A Mixed-Integer Distributionally Robust Chance-Constrained Model for Optimal Topology Control in Power Grids with Uncertain Renewables

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***Abstract*—This paper proposes a distributionally robust chance-constrained (DRCC) optimization model for optimal topology control in power grids overwhelmed with significant renewable uncertainties. A novel moment-based ambiguity set is characterized to capture the renewable uncertainties with no knowledge on the probability distributions of the random parameters. A distributionally robust optimization (DRO) formulation is proposed to guarantee the robustness of the network topology control plans against all uncertainty distributions defined within the moment-based ambiguity set. The proposed model minimizes the system operation cost by co-optimizing dispatch of the lower-cost generating units and network topology—i.e., dynamically harnessing the way how electricity flows through the system. In order to solve the problem, the DRCC problem are reformulated into a tractable mixed-integer second order cone programming problem (MISOCP) which can be efficiently solved by off-the-shelf solvers. Numerical results on the IEEE 118-bus test system verify the effectiveness of the proposed network reconfiguration methodology under uncertainties.**

***Index Terms*—Chance-constrained programming; distributionally robust optimization (DRO); optimal topology control; renewable; uncertainty characterization.**

I. Introduction

Electric power transmission system has been traditionally designed, planned, and operated with a fixed topological configuration, characterizing its infrastructure as static assets. The network topology changes only in the cases of (i) faults and forced outages, (ii) maintenance operations on transmission lines, (iii) seasonal switching actions, and (iv) other operator-driven ad hoc circumstances. Power grid topology control through transmission line switching harnesses the network built-in flexibility by temporarily removing lines out of service and changing the way how electricity flows through the grid. Topological reconfiguration of the transmission grid brings about potentials for a cost-effective operation and higher economic benefits by enabling dispatch of the lower-cost generating units along with a dynamically-routed electricity flow [1]. Utilizing the network existing infrastructure with minimum additional cost, network topology control has been proven to be effective in emergency scenarios as well primarily for enhancing the network reliability and resilience against a sheer number of possible disruptions [2]–[5].

The increasing proliferation of uncertainties in power grids—either through intermittent renewable generation portfolios or stochastic load profiles—has introduced significant challenges to today's grid operation and control paradigms. Such uncertainties, if not properly modeled and accounted for, may change the underlying principles of the network topology control optimization mechanisms and, at times, may render this technology inefficient. Different approaches have been proposed to tackle the grid uncertainties through characterization, modeling, and uncertainty-driven formulations: some are centered on deterministic assumptions for uncertain parameters and may not result in accurate and reliable solutions [1], [6] and some employ scenario-based stochastic programming (SBSP) approaches [7]–[10] where the system uncertainties are characterized through a finite number of scenarios. The SBSP approach has been criticized in recent studies to be computationally demanding as it requires a large number of scenarios to precisely characterize the uncertainties [11]–[14].

Robust optimization (RO) is another approach in which a suitable uncertainty space is defined for stochastic variables through which the optimal solution can be found in the *worst-case* uncertainty scenario [15]–[17]. While principally not dependent to detailed knowledge of the probability distributions for uncertain variables (in contrast to the probabilistic approaches) and computationally more attractive than the previous alternatives (e.g., the SBSP approach), it often leads to *over-conservative* solutions as it cannot effectively tackle the degree of conservatism. Besides stochastic and robust optimization techniques, distributionally robust optimization (DRO) is an intermediate generalized variant of the classical RO frameworks [18]–[20], which treats the uncertain parameters as random variables with an *unknown* probability distribution. Different from the stochastic optimization, DRO realistically accounts for all possible uncertainty distributions according to the available uncertainty information (e.g., statistical moments). Indeed, DRO assumes that the true distribution lies in an ambiguity set and immunizes the operation strategies against all distributions within the ambiguity set.

In this paper, power grid uncertainties (e.g., those driven by the intermittent wind generation) are formulated and incorporated in the optimal topology control optimization through a distributionally robust chance constrained (DRCC) programming approach, which is a powerful technique for risk-informed decision making under uncertainty [21]. The

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uncertainties are characterized through a number of distributionally robust (DR) chance constraints, ensuring that the constraints subject to uncertainty will be satisfied with a certain probability level prescribed by the decision maker. The only challenge in dealing with the DRCC approach is to reformulate the implicit DR chance constraints into explicit constraints. In most of the existing research works (e.g., [22], [23]), this reformulation is accomplished assuming that the random variables involved in chance constraints (CCs) are Gaussian distributed. In practice, however, this assumption is quite unrealistic. Some others (e.g., [24], [25]) do not assume any specific PDF for random variables, but propose and employ approximate—i.e., not exact—reformulations of CCs, which may adversely affect the reliability and dependability of the chance-constrained programming (CCP) approach. We first construct a moment-based ambiguity set which captures all PDFs with the first two moments lying within its confidence intervals. This ambiguity set is then utilized to derive the DR variants of CCs, based on which, the chance constraint can be processed with DRO. The optimal topology control problem is eventually reformulated as a tractable mixed-integer second-order cone programming (MISOCP) problem that is efficiently solved using off-the-shelf optimization tools.

The proposed approach driven by the DRCC model (i) offers an attractive computational demand, which in turn, enables its application in large-scale systems with high-dimensional uncertainties, (ii) requires limited knowledge on uncertainty distributions of the random variables, (iii) immunizes the solution of the topology control problem against all the uncertainty distributions defined within a moment-based ambiguity set, and (iv) enables decision makers to efficiently control the degree of conservatism for the solutions implementation.

II. Modelling Forecast Uncertainty

Uncertainties arisen from intermittent wind is primarily driven by the forecast errors. Wind forecast can be described using continuous probability distributions. In this paper, wind uncertainties $\tilde{u} \in \mathbb{R}^m$ are modeled as the sum of the forecasted active power $u_f \in \mathbb{R}^m$ and a random fluctuation δu:

$$\tilde{u} = u_f + \delta u \tag{1}$$

Full probability distribution of δu is generally unknown. Nonetheless, it is assumed that at least partial information on the probability distribution (e.g., the first moment μ and second moment $\sigma_W \in \mathbb{R}^{m\times m}$) is available and can be estimated either based on historical data or through forecasting methods. Given this condition, (1) can be rewritten in terms of the expected power generation $u = u_f + \mu$ and a zero-mean fluctuating component $w \in \mathbb{R}^m$:

$$\tilde{u} = u_f + \mu + w \; = u + w \tag{2}$$

Note that the mean μ is zero since the forecasts are based on the expectation of $\tilde{u}$. The total power mismatch $\Omega \in \mathbb{R}$ based on the zero-fluctuation component can be defined as follows:

$$\Omega = \sum_{i\in\kappa} w_i, \quad \text{with} \quad \sigma_\Omega = \sqrt{\mathbf{1}_{1,m}\sigma_W \mathbf{1}_{1,m}^T} \tag{3}$$

where $\sigma_\Omega \in \mathbb{R}$ is the standard deviation of ω and $\mathbf{1}_{1,m} \in \mathbb{R}^{1\times m}$ is an m-dimensional row vector of ones. κ is the set of nodes in the grid with uncertain wind generations.

In a secured power system operation, a balance between the load demand and power generations should be maintained at all times. With the forecast errors, the total power mismatch Ω must be balanced by adjustments in outputs of the controllable generation, reflecting the actions enforced by the automatic generation control (AGC) [26]. In this paper, an affine control policy is utilized in order to model the reserve activation in each generating unit [27].

$$\tilde{p}_G(\Omega) = p_g - \alpha\Omega \tag{4}$$

where p_G, $\tilde{p}_G(\Omega) \in \mathbb{R}^m$ represent the scheduled and actual generation set-point, respectively, and $\alpha \in \mathbb{R}^m$ is the vector of participation factors illustrating the contribution of each generating unit toward the power balance requisites. Note that sum of the α elements must be one to ensure that any given fluctuation Ω is balanced by the same amount $\Sigma_{i\in\mathcal{G}}(\alpha_i\Omega) = \Omega$. Here, the participation factor of each generating unit i is defined by [28]:

$$\alpha_i = \frac{p_{G,i}^{max}}{\sigma_{j\in\mathfrak{g}}\, p_{G,j}^{max}} \;\forall i \in \mathfrak{g} \tag{5}$$

III. Distributionally Robust Chance-Constrained Topology Control Formulation

Power system topology control optimization based on the DC optimal power flow (DCOPF) foundations is formulated with the main objective to minimize the system operation cost [29]. Each transmission line is assigned a binary variable, z_{ij}, representing its ON or OFF status, thereby enabling an opportunity for a network topology reconfiguration. The optimization problem would either return the results with no line switching actions (no cost saving) or with some line switching solutions with an improved objective function. In order to incorporate the wind uncertainties, a DRCC model is considered in order to guarantee that the constraints subject to uncertainty will be satisfied with a certain probability. The DRCC model will be then reformulated in such a way that the constraints would be robust against all PDFs corresponding to the random variables. We suggest a DRCC formulation of the grid topology control optimization (DRCC-OTC) as follows:

$$TC_\varphi = \min_{p_G} \sum_{i\in\mathfrak{g}} (c_i p_{G,i}) \tag{6}$$

$$\sum_{i\in\mathcal{N}} (p_{G,i} - d_i + u_i) = 0 \tag{7}$$

$$\boldsymbol{M}_{(ij)}\Big(p_G - \alpha\Omega + u + w - d\Big) - P_{ij} - (1 - z_{ij})\gamma \leq 0 \qquad \forall ij \in \ell \tag{8}$$

$$\boldsymbol{M}_{(ij)}\Big(p_G - \alpha\Omega + u + w - d\Big) - P_{ij} + (1 - z_{ij})\gamma \geq 0 \qquad \forall ij \in \ell \tag{9}$$

$$\sum_{ij}(1 - z_{ij}) \leq \varphi \tag{10}$$

$$\inf_{\nu \in D} \mathbb{P}\Big[p_{G,i} - \alpha_i \Omega \leq P_{G,i}^{max}\Big] \geq 1 - \epsilon \quad \forall i \in \mathfrak{g} \tag{11}$$

$$\inf_{\nu \in D} \mathbb{P}\Big[p_{G,i} - \alpha_i \Omega \geq P_{G,i}^{min}\Big] \geq 1 - \epsilon \quad \forall i \in \mathfrak{g} \tag{12}$$

$$\inf_{\nu \in D} \mathbb{P}\Big[\boldsymbol{M}_{(ij)}\big(p_G - \alpha\Omega + u + w - d\big) \leq P_{ij}^{max} z_{ij}\Big] \geq 1 - \epsilon \qquad \forall ij \in \ell \tag{13}$$

$$\inf_{\nu \in D} \mathbb{P}\Big[\boldsymbol{M}_{(ij)}\big(p_G - \alpha\Omega + u + w - d\big) \geq -P_{ij}^{max} z_{ij}\Big] \geq 1 - \epsilon \qquad \forall ij \in \ell \tag{14}$$

where, p_G and z_{ij} are non-negative decision variables; υ is the distribution of random variables; and D is a moment-based ambiguity set. Note that the power flows p_{ij} on the lines are considered based on the DC approximations [30]; $\boldsymbol{M} \in \mathbb{R}^{(l \times m)}$ is the matrix of power transfer distribution factors (PTDFs); $\mathcal{N}$ and $\mathcal{L}$ denote the sets of nodes and lines; ℓ, m, and $|\mathfrak{g}| \subseteq m$ are the number of lines, nodes, and conventional generators, respectively; matrix $\boldsymbol{M}$ relates the line flows to the nodal power injections and is expressed as the susceptance matrix. Note that $p - \alpha\Omega$ is the sum of conventional generations; $u+w$ represent the wind generation, and d is demand power. Besides, γ in (8) and (9) is a large number greater than or equal to $B_{ij}(\theta_i^{max} - \theta_j^{max})$. We defined φ as a generalized upper bound constraint in (10) through which the number of open lines in the network is limited.

Objective function (6) minimizes the total generation costs where $c_i \in \mathbb{R}^m$ is the cost factor of conventional generators. Power balance constraint is enforced in (7) in which u_i is the expected active power generation from uncertain sources. Constraints (11), (12) express the generator outputs to remain within the limits. Similarly, constrains (13), (14) limit the power flows across the lines within their capacities. As the reserve activation depends on the random variable ω, constraints (11)-(14) cannot be enforced deterministically. Instead, they are formulated as chance constraints where each constraint is required to stay within a predetermined confidence level $1 - \epsilon$. ϵ is a controllable risk parameter, enabling the decision maker to adjust the degree of conservatism. The lower the ϵ is, the more conservative the solutions are.

IV. Analytics for DRCC Reformulation

Unfortunately, constraints (11)-(14) are implicit in nature, making them very intractable and challenging to handle. Such implicitness arises from the fact that the assessment of the probability statements on the left-hand side of such constraints is not determined due to the unknown PDFs of the random variables. Unlike several number of reformulation efforts in the past assuming a known Gausian distribution of random variables [31], [22], we suggest a reformulation of DRCC problem in which no such assumption is enforced [32]. The explicit counterparts of DR chance constraints are proposed according to [28] in such a way that DR chance constraints are satisfied irrespective of the PDFs of the random variables. In order to find a solution to the distributionally robust chance-constrained optimal topology control (DRCC-OTC) problem, constraints (11)-(14) must be reformulated to deterministic and tractable constraints. To simplify, each constraint (11)-(14) can be expressed as:

$$\mathbb{P}\Big[\Xi(p_G) + \Upsilon(p_G)\delta u \leq \chi\Big] \geq 1 - \epsilon \tag{15}$$

where $\Xi(p_G) \in \mathbb{R}$ states the generation output or the line flows with no forecast errors; $\Upsilon(p_G) \in \mathbb{R}^{1 \times m}$ is expressed as functions of the decision variables by which the influence of the forecast errors δu on the respective constraint can be captured. χ is a constant which represents the generation or line flow limit. Regardless of the exact expression for $\Upsilon(p_G)$, and for any distribution and dimension of the random vector δu, the left hand side of (15) is a scalar random variable with a mean $\mu_\delta(p_g)$ and variance $\sigma_\delta(p_g)$ expressed as follows [28]:

$$\mu_\delta(p_g) = \Xi(p_G) + \Upsilon(p_G)\mu \tag{16}$$

$$\sigma_\delta(p_g) = \sqrt{\Upsilon(p_G)\Sigma_\upsilon \Upsilon(p_G)^T} = ||\Upsilon(p_G)\sqrt{\Sigma_\upsilon}||_2 \tag{17}$$

Hence, constraint (15) can be represented as:

$$\inf \mathbb{P} = \left[\frac{\delta - \mu_\delta(p_G)}{\sigma_\delta(p_G)} < \frac{\chi - \mu_\delta(p_G)}{\sigma_\delta(p_G)}\right] = \inf \mathbb{P}\left[\delta_n < \frac{\chi - \mu_\delta(p_G)}{\sigma_\delta(p_G)}\right] \geq 1 - \epsilon \quad \forall \nu \in D \tag{18}$$

where the scaled random variable δ_n has a zero mean and unit variance. Note that the distribution ν of δ_n must belong to an ambiguity set which can be described as the family of all distributions with the same structural properties—e.g., mean, variance, and co-variance [33]. By defining $\Theta_D(k)$ as the the worst-case probability distribution in the ambiguity set D, equation (18) can be re-written as follows:

$$\Theta_D\big(\frac{\chi - \mu_\delta(p_G)}{\sigma_\delta(p_g)}\big) \geq 1 - \epsilon \tag{19}$$

Since the function $\Theta_D(k)$ is increasing, it can be reformulated as a well-defined inverse function:

$$\Theta_D^{-1}(k) = \inf\{k \,|\, \Theta_D(k) \geq \lambda\} \tag{20}$$

$$\big(\frac{\chi - \mu_\delta(p_G)}{\sigma_\delta(p_g)}\big) \geq \Theta_D^{-1}(1 - \epsilon) \tag{21}$$

Employing (16), (17), the CC (15) can be reformulated to the following analytical expression for all PDFs that exist within the ambiguity set:

$$\Xi(p_G) \leq \chi - \Upsilon(p_G)\mu - \Theta_D^{-1}(1 - \epsilon)||\Upsilon(p_G)\sqrt{\Sigma_\upsilon}||_2 \tag{22}$$

Characterization of the ambiguity set D directly depends on the dominant source of the uncertainty, the forecast interval, and the type and quality of available data. Among two main types of the ambiguity sets in the literature—e.g., moment-based ambiguity sets [34] and distance-based ambiguity sets

[35], we use the moment-based ambiguity set with known moment information [34]. To reformulate (22) into a deterministic counterpart, the set Λ containing all distributions with zero mean and unit variance is defined in (24):

$$\begin{aligned}\Theta_\Lambda(\rho) &= 1 - \sup_{\mathbb{P}\in\Lambda} \mathbb{P}[\delta_n \geq \rho] \\ &= 1 - \begin{cases} \frac{1}{1+\rho^2} & \text{if } \rho \geq 0 \\ 1 & \text{otherwise} \end{cases} \\ &= \begin{cases} \frac{\rho^2}{1+\rho^2} & \text{if } \rho \geq 0 \\ 0 & \text{otherwise} \end{cases}\end{aligned} \tag{23}$$

Taking the inverse of (23) gives the following equation:

$$\Theta_\Lambda^{-1}(1-\epsilon) = \sqrt{\frac{1-\epsilon}{\epsilon}} \quad \text{for } 0 \leq \epsilon \leq 1 \tag{24}$$

Considering (22) and (24), the reformulation of (15) is achieved as follows:

$$\Xi(p_G) \leq \chi - \Upsilon(p_G)\mu - \Theta_\Lambda^{-1}(1-\epsilon)\,||\Upsilon(p_G)\sqrt{\Sigma_\nu}||_2 \tag{25}$$

The first part in (25), i.e., $\Xi(p_G) \leq \chi$, is the nominal constraint in which the uncertainty is entirely neglected; the second part, $\Upsilon(p_G)\mu$, is a modification factor related to the forecast error bias μ; and the third part, $\Theta_\Lambda^{-1}(1-\epsilon)\,||\Upsilon(p_G)\sqrt{\Sigma_\nu}||_2$, is a security margin against the uncertainty. To sum up, the second and the third parts can be interpreted as an adjustment of the nominally available capacity to secure the CC against the forecast deviations. Therefore, equation (25) will be satisfied for all PDFs covered by the ambiguity set D. As a result, DRCC problem is able to control the conservatism degree and the robustness level of the solution by adjusting the risk parameter ϵ in the decision making process.

According to the above discussions, constraints (11)-(14) can be directly transferred into an equivalent deterministic constraint as follows:

$$p_{G,i} \leq P_{G,i}^{max} - \alpha_i \mathbf{1}_{1,m}\mu - \sqrt{\frac{1-\epsilon}{\epsilon}} \left\| \alpha_i \mathbf{1}_{1,m} \sqrt{\Sigma_\nu} \right\|_2 \;\; \forall i \in \mathfrak{g} \tag{26}$$

$$p_{G,i} \geq P_{G,i}^{min} - \alpha_i \mathbf{1}_{1,m}\mu + \sqrt{\frac{1-\epsilon}{\epsilon}} \left\| \alpha_i \mathbf{1}_{1,m} \sqrt{\Sigma_\nu} \right\|_2 \;\; \forall i \in \mathfrak{g} \tag{27}$$

$$\begin{aligned}\boldsymbol{M}_{ij}(p_G + u_f - d) &\leq P_{ij}^{max} z_{ij} - \boldsymbol{M}_{ij}(\boldsymbol{I} - \alpha \mathbf{1}_{1,m})\mu \\ &- \sqrt{\frac{1-\epsilon}{\epsilon}} \left\| \boldsymbol{M}_{ij}(\boldsymbol{I} - \alpha \mathbf{1}_{1,m})\sqrt{\Sigma_\nu} \right\|_2 \quad \forall ij \in \ell\end{aligned} \tag{28}$$

$$\begin{aligned}\boldsymbol{M}_{ij}(p_G + u_f - d) &\geq -P_{ij}^{max} z_{ij} - \boldsymbol{M}_{ij}(\boldsymbol{I} - \alpha \mathbf{1}_{1,m})\mu \\ &+ \sqrt{\frac{1-\epsilon}{\epsilon}} \left\| \boldsymbol{M}_{ij}(\boldsymbol{I} - \alpha \mathbf{1}_{1,m})\sqrt{\Sigma_\nu} \right\|_2 \quad \forall ij \in \ell\end{aligned} \tag{29}$$

where $\boldsymbol{I} \in \mathbb{R}^{m\times m}$ is the identity matrix and $\Sigma_\nu = e^T(\text{diag}(\Sigma_0))$, in which e is the vector of all ones and Σ_0 is the statistical co-variance of the forecast error associated with the random variable. Therefore, such constraints can be handled and tracked as a MISOCP model. Besides, the forecast bias correction and the uncertainty margin on the right hand side result in a reduction in the available output power of each generating unit and the power flow through each line. Thus, the higher the uncertainty margin, the lower the violation probability and the higher the objective cost. The risk parameter, ϵ, plays an important role to achieve a reasonable trade-off between the security level against the forecast errors and the objective cost function.

V. Simulation Results and Discussions

In order to evaluate and verify the effectiveness of the proposed DRCC-OTC formulation, a modified IEEE 118-bus test system penetrated with wind generation is employed as the test-bed. All simulations have been performed on a Laptop with a 3.40 GHz Intel Core i7-2620 processor and 8 GB of RAM using CPLEX 12.6.1 [36].

A. Test System Description

In the modified IEEE 118-bus test system, the cost functions for the system generating units are modeled through linear functions, network losses and reactive power are ignored (i.e., simulations are conducted in a DC setting), and the resistance and shunt capacitance of transmission lines are assumed zero. The generator variable costs and transmission line characteristics are borrowed from [37], with the corresponding data coming from the University of Washington Power System Test Case Archive [38]. The system consists of 118 buses, 186 transmission lines (i.e., 186 binary variables), 19 conventional generating units with a total capacity of 5859.2 MW, and 99 load buses with a total demand power of 4519 MW. As for the wind power, a wind farm with the capacity of 2.6 percent of the maximum system generation capacity is assumed to be located at bus 111. The forecast error vector is the difference between the mean of a 10-month historical wind data and the corresponding actual data.

B. Discussions on Topology Control Solutions

We first investigate the DRCC-DCOPF problem in which no transmission line switching action is allowed (φ = 0). This is referred to as the ”base-case” scenario (DCOPF) where the network configuration is fixed. The system total operation cost in this case is achieved $1916.923 considering a risk parameter of $\epsilon = 0.1$, meaning that DR chance constraints must be satisfied with the probability of 90%.

In the second scenario, we implement the DRCC-OTC problem when constraint (10) is relaxed, i.e., the optimization model can render an unconstrained number of lines to be switched open. The optimal solution reveals that the system cost is significantly improved by 30% (i.e., $575.27) over the base-case scenario. In this case, 43 number of lines are found open and this verifies the role of topology change for an enhanced economic efficiency of the power grid.

In the third scenario, we limit the number of open lines by enforcing constraint (10). Allowing only one transmission line to be open (by adjusting $\varphi = 1$) would result in a lower system operation cost and higher economic benefits compared to the base-case scenario. In particular, by temporarily removing

TABLE I
CHANGES IN GENERATION DISPATCH FOLLOWING LINE 133 SWITCHING

Generator Bus	Output Change [MW]	Generator Variable Cost [$/MWh]
25	177.967	0.434
49	-112.606	0.467
87	-7.159	7.142
112	-57.402	2.173

TABLE II
TOPOLOGY CONTROL SOLUTIONS AND COST SAVINGS

Number of Switchable Lines (φ)	Switching Lines	Percent Cost Saving	Time (Sec)
0	-	-	2.4
1	133	8.671%	3.7
2	133, 152	18.42%	14.4
3	133, 152, 164	21.02%	18.5
4	142, 148, 150, 152	25.49%	53.2
5	128, 142, 148, 150, 152	27.43%	223
No restriction	[...]	30%	20.6

transmission line 133 connecting bus 77 to bus 82 out of service and by changing the way electricity flows in the network, the system operation cost would decrease from $1916.92 (corresponding to the base-case condition) to $1763.97. This economic benefit is achieved by co-optimizing the network topology and generation dispatch. The changes in the output power of the generating units are illustrated in Table I. One can realize from Table I that some cheaper generating units have increased their generation, while some more expensive ones have seen a decrease in their generation portfolios. The increased generation cost at bus 25 is $77.24 and the total cost saving realized through contributions from several generating units at buses 49, 87, and 112 is equivalent to $-228.4511.

When the risk parameter is set to $\epsilon = 0.1$, the DRCC-OTC optimization problem is simulated considering a flexible number of line switching possibilities through $\varphi = \{1, \ldots, 5\}$. The results on the optimal switching lines, total cost saving, and the computational run-time for each scenario are demonstrated in Table II. The results imply that switching a small number of lines out of service can have a significant impact on the economic dispatch of the generating units and consequently the cost saving in the network operation (e.g., switching out transmission line 133 and line 152 will result in 18.42% cost saving compared to the base-case operation scenario); nonetheless, a large number of lines would not necessarily result in an enormous cost saving. For instance, it is observed that there is not a significant difference in the total cost saving when φ changes from 3 to 5.

C. Discussions on the Proposed DRCC-OTC Model

In this section, we demonstrate that the conservatism degree of the optimal topology control solutions and the realized economic benefits are highly dependent to the risk parameter ϵ. In other words, the smaller the ϵ is, the more conservative the solution and the higher system operation cost will be. It can be shown that selecting various values of risk parameter

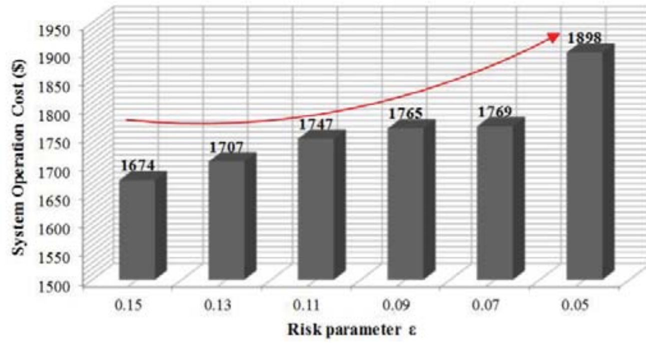

Fig. 1. System cost with different ϵ.

TABLE III
PERFORMANCE COMPARISON OF DIFFERENT TEST CASES FROM THE VIEWPOINT OF SYSTEM OPERATION COST

Test Cases	TC#1	TC#2	TC#3		TC#4	
$1-\epsilon(\%)$	N/A	N/A	85	95	85	95
System Cost ($)	1644	1455	1809	2155	1674	1898

ϵ will result in different topology control solutions with different cost savings. The system operation cost considering $\varphi = 1$ with different risk parameters ϵ, ranging from 0.15, 0.13, 0.11, 0.09, 0.07 and 0.05, are summarized in Fig. 1. One can realize from the results presented in this figure that the wind generation uncertainty can be tackled in the grid topology control decision making process by adjusting the risk parameter: the smaller the ϵ is, the higher system operation cost will be. Besides, according to Fig. 1, the marginal cost would increase remarkably when parameter ϵ decreases from 0.07 to 0.05, reflecting that the marginal cost increases as the confidence level $(1-\epsilon)$ increases. In practice, the risk parameter should be selected properly in order to avoid the significant costs imposed to the grid operation when a super-strict conservative solution is approached. In order to demonstrate how considering DRCC-OTC model will result in a higher system operation cost compared to the deterministic OTC model (base-case scenario), we consider and study four test cases as follows:

- *Test Case 1:* Deterministic model in which transmission lines are not switchable and the renewable uncertainties are totally ignored.
- *Test Case 2:* Deterministic OTC model in which only one transmission line is allowed to be switched off the grid and the renewable uncertainties are totally ignored.
- *Test Case 3:* DRCC model in which the uncertainty distributions are assumed unknown and transmission lines are considered not switchable.
- *Test Case 4:* DRCC-OTC model in which the uncertainty distributions are assumed unknown and only one transmission line is allowed to be switched off the grid.

Table III compares the system operation costs evaluated in different studied test cases. One can see from Table III that all DRCC solutions have resulted in a higher operation cost compared to their deterministic counterparts, reflecting the role of wind uncertainties in the DCOPF decision making. Moreover, if the number of switchable lines φ changes from 0 to 1, the system cost decreases in both deterministic and DRCC-OTC formulations, resulting in a significant cost saving.

VI. Conclusion

In this paper, a novel DRCC-OTC optimization model was proposed and formulated which could well capture the wind power uncertainties on the topology control decisions in power grid. The proposed model assumes no prior knowledge of the uncertainty distribution functions for the random variables and parameters. Simulation results demonstrated the potentially-high cost saving of the topology control practice in uncertainty-hosted power grids by harnessing the network existing infrastructure. We also demonstrated how the proposed decision making model under uncertainty is able to significantly increase the solution robustness; unlike the robust optimization models, a decision maker can efficiently harness the conservatism degree of the solution by adjusting a risk parameter embedded in the proposed DRCC-OTC formulation.

Real-world application of the proposed approach should consider the N–1 security criterion. Future research may take into account such security constraints that may otherwise hinder its practical implementation.

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