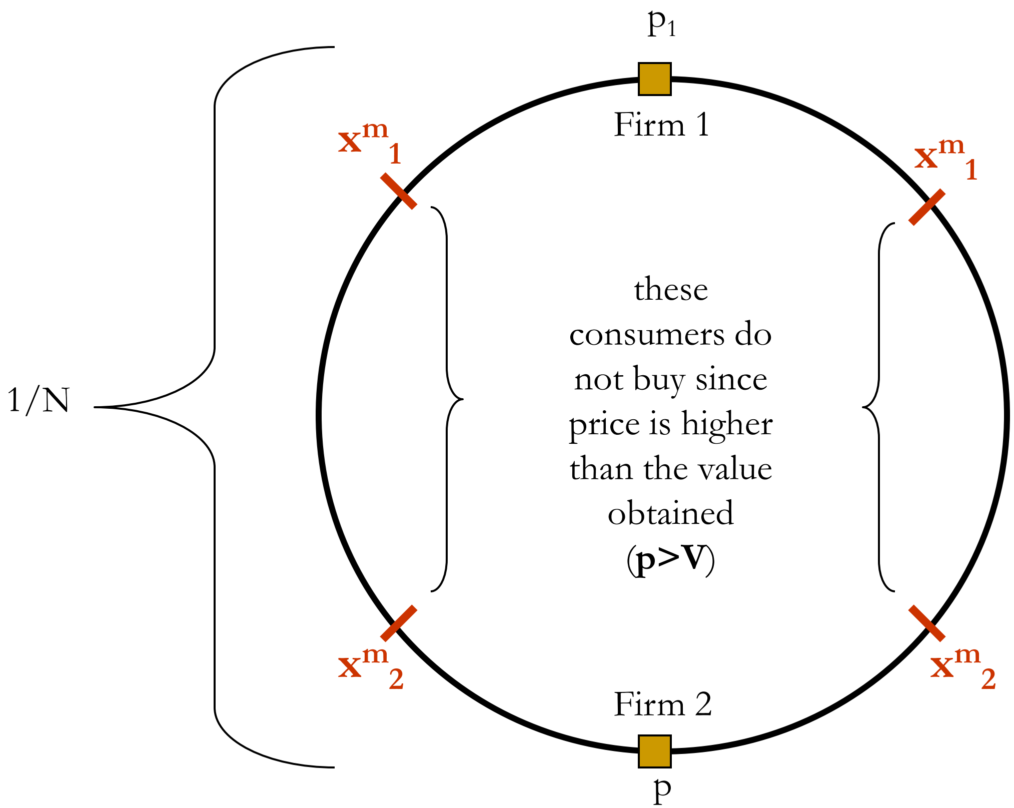
- Under free-entry, an increase in transportation cost (t) increases both profit margin (p-c) and the number of firms (N).

- When fixed cost (f) falls to zero (0), the number of firms tends to be very large (N→ infinity).

$$N^c = \lim_{f \to 0} \sqrt{\frac{t}{f}} = \infty$$

- So far, we have been discussing the case in which **firms are located sufficiently close to each other** and **compete for the same consumers** → a firm must take into account the price of rivals → **competitive region** (Salop 1979).
- If there are only **few firms** such that they **don't compete for the same consumers** → each firm is a **local monopoly**.

Salop's Circle Model…



market is uncovered.

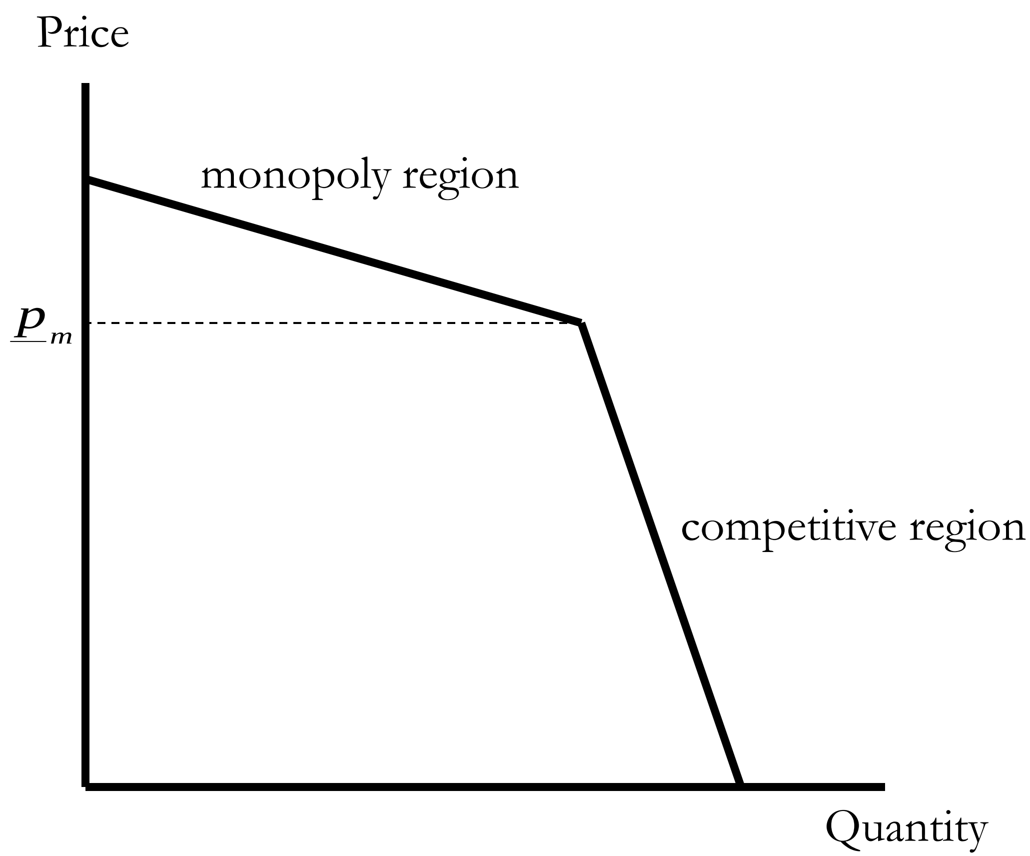
Indifferent consumer between **buying** and **not buying**:

$$V - p_1 - tx = 0 \qquad x = \frac{V - p_1}{c}$$

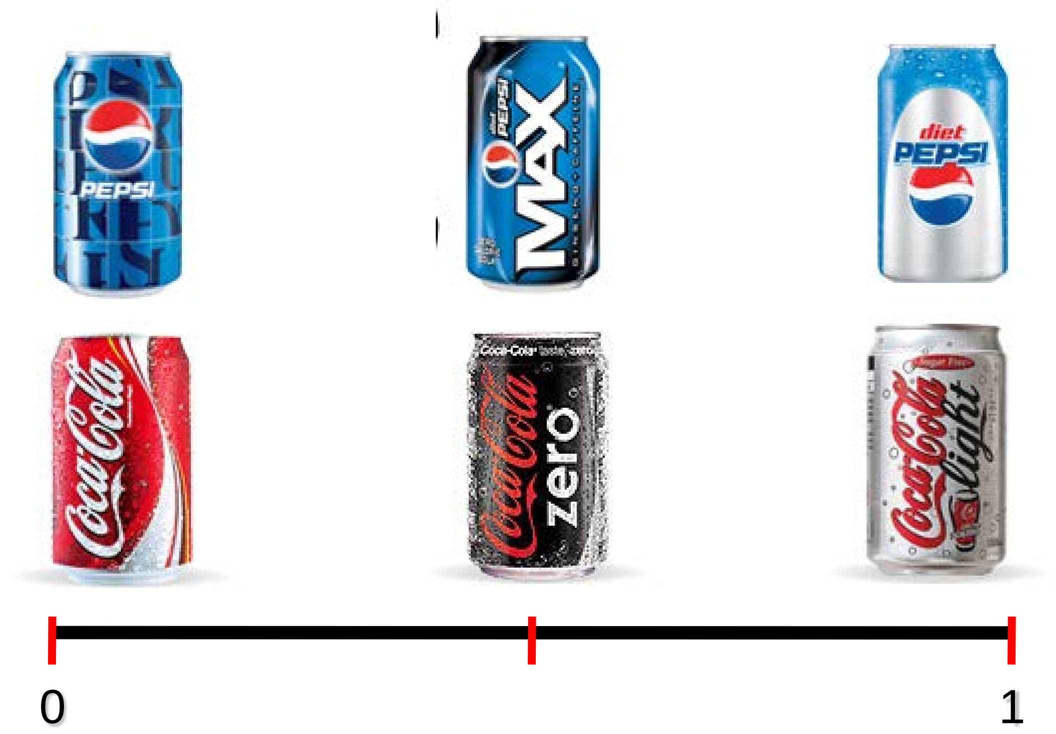
$$D^1(p_1) = 2x = \frac{2}{c}(V - p_1)$$ monopoly demand

Salop's Circle Model…

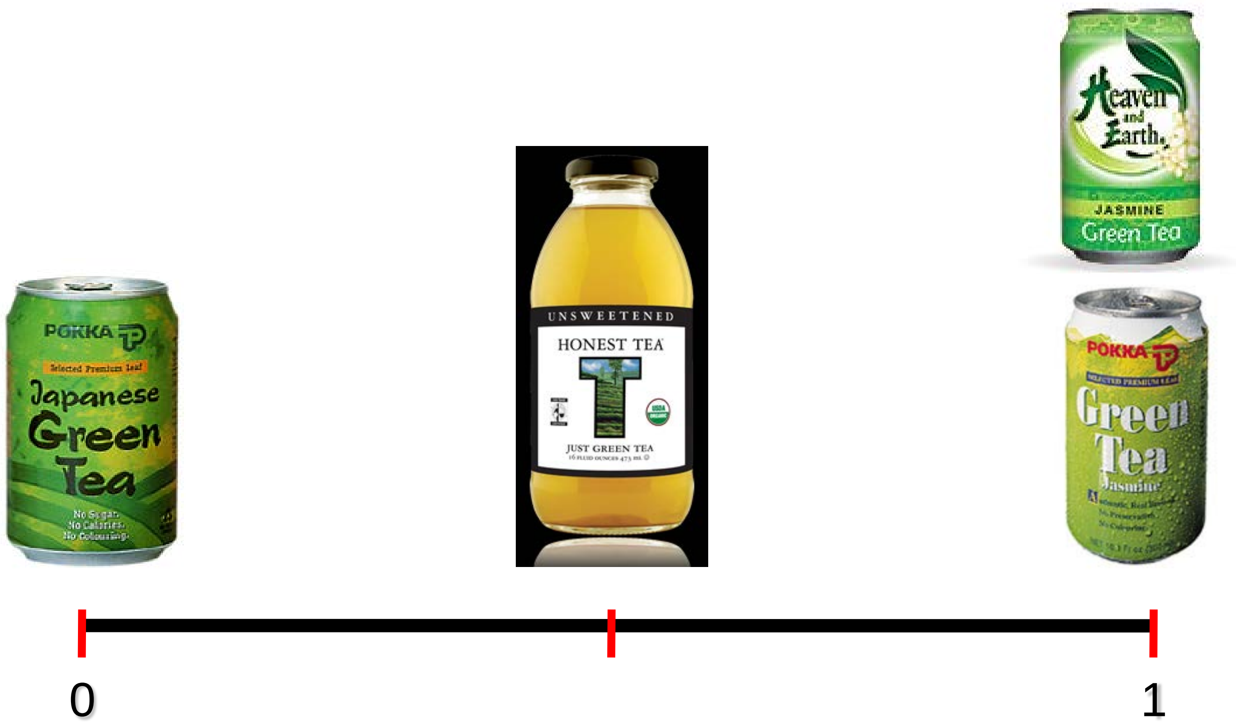
Demand in Salop's Circle Model



Example (Horizontal Prod. Diff.)



Example (Horizontal Prod. Diff.)



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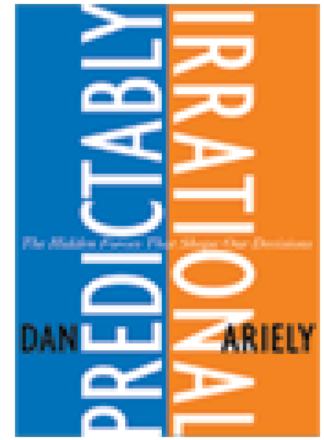
Types of Subscription Plan	**Number of Students**	**%**
Economist.com Subscription US$59	10	**43.5%**
Print Subscription US$125	1	**4.3%**
Print & Web Subscription US$125	12	**52.2%**
TOTAL	23	**100%**

DW 2 Tutorial Group		
Types of Subscription Plan	**Number of Students**	**%**
Economist.com Subscription US$ 59	14	**77.8%**
Print & Web Subscription US $125	4	**22.2%**
TOTAL	18	**100%**



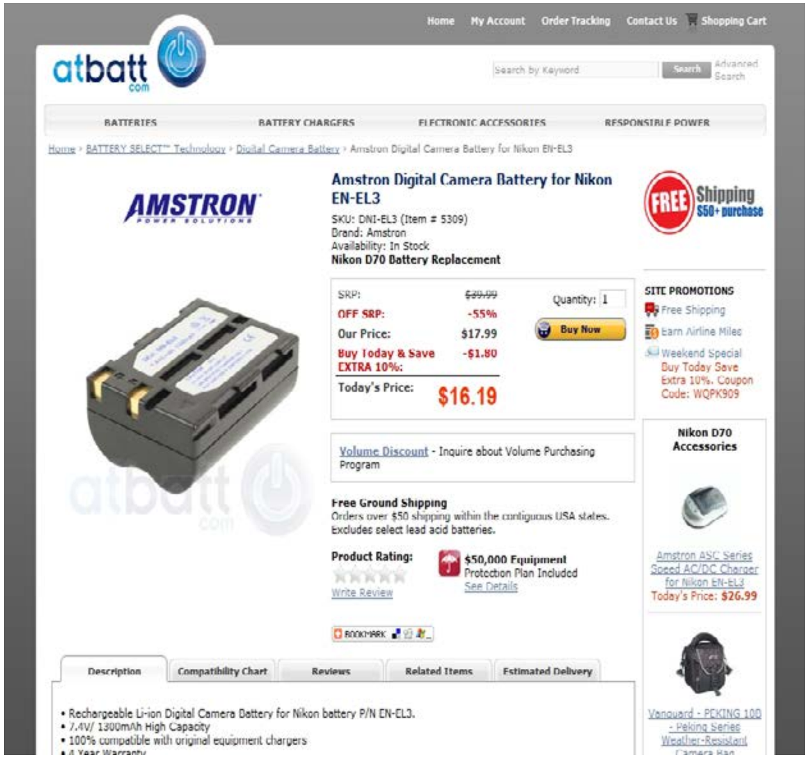
Suppose you are a salesman and puts the following display:

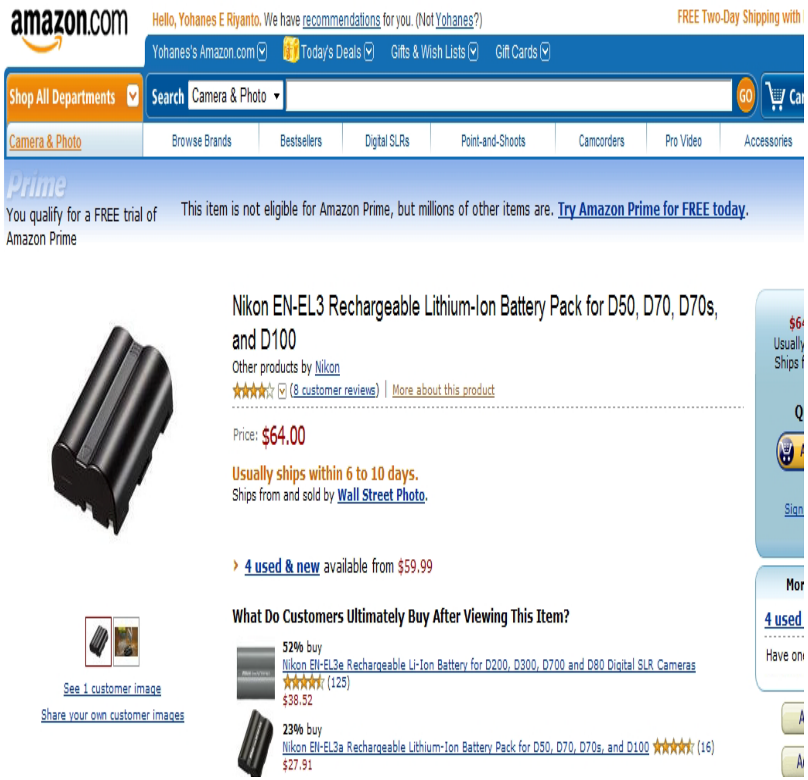
36-inch Panasonic → $690

42-inch Toshiba → $850

50-inch Phillips → $1480

Example (Vertical Prod. Diff.)



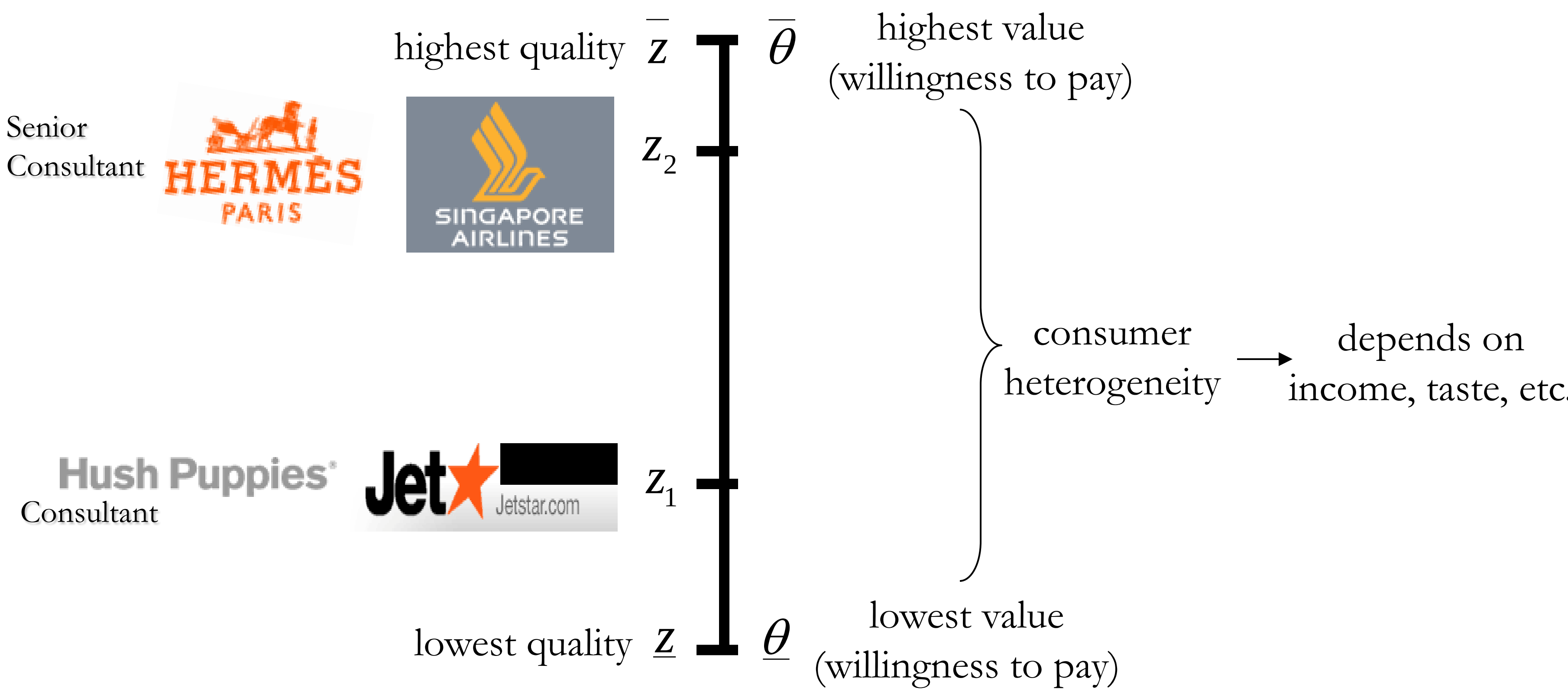


Vertical Product Differentiation

- Under **vertical differentiation** → consumers agree that there are **quality differences** among products → and they have **different willingness to pay for different quality.**

- <u>**Setup**</u>:
  - Each consumer buys one unit provided that $p \leq V$ (there is non-negative surplus).
  - The product (brand) produced by a firm is characterized by a quality index, $z \in \left(\underline{z}, \overline{z}\right)$
  - There are 2 firms, *i*=1,2, produces a good with quality respectively $\mathbf{z_1}$ and $\mathbf{z_2}$ with $\mathbf{z_2} > \mathbf{z_1}$. The unit cost of production is the same for both qualities, **c**.
  - There is a continuum of consumers with measure N whose preference for quality (θ) is uniformly distributed on the quality interval $\theta \in \left(\underline{\theta}, \overline{\theta}\right)$.

Vertical Product Differentiation…

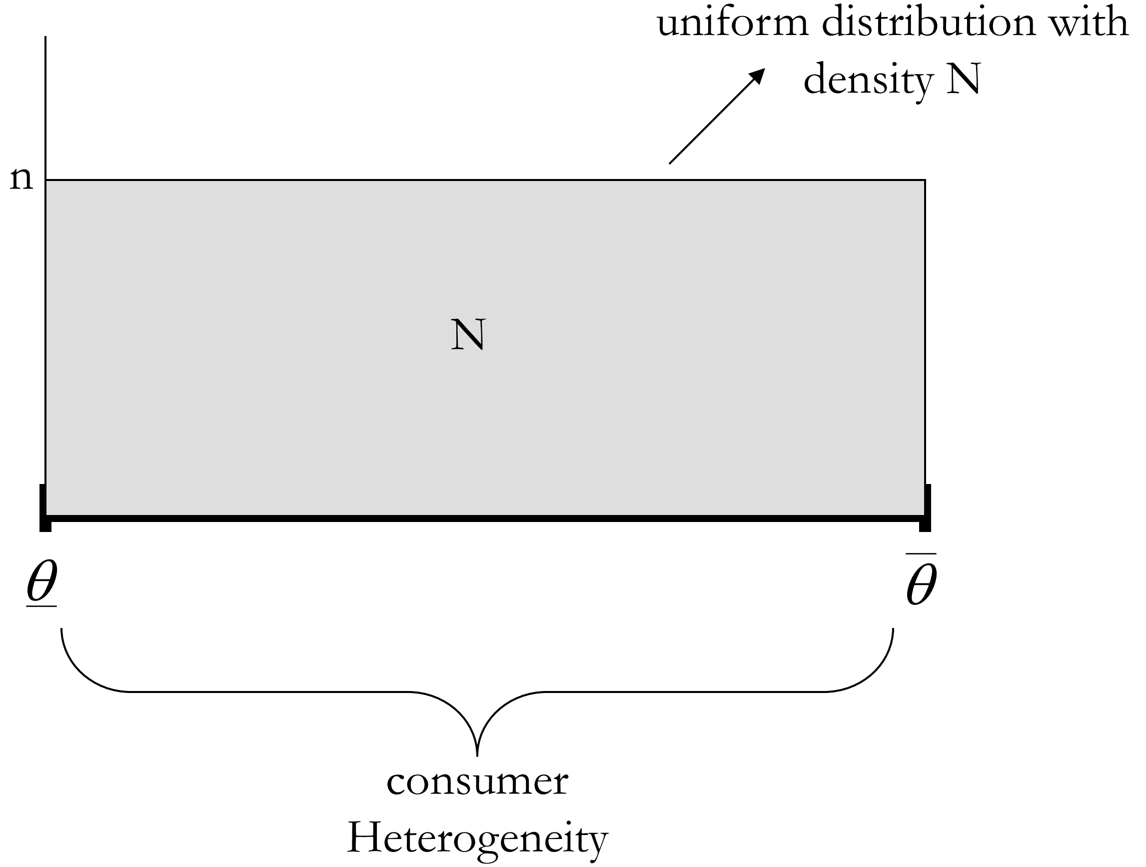


consumer taste/ preference

&

the range of quality

Vertical Product Differentiation…



Vertical Product Differentiation…

- **Setup**:
  - Denote the quality differential between the products of the two firms as $\Delta_z \equiv z_2 - z_1$
  - Both firms engage in price competition.
  - The utility of a consumer from buying a brand,

$$U = \begin{cases} \theta z_i - p & \text{if she buys a brand with quality } z_i \\ 0 & \text{if she does not buy} \end{cases}$$

  - We are going to assume that the whole market is "**covered**" (consumers will always buy a product).

$$U = \theta z_i - p \geq 0 \quad \text{or} \quad p \leq \theta z_i$$

  - High θ consumers buy the high quality good z_2, and low θ consumers buy the low quality good z_1 (which must be priced lower to attract any consumers).

Vertical Product Differentiation…

- **Setup**:
  - A consumer with taste (preference) θ is indifferent between buying the high quality and the low quality good if:

$$\theta z_2 - p_2 = \theta z_1 - p_1$$

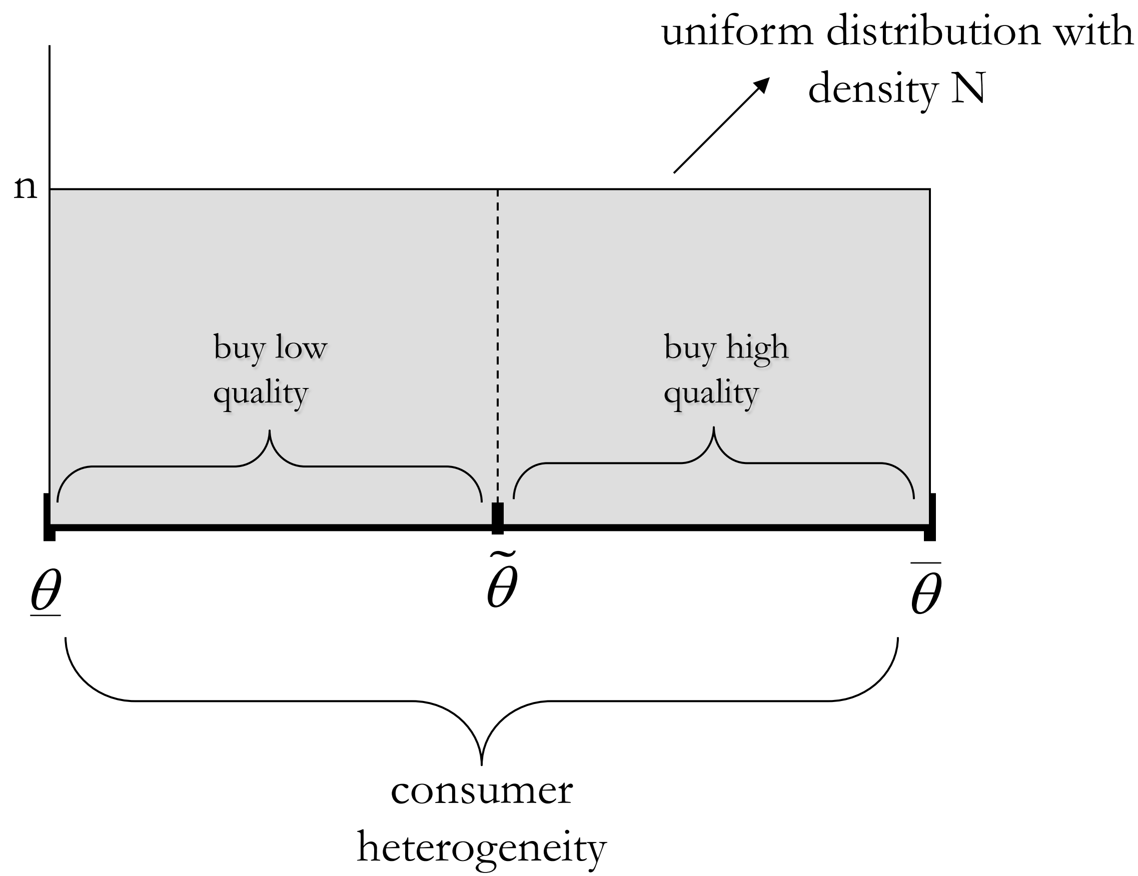
$$\tilde{\theta} = \frac{(p_2 - p_1)}{(z_2 - z_1)} = \frac{(p_2 - p_1)}{\Delta_z}$$

  - This implies that all consumers with taste in the interval of $\theta \in \left(\underline{\theta}, \tilde{\theta}\right)$ will buy the **low quality** good from **firm 1**. While consumers with taste in the interval of $\theta \in \left(\tilde{\theta}, \overline{\theta}\right)$ will buy the **high quality** good from **firm 2**.
  - The **demand functions for both firms** can be derived:

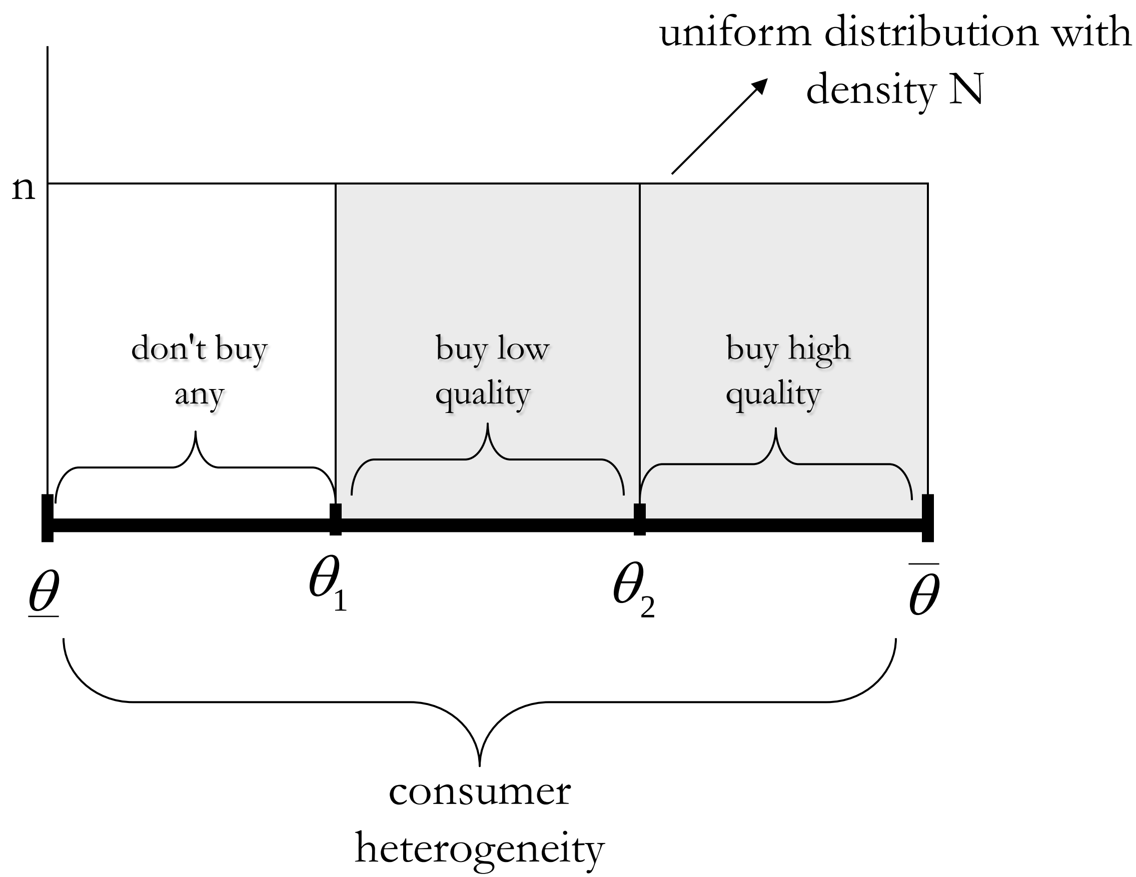
$$D^1(p_1, p_2) = N\left(\left(\frac{p_2 - p_1}{\Delta_z}\right) - \underline{\theta}\right)$$

$$D^2(p_1, p_2) = N\left(\overline{\theta} - \frac{p_2 - p_1}{\Delta_z}\right)$$

Vertical Product Differentiation…



Vertical Product Differentiation…



Vertical Product Differentiation…

- Each firm will maximize its profit.

$$\pi_1 = (p_1 - c) D^1 (p_1, p_2) = (p_1 - c) N \left(\frac{(p_2 - p_1)}{\Delta_z} - \underline{\theta} \right)$$

$$\pi_2 = (p_2 - c) D^2 (p_1, p_2) = (p_2 - c) N \left(\overline{\theta} - \frac{p_2 - p_1}{\Delta_z} \right)$$

$$\frac{\partial \pi_1}{\partial p_1} = \frac{N (c + p_2 - 2 p_1 - \underline{\theta} \Delta_z)}{\Delta_z} = 0$$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{N \left(c + p_1 - 2 p_2 + \overline{\theta} \Delta_z \right)}{\Delta_z} = 0$$

$$p_1 = \frac{c - \underline{\theta} \Delta_z}{2} + \frac{1}{2} p_2 \qquad p_1 = c + \frac{\left(\overline{\theta} - 2\underline{\theta}\right)}{3} \Delta_z = c + \frac{\left(\overline{\theta} - 2\underline{\theta}\right)}{3} (z_2 - z_1)$$

$$p_2 = \frac{c + \overline{\theta} \Delta_z}{2} + \frac{1}{2} p_1 \qquad p_2 = c + \frac{\left(2\overline{\theta} - \underline{\theta}\right)}{3} \Delta_z = c + \frac{\left(2\overline{\theta} - \underline{\theta}\right)}{3} (z_2 - z_1) > p_1$$

$$D^1 = \left(\frac{\overline{\theta} - 2\underline{\theta}}{3} \right) N \qquad \pi_1 = \left(\frac{\left(\overline{\theta} - 2\underline{\theta}\right)^2}{9} \Delta_z \right) N$$

$$D^2 = \left(\frac{2\overline{\theta} - \underline{\theta}}{3} \right) N > D^1 \qquad \pi_2 = \left(\frac{\left(2\overline{\theta} - \underline{\theta}\right)^2}{9} \Delta_z \right) N > \pi_1$$

Vertical Product Differentiation…

- Recall that the condition for consumers to buy any product is:

$$U = \theta z_i - p \geq 0 \qquad \text{or} \qquad p \leq \theta z_i$$

- Thus, to ensure that the market is covered, all consumers will always buy:

$$p_1 \leq \underline{\theta} z_1$$

$$c + \frac{(\overline{\theta} - 2\underline{\theta})}{3}(z_2 - z_1) \leq \underline{\theta} z_1$$

- We require that consumers are sufficiently heterogeneous in taste

$$\overline{\theta} \geq 2\underline{\theta} \quad \text{so that } D^1 = \left(\frac{\overline{\theta} - 2\underline{\theta}}{3}\right) N > 0$$

$$p_1 = c + \frac{(\overline{\theta} - 2\underline{\theta})}{3}(z_2 - z_1)$$

$$p_2 = c + \frac{(2\overline{\theta} - \underline{\theta})}{3}(z_2 - z_1)$$

- **Summary:**
  - **Undifferentiated firms (z_1=z_2)** will charge $p_1 = p_2 = c$ and make **no profit**.
  - **Profits are increasing in the quality differential (z_2-z_1)**.
  - What happen when **quality choice is endogenous**?

Vertical Product Differentiation…

- **Endogenous choice of quality**.
  - Suppose now, firms play a **two-stage game**, in which firms first choose quality (one per firm) and then compete in price.
  - Assume for simplicity that the choice of quality is costless. Firms choose z_i from the interval $z_i \in \left(\underline{z}, \overline{z}\right)$
  - Since the size of profit depends on $(z_2 - z_1)$, for a given z_2 firm 1 wants to choose z_1 as lowest as possible ($z_1 = \underline{z}$). For a given z_1, firm 2 wants to set z_2 is high as possible ($z_2 = \overline{z}$).
  - A similar result is obtained if instead firm 1 is the one which produces high quality.
  - There are **two pure strategy Nash equilibria,** the first one is **(**$z_1 = \underline{z}$, $z_2 = \overline{z}$ **)**, the second one is **(**$z_1 = \overline{z}$, $z_2 = \underline{z}$ **).**
  - In equilibrium, we have **maximal differentiation → relaxing price competition** through product differentiation.

Vertical Product Differentiation…

- **Endogenous choice of quality**.
  - In a **Stackelberg setting**, the first mover always wants to enter with high quality, and given this the second mover will choose low quality → this gives us a **unique equilibrium**.

We have a setting in which choice of **quality is costless →** and yet, the low quality firm **gains from reducing its quality level to the minimum →** **trade-off**: **demand reduction** because of lower quality vs. **softer price competition**.