# The Real Exchange Rate, Real Interest Rates, and the Risk Premium

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The famous uncovered interest parity arises from this regression:

Regression: $s_{t+1} - s_t = \beta_0 + \beta_1(i_t - i_t^*) + u_{t+1}$.

--(Log of) nominal exchange rate: $s$

(Price of Foreign currency – increase means Home depreciation)

UIP says $E_t s_{t+1} - s_t = i_t - i_t^*$

Null is $\beta_0 = 0$ and $\beta_1 = 1$.

Typical finding: $\beta_1 < 1$ (often $\beta_1 < 0$), $\beta_0 = 0$.

--(Log) of Home and Foreign prices: $p, p^*$

-- Home and Foreign inflation: $\pi_{t+1} = p_{t+1} - p_t$, $\pi^*_{t+1} = p^*_{t+1} - p^*_t$

--(Log of) real exchange rate: $q = s + p^* - p$

(Increase means Home real depreciation)

-- Home and Foreign real interest rates: $r_t = i_t - E_t\pi_{t+1}$, $r^*_t = i^*_t - E_t\pi^*_{t+1}$

We will show the same puzzle holds for real exchange rates and real interest rates

$q_{t+1} - q_t = \beta_0 + \beta_1(r_t - r^*_t) + u_{t+1}$, we find $\beta_1 < 1$ (often $\beta_1 < 0$)

"Risk premium": $\lambda_t \equiv i^*_t + E_t s_{t+1} - s_t - i_t = r^*_t + E_t q_{t+1} - q_t - r_t$

UIP: $r^*_t + E_t q_{t+1} - q_t = r_t$ (i.e., $\lambda_t \equiv 0$)

The interest-parity puzzle: relation between $E_t q_{t+1} - q_t$ and $r_t - r_t^*$

What about relation between $q_t$ and $r_t - r_t^*$?

Under interest parity, many models predict $\text{cov}(q_t^{IP}, r_t - r_t^*) < 0$.

We find $\text{cov}(q_t, r_t - r_t^*) < \text{cov}(q_t^{IP}, r_t - r_t^*) < 0$. Overreaction, in a sense.

Are these two findings:

$$\text{cov}(q_{t+1} - q_t, r_t - r_t^*) < 0$$
$$\text{cov}(q_t, r_t - r_t^*) < \text{cov}(q_t^{IP}, r_t - r_t^*) < 0$$

arising from the same source?

No. They seem to say the opposite.

$$\text{cov}(q_{t+1} - q_t, r_t - r_t^*) < 0$$

means when home $r_t$ is high (relative to $r_t^*$, relative to average), expected returns on home assets are high – i.e., home deposits are *riskier*.

$$\text{cov}(q_t, r_t - r_t^*) < \text{cov}(q_t^{IP}, r_t - r_t^*) < 0$$

means when home $r_t$ is high (relative to $r_t^*$, relative to average), the value of the home currency is higher than it would be under interest parity. Why? Because home deposits are *less risky*.

Plan of the seminar:

1. Empirical methodology
2. Empirical results
3. Why findings are a puzzle
   -- not readily explainable by complete-market risk-premium models
   -- not readily explainable by "delayed overshooting" models
4. Outline of a model that can resolve the puzzle

Should only international finance specialists be interested in this?

Real interest rates and real exchange rates.

Rewrite: $q_t - E_t q_{t+1} = -(r_t - r_t^* - \bar{r}) - (\lambda_t - \bar{\lambda})$

Iterate forward to get:

$$q_t - \lim_{j\to\infty}\left(E_t q_{t+j}\right) = -R_t - \Lambda_t$$

where

$$R_t \equiv \sum_{j=0}^{\infty} E_t(r_{t+j} - r_{t+j}^* - \bar{r}) \qquad \Lambda_t \equiv \sum_{j=0}^{\infty} E_t(\lambda_{t+j} - \bar{\lambda})$$

$\Lambda_t$ - “level risk premium”

We find evidence for long run purchasing power parity: $\lim_{j\to\infty} E_t q_{t+j} = \bar{q}$

$q_t = q_t^{IP} - \Lambda_t$

## Data

U.S., Canada, France, Germany, Italy, Japan, U.K., and “G6”

G6 is like doing panel regressions

Exchange rates – last day of month (noon buy rates, NY)
Prices – consumer price indexes
Interest rates – 30-day Eurodeposit rates (last day of month)

Monthly, June 1979 – October 2009

(Unit root test for real exchange rates, uses data back to June, 1973)

Preliminary – unit root tests

| *Country* | *ADF* | *DF-GLS* |
|---|---|---|
| Canada | -1.771 | -1.077 |
| France | -2.033 | -2.036* |
| Germany | -2.038 | -2.049* |
| Italy | -1.888 | -1.914† |
| Japan | -2.071 | -0.710 |
| United Kingdom | -2.765† | -2.076* |
| G6 | -2.052 | -1.846† |

Panel Unit Root Test, 1973:3-2009:10

| *Model* | *Estimated Coefficient* | *1%* | *5%* | *10%* |
|---|---|---|---|---|
| No Covariates | -0.01705* | -0.02199 | -0.01697 | -0.01485 |
| With Covariates | -0.01688* | -0.02126 | -0.01638 | -0.01411 |

Fama Regressions: $i_t^* + s_{t+1} - s_t - i_t = \zeta_s + \beta_s(i_t^* - i_t) + u_{s,t+1}$

1979:6-2009:10

| *<u>Country</u>* | $\hat{\beta}_s$ | *90% c.i.* ( $\hat{\beta}_s$ ) |
|---|---|---|
| Canada | 2.271 | (1.186,3.355) |
| France | 1.216 | (-0.171,2.603) |
| Germany | 2.091 | (0.599,3.583) |
| Italy | 0.339 | (-0.680,1.359) |
| Japan | 3.713 | (2.390,5.036) |
| U.K. | 3.198 | (1.170,5.225) |
| G6 | 2.467 | (0.769,4.164) |

VAR methodology

Two different VAR models:

Model 1: $\left[ q_t, i_t - i_t^*, i_{t-1} - \pi_t - (i_{t-1}^* - \pi_t^*) \right]$

Model 2: $\left[ q_t, i_t - i_t^*, \pi_t - \pi_t^* \right]$

(Extensions include long-term bond yields and stock returns.)

Estimate VAR with 3 lags. (Extension with 12 lags.)

Use standard projection measures to estimate

$r_t - r_t^* = i_t - i_t^* - (E_t \pi_{t+1} - E_t \pi_{t+1}^*)$, and $q_t^{IP} \equiv -\sum_{j=0}^{\infty} E_t (r_{t+j} - r_{t+j}^* - \bar{r}) + \bar{q}$

Then $\lambda_t$ is constructed as $\lambda_t \equiv r_t^* + E_t q_{t+1} - q_t - r_t$

$\Lambda_t$ estimate is constructed from $\Lambda_t = q_t^{IP} - q_t$

## Fama Regression in Real Terms: $q_{t+1} - q_t - \hat{r}_t^d = -\zeta_q - \beta_q \hat{r}_t^d + u_{q,t+1}$

| *Country* | $\hat{\beta}_1$ | *90% c.i.(* $\hat{\beta}_1$*)* |
|---|---|---|
| Canada | 0.862 | (-0.498,2.222) |
| | | (-0.632,2.908) |
| | | (-0.676,2.800) |
| France | 1.576 | (-0.117,3.269) |
| | | (0.281,3.240) |
| | | (-0.125,3.602) |
| Germany | 1.837 | (-0.015,3.689) |
| | | (0.687,4.458) |
| | | (0.589,4.419) |
| Italy | 0.360 | (-1.336,2.056) |
| | | (-1.087,2.136) |
| | | (-1.358,2.328) |
| Japan | 2.314 | (0.768,3.860) |
| | | (0.746,4.300) |
| | | (0.621,4.441) |
| United Kingdom | 2.448 | (0.854,4.042) |
| | | (0.873,4.614) |
| | | (1.039,4.846) |
| G6 | 1.933 | (0.318,4.548) |
| | | (0.510,3.932) |
| | | (0.473,4.005) |

## Regression of $q_t$ on $\hat{r}_t - \hat{r}_t^*$ : $q_t = \beta_0 + \beta_1(\hat{r}_t - \hat{r}_t^*) + u_{t+1}$

| *Country* | $\hat{\beta}_1$ | *90% c.i.( $\hat{\beta}_1$)* |
|---|---|---|
| Canada | -48.517 | (-62.15,-34.88) |
| | | (-94.06,-31.41) |
| | | (-140.54,-27.34) |
| France | -20.632 | (-32.65,-8.62) |
| | | (-44.34,-1.27) |
| | | (-54.26,1.75) |
| Germany | -52.600 | (-67.02.-38.18) |
| | | (-85.97,-25.35) |
| | | (-105.29,-19.38) |
| Italy | -39.101 | (-51.92,-26.28) |
| | | (-67.63,-16.36) |
| | | (-90.01,-13.70) |
| Japan | -19.708 | (-29.69,-9.72) |
| | | (-42.01,-1.05) |
| | | (-46.53,-4.33) |
| United Kingdom | -18.955 | (-31.93,-5.98) |
| | | (-40.19,-3.08) |
| | | (-55.94,4.08) |
| G6 | -44.204 | (-55.60,-32.80) |
| | | (-73.17,-23.62) |
| | | (-82.87,-21.74) |

## Regression of $\hat{\Lambda}_t$ on $\hat{r}_t - \hat{r}_t^*$: $\hat{\Lambda}_t = \beta_0 + \beta_1(\hat{r}_t - \hat{r}_t^*) + u_{t+1}$

| Country | $\hat{\beta}_1$ | *90% c.i.( $\hat{\beta}_1$)* |
|---|---|---|
| Canada | 23.610 | (15.12,32.10) |
| | | (12.62,51.96) |
| | | (11.96,63.71) |
| France | 13.387 | (1.06,25.72) |
| | | (-2.56,36.25) |
| | | (-6.98,42.40) |
| Germany | 34.722 | (19.66,49.78) |
| | | (9.34,57.59) |
| | | (3.68,69.36) |
| Italy | 27.528 | (17.58,37.48) |
| | | (14.98,48.32) |
| | | (12.51,58.54) |
| Japan | 15.210 | (4.76,25.66) |
| | | (-0.45,37.08) |
| | | (0.91,38.87) |
| United Kingdom | 14.093 | (0.33,27.86) |
| | | (0.39,34.46) |
| | | (-8.70,46.45) |
| G6 | 31.876 | (20.62,43.13) |
| | | (16.89,54.62) |
| | | (16.78,60.89) |

Implications:

$\text{cov}(\lambda_t, r_t - r_t^*) < 0$ (Fama regression in real terms)

$\text{cov}(\Lambda_t, r_t - r_t^*) = \text{cov}(\sum_{j=0}^{\infty} E_t \lambda_{t+j}, r_t - r_t^*) > 0$ (from VAR estimates)

→ $\text{cov}(E_t \lambda_{t+j}, r_t - r_t^*) > 0$ for some *j* (as in previous figure)

Explaining $\text{cov}(\lambda_t, r_t - r_t^*) < 0$ and $\text{cov}(E_t \lambda_{t+j}, r_t - r_t^*) > 0$ is a challenge for risk premium models – when $r_t - r_t^*$ is high, the home currency is both riskier than average and expected to be less risky than average.

Notation: $d_{t+1} = q_{t+1} - q_t$

$$E_t\hat{d}_{t+k} = \beta_{dk}(\hat{r}_t - \hat{r}_t^*),\ E_t(\hat{r}_{t+k} - \hat{r}_{t+k}^*) = \beta_{rk}(\hat{r}_t - \hat{r}_t^*),\ \text{and}\ \ E_t\hat{\lambda}_{t+k} = \beta_{\lambda k}(\hat{r}_t - \hat{r}_t^*)$$

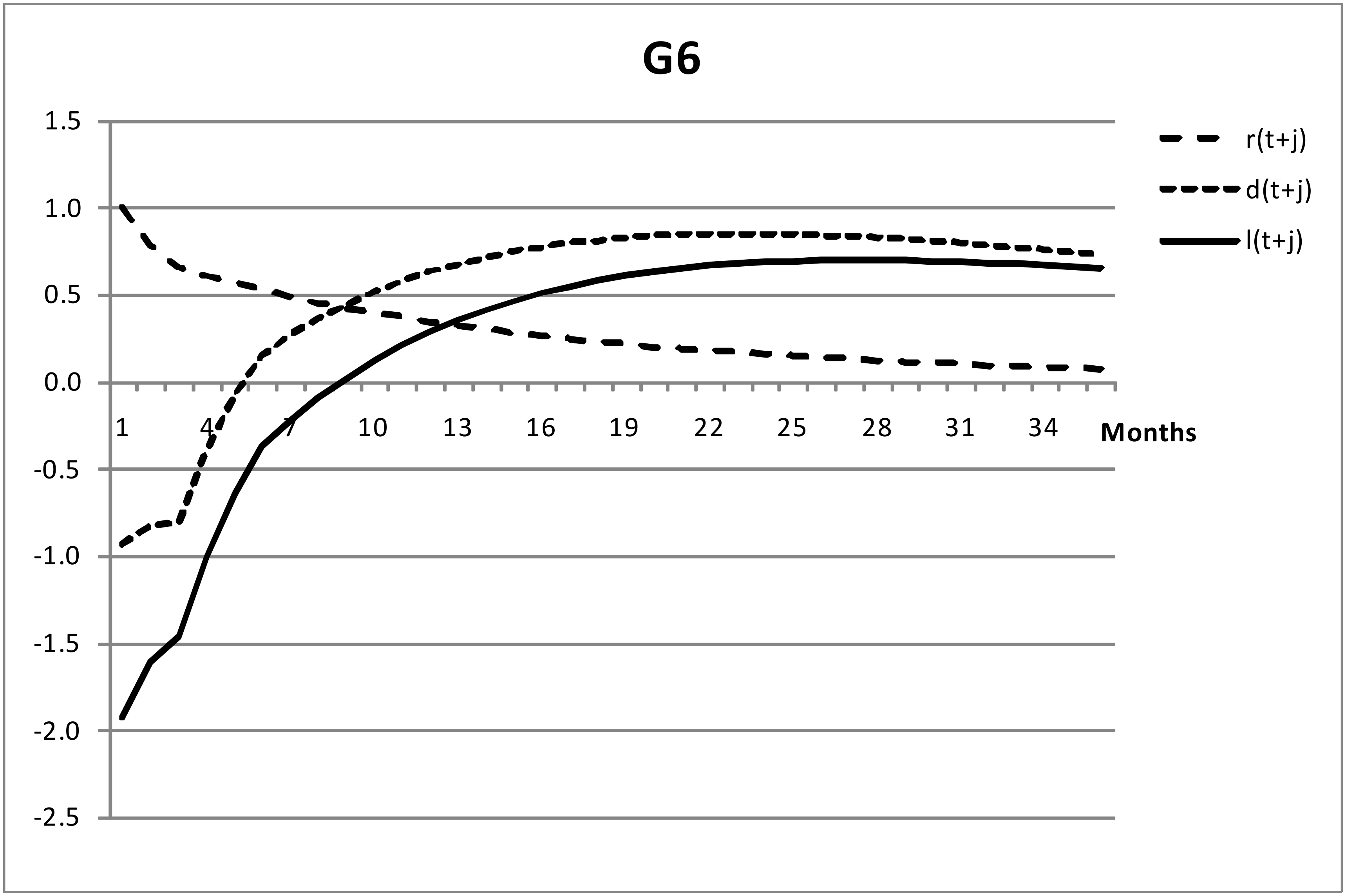


Figure 1

Figure 2 plots slope coefficients from the following regressions (Data are monthly, interest rates are 1-month, end-of-month. For this slide, U.S. relative to weighted average of rest of G7):

$$q_{t+k} = \alpha_{qk} + \beta_{qk}(r_t - r_t^*) \qquad \beta_{qk} = \text{cov}(q_{t+k}, r_t - r_t^*) / \text{var}(r_t - r_t^*)$$

$$q_{t+k}^{IP} = \alpha_{Rk} + \beta_{Rk}(r_t - r_t^*) \qquad \beta_{Rk} = \text{cov}(q_{t+k}^{IP}, r_t - r_t^*) / \text{var}(r_t - r_t^*)$$

(Real interest rates themselves are estimates)

Difference between $q_{t+k}^{IP}$ and $q_{t+k}$ is $\Lambda_{t+k}$:

$$\Lambda_{t+k} = q_{t+k}^{IP} - q_{t+k}.$$

So difference in lines is $\beta_{\Lambda k} = \text{cov}(\Lambda_{t+k}, r_t - r_t^*) / \text{var}(r_t - r_t^*)$

$$q_{t+k} = \alpha_{qk} + \beta_{qk}(r_t - r_t^*),\ q_{t+k}^{IP} = \alpha_{Rk} + \beta_{Rk}(r_t - r_t^*), \text{ and Model } q_{t+k} - \lim_{j\to\infty} E_t q_{t+j}$$

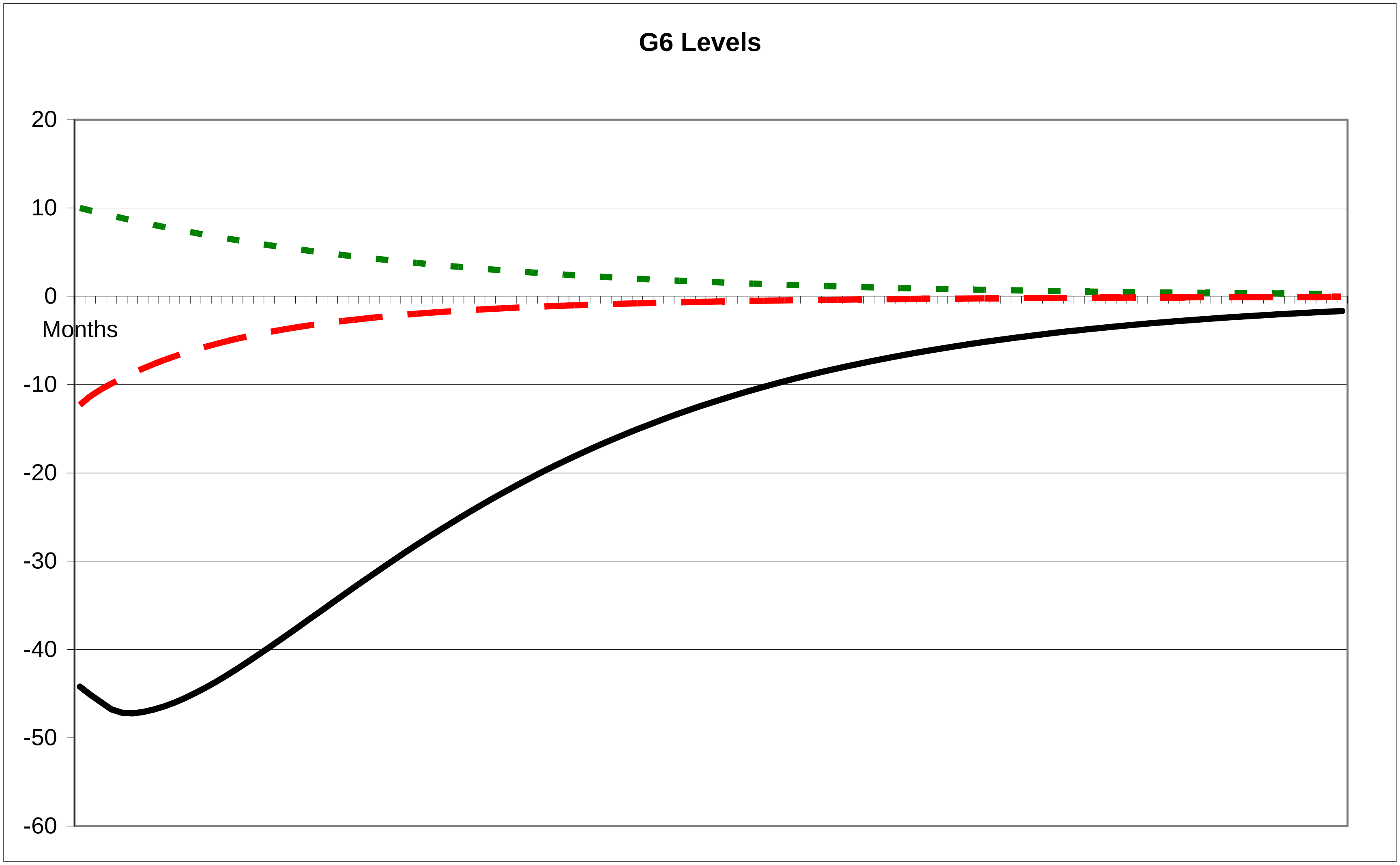


Figure 2

Puzzle is $\mathrm{cov}(\lambda_t, r_t^d) < 0$ but $\mathrm{cov}(\Lambda_t, r_t^d) > 0$

Models with a single economic variable driving $r_t^d$ and $\lambda_t$:

$$r_t^d = \sum_{j=0}^{\infty} a_j \varepsilon_{t-j} \qquad \lambda_t = \sum_{j=0}^{\infty} c_j \varepsilon_{t-j}$$

1. Single factor models: $r_t^d = k\lambda_t$

2. Unidirectional models: $a_j$ same sign $\forall j$, and $c_j$ same sign $\forall j$.

These are common assumptions. Sometimes both are made.
Assumption 2, especially, seems sensible.

But then we cannot get $\mathrm{cov}(\lambda_t, r_t^d) < 0$ and $\mathrm{cov}(\Lambda_t, r_t^d) > 0$

Assumption 1, $r_t^d = k\lambda_t$, requires $k < 0$ for $\text{cov}(\lambda_t, r_t^d) < 0$.

But then $q_t^{IP} = -k\Lambda_t$, so $\text{cov}(\Lambda_t, r_t^d) = -k\,\text{cov}(q_t^{IP}, r_t^d) < 0$

Assumption 2, $a_j$ same sign $\forall j$, and $c_j$ same sign $\forall j$

Normalize $a_j \geq 0$ .

Then $c_j \leq 0$ to get $\text{cov}(\lambda_t, r_t^d) < 0$ since $\text{cov}(\lambda_t, r_t^d) = \sum_{j=0}^{\infty} a_j c_j$

But in that case $\text{cov}(\Lambda_t, r_t^d) = \sum_{j=0}^{\infty} a_j \sum_{i=j}^{\infty} c_i < 0$

We need at least two driving variables:

$$r_t^d = \sum_{j=0}^{\infty} a_{1j}\varepsilon_{1,t-j} + \sum_{j=0}^{\infty} a_{2j}\varepsilon_{2,t-j} \qquad \lambda_t = \sum_{j=0}^{\infty} c_{1j}\varepsilon_{1,t-j} + \sum_{j=0}^{\infty} c_{2j}\varepsilon_{2,t-j}$$

Necessary to have $a_{1j}c_{1j}$ and $a_{2j}c_{2j}$ of opposite signs

What models have been put forward to explain UIP puzzle?

1. Risk premium models
2. Delayed overshooting/delayed reaction

Generally these assume a single driving variable, so cannot explain $\text{cov}(\lambda_t, r_t^d) < 0$ and $\text{cov}(\Lambda_t, r_t^d) > 0$

Risk premium models might have multiple sources of risk, but $a_{ij}c_{ij}$ always the same sign for all *i* – determined by preferences.

I will then sketch a model of a liquidity premium that has two sources of shocks (monetary policy shocks and liquidity shocks) that can account for the puzzle.

Review of foreign exchange risk premium

$m_{t+1}, m^*_{t+1}$ are logs of home, foreign stochastic discount factors

Under complete markets,

$$d_{t+1} = m^*_{t+1} - m_{t+1}$$

Since $r_t = -E_t m_{t+1} - \frac{1}{2}\text{var}_t(m_{t+1})$ and $r^*_t = -E_t m^*_{t+1} - \frac{1}{2}\text{var}_t(m^*_{t+1})$

Then $r^d_t = -E_t(m_{t+1} - m^*_{t+1}) - \frac{1}{2}(\text{var}_t\, m_{t+1} - \text{var}_t\, m^*_{t+1})$

$\rightarrow \quad \lambda_t = \frac{1}{2}(\text{var}_t\, m_{t+1} - \text{var}_t\, m^*_{t+1})$

$\rightarrow \quad E_t d_{t+1} = -E_t(m_{t+1} - m^*_{t+1})$

I will quickly review two well-known recent general equilibrium models of the risk premium, with representative agents in each country.

1. Campbell-Cochrane preferences (Verdelhan, 2010)
2. Long-Run Risks (Bansal and Shaliastovich, 2010)

Both models have risk premium driven by single factor. For that reason alone, they cannot account for the puzzle. ("Model" in Figure 2)

Can generalize the second model as on previous slide.

Campbell-Cochrane preferences (Verdelhan, 2010):

Factor drives $\text{var}_t(m_{t+1})$ and $E_t(m_{t+1})$ is $\phi_{1t} = -\log\left(\frac{C_t - H_t}{C_t}\right)$.

"Long-run Risks" (Bansal-Yaron (2004), Bansal-Shaliastovich (2010))

Consumption in each country subject to shocks to the level and growth rate of consumption. The model assumes Epstein-Zin preferences.

Possibly multiple factors that drive $\text{var}_t(m_{t+1})$ and $E_t(m_{t+1})$, given by $\text{var}(g_{it})$, where $g_{it}$ is a component of consumption growth.

Symmetric Model (e.g., long-run risks)

$$\lambda_t = \tfrac{1}{2}(\text{var}_t\, m_{t+1} - \text{var}_t\, m^*_{t+1}) = \sum \alpha_i(\sigma^2_{it} - \sigma^{*2}_{it})$$
$$r^d_t = -E_t(m_{t+1} - m^*_{t+1}) - \tfrac{1}{2}(\text{var}_t\, m_{t+1} - \text{var}_t\, m^*_{t+1}) = -\sum(\alpha_i + \gamma_i)(\sigma^2_{it} - \sigma^{*2}_{it})$$

$\alpha_i$ and $\gamma_i$ determined by assumption on preferences. $\sigma^2_{it} - \sigma^{*2}_{it}$ are AR(1)

The long-run risks model can account for a large number of asset-pricing puzzles when inverse of IES > CRRA > 1.

This implies $\alpha_i > 0$ and $\alpha_i + \gamma_i > 0$ for all *i*.

**Intuition:** If there is an increase in uncertainty, precautionary motive prevails and $r_t$ falls.

Gives us $\text{cov}(\lambda_t, r^d_t) < 0$ but $\text{cov}(\Lambda_t, r^d_t) < 0$

Asymmetric model

$\sigma_{iWt}^2$ is a common component to world consumption growth.

$$\lambda_t = \tfrac{1}{2}(\text{var}_t\, m_{t+1} - \text{var}_t\, m_{t+1}^*) = \sum (\alpha_i - \alpha_i^*)\sigma_{iWt}^2$$
$$r_t^d = -E_t(m_{t+1} - m_{t+1}^*) - \tfrac{1}{2}(\text{var}_t\, m_{t+1} - \text{var}_t\, m_{t+1}^*) = -\sum (\alpha_i + \gamma_i - (\alpha_i^* + \gamma_i^*))\sigma_{iWt}^2$$

As long as country with larger precautionary effect (larger $\alpha_i$) has largest fall in interest rate (larger $\alpha_i + \gamma_i$), cannot account for $0 < \text{cov}(\Lambda_t, r_t^d)$.

Figure 2 looks like delayed overshooting. But it is not.

Delayed overshooting – pattern of impulse responses to an identified monetary shock.

1. Our calculations are not for an identified shock.

2. Figure 2 plots $\text{cov}(q_{t+j}, r_t^d)/\text{var}(r_t^d)$, which is different than an impulse response function.

3. Our main results concern the difference between $\text{cov}(q_{t+j}^{IP}, r_t^d)/\text{var}(r_t^d)$ and $\text{cov}(q_{t+j}, r_t^d)/\text{var}(r_t^d)$.

Delayed overshooting to monetary shocks has been explained in models of delayed reaction in the foreign exchange market
Froot and Thaler (1990), Bacchetta and van Wincoop (2010)

Suppose impulse response of $q_t$ is $b_0 < 0$ and of $q_t^{IP}$ is $f_0 < 0$.
Delayed overshooting implies underreaction of $q_t$: $b_0 > f_0$, then gradual catching up:
$q_t - q_t^{IP} = \delta(q_{t-1} - q_{t-1}^{IP}) + (b_0 - f_0)\varepsilon_t$

Suppose as in BvW, interest differential is AR(1):
$r_t^d = \rho r_{t-1}^d + \varepsilon_t$

Then we find $\lambda_t = \delta\lambda_{t-1} - (1-\delta)(b_0 - f_0)\varepsilon_t$. Gives us $\text{cov}(\lambda_t, r_t^d) < 0$.

But this is a case of single driving variable, and unidirectional effects of shocks on $r_t^d$ and $\lambda_t$, so implies $\text{cov}(\Lambda_t, r_t^d) < 0$

This model of delayed reaction does produce delayed overshooting

That is, the impulse response function for $q_t$ starts off negative, declines for awhile, and then increases (for certain parameters)

Impulse response *j*: $\delta^j(b_0 - f_0) + \rho^j f_0$
IRF will have the right shape if $f_0 < b_0 < 0$, $\delta < \rho$, and $(\rho - \delta) f_0 < (1-\delta) b_0$

Model can give us $\text{cov}(\lambda_t, r_t^d) < 0$, and even $\text{cov}(d_{t+1}, r_t^d) < 0$,
but implies $\text{cov}(\Lambda_t, r_t^d) < 0$

The real exchange rate underreacts to the increase in real interest rates, rather than overreacting.

An example of a model that would work:

Standard New Keynesian, except u.i.p. does not hold (good starting place because of implications under u.i.p.):

Open-economy Phillips curve:

$$\pi_t - \pi_t^* = \delta q_t + \beta E_t(\pi_{t+1} - \pi_{t+1}^*)$$

Taylor rule:

$$i_t - i_t^* = \sigma(\pi_t - \pi_t^*) + \varepsilon_t, \quad \varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \varsigma_t$$

“Liquidity” premium – short-term bonds have value as collateral

$$\lambda_t = \alpha\left[i_t - E_t\pi_{t+1} - (i_t^* - E_t\pi_{t+1}^*)\right] - \eta_t, \qquad \alpha > 0$$

$\eta_t$ -- exogenous increase in value of Foreign bonds

$\alpha\left[i_t - E_t\pi_{t+1} - (i_t^* - E_t\pi_{t+1}^*)\right]$ -- Home bonds are more valued as collateral during Home monetary policy contraction

This can account for $\text{cov}(E_t d_{t+1}, r_t^d) < 0$ and $\text{cov}(\Lambda_t, r_t^d) > 0$ when $\eta_t$ is more volatile but less persistent than $\alpha\varepsilon_t$.

$$\lambda_t = \alpha\left[ i_t - E_t\pi_{t+1} - (i_t^* - E_t\pi_{t+1}^*) \right] - \eta_t$$

$\eta_t \uparrow \Rightarrow$ Foreign assets more valuable. Foreign currency appreciates, increasing inflationary pressure in Home. $\Rightarrow r_t^d \uparrow$.

This tends to give us $\text{cov}(\lambda_t, r_t^d) < 0$ and $\text{cov}(E_t d_{t+1}, r_t^d) < 0$ as in u.i.p. puzzle

$\varepsilon_t \uparrow \Rightarrow$ Home monetary contraction, $r_t^d \uparrow$. Relative liquidity value of Home assets rises, tends to make $\text{cov}(\lambda_t, r_t^d) > 0$.

If $\eta_t$ is more volatile, it dominates short run behavior of $\text{cov}(\lambda_t, r_t^d)$. If $\varepsilon_t$ is sufficiently persistent, it dominates long run behavior and determines $\text{cov}(\Lambda_t, r_t^d)$.

$$q_t = \frac{-(1+\alpha)(1-\rho_\varepsilon \beta)}{\delta(1+\alpha)(\sigma-\rho_\varepsilon)+(1-\rho_\varepsilon \beta)(1-\rho_\varepsilon)}\varepsilon_t + \frac{1}{1+\sigma\delta(1+\alpha)}\eta_t$$

$$\lambda_t = \frac{\alpha(1-\rho_\varepsilon \beta)(1-\rho_\varepsilon)}{\delta(1+\alpha)(\sigma-\rho_\varepsilon)+(1-\rho_\varepsilon \beta)(1-\rho_\varepsilon)}\varepsilon_t - \left(\frac{1+\sigma\delta}{1+\sigma\delta(1+\alpha)}\right)\eta_t$$

$$\Lambda_t = \frac{\alpha(1-\rho_\varepsilon \beta)}{\delta(1+\alpha)(\sigma-\rho_\varepsilon)+(1-\rho_\varepsilon \beta)(1-\rho_\varepsilon)}\varepsilon_t - \left(\frac{1+\sigma\delta}{1+\sigma\delta(1+\alpha)}\right)\eta_t$$

$$r_t^d = \frac{(1-\rho_\varepsilon \beta)(1-\rho_\varepsilon)}{\delta(1+\alpha)(\sigma-\rho_\varepsilon)+(1-\rho_\varepsilon \beta)(1-\rho_\varepsilon)}\varepsilon_t + \frac{\sigma\delta}{1+\sigma\delta(1+\alpha)}\eta_t$$

Conclusions:

A new puzzle. Our models for the UIP puzzle don't seem adequate.

Visually, Figure 2 implies that the finding of $\text{cov}(\Lambda_t, r_t^d) > 0$ may be more important than the UIP puzzle, $\text{cov}(\lambda_t, r_t^d) < 0$.

Understanding this matters:

1. For understanding exchange rates
2. For understanding macroeconomics and finance.

$$q_{t+k} = \alpha_{qk} + \beta_{qk}(r_t - r_t^*),\ q_{t+k}^{IP} = \alpha_{Rk} + \beta_{Rk}(r_t - r_t^*),\ \text{and Model } q_{t+k} - \lim_{j\to\infty} E_t q_{t+j}$$