Lecture 11: Revenue Management

What is Revenue Management?

The science of selling the right item to the right person (= at the right price)

Limited Capacity
Uncertain Demand

What are the levers we have?
1. allocation
2. pricing
The Origins of Revenue Management

- American Airlines implements DINAMO (Dynamic Inventory and Maintenance Optimizer)
- Led to Ultimate Super Saver Fares in 1985
- Revenues increased 14.5% and profits increased 47.8%
- Drove low-fare competitor PEOPLExpress out of business in 1986

Where is RM?

<table>
<thead>
<tr>
<th>Airlines</th>
<th>MASSIVE DATA SETS</th>
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<tbody>
<tr>
<td>Freight &amp; Cargo</td>
<td>Advertising</td>
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<tr>
<td>Cruise Lines</td>
<td>Entertainment</td>
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<td>Car Rental</td>
<td>Oil and Gas</td>
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<td>Hotels</td>
<td>E-Commerce</td>
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<td>Apparel</td>
<td>Insurance</td>
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<td>Retail</td>
<td>Public policy</td>
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<tr>
<td>(Mature)</td>
<td>(Emerging)</td>
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Sports Tickets

Dynamic pricing is new trend in ticket sales

1

Doug Williams

The seat in Section U of the Skyline Deck at Target Field is high above the left-field line and provides a panoramic view of the Minneapolis skyline.

From Seat 1 in Row 5, a Twins fan can watch Joe Mauer rip line drives

Taxis (aka Ubers)

INTRODUCING DYNAMIC PRICING FOR UBERX

INTRODUCING DYNAMIC PRICING FOR UBERX

50.0X

CANCER NORMAL TAXA

5 000 KR MINSTA TAXA

+ 425.00 KR / MIN s 589.99 KR / KM
Parking Spots

Dynamic Pricing Parking Meters Climb Above $5/Hour in SF

Thursday, August 30, 2018 - 07:40 AM
By Julie Caine

Seattle to install new parking meters that adjust price based on demand

Seattle is getting smarter with its parking rates.

The Department of Transportation received approval from the City Council on Monday to begin a 5-year project that will upgrade each of Seattle’s 2,200 parking meters with dynamic pricing technology.

That means, by 2018, you may see two different prices charging the day on the same parking meter depending on demand for that specific street.

“We decided it’s time to make an investment and upgrade the technology,” said Mike Riley, Manager of Parking Operations and Traffic Planning for Seattle’s Department of Transportation (SDOT).

The new technology will let the

Dynamic pricing in Alinea

<table>
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<tbody>
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<td>5:30 PM</td>
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<td>6:00 PM</td>
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<tr>
<td>6:30 PM $265.00 per person 1 lift</td>
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<tr>
<td>7:00 PM $265.00 per person 1 lift</td>
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<tr>
<td>8:30 PM</td>
<td>Not Offered</td>
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Congestion/queue pricing: charging to control input

The Consumers – Fairness

• Amazon’s test of dynamic pricing strategies
  “This is a very strange business model, to charge customers more when they buy more or come back to the site more. ... This is definitely not going to earn customer loyalty.”
  “I find this extremely sneaky and unethical.”
  “I will never buy another thing from those guys!!!”
Outline for next lectures

- Yield management
- Basic pricing
- Dynamic pricing
- Choice models
- Auctions

Yield Management:
Models of capacity allocation
Operationalizing segmentation: booking limits and protection levels

- Suppose we are selling two fare classes: Low and High.
- We know the demand distribution for rooms in each class (leisure and business) but not the actual numbers.
- Manage access through booking limits or protection levels.
  - Booking limit = Capacity – protection level

Littlewood’s 2-Fare Model

- C seats
- Leisure demand $D_L >> C$
- Business demand uncertain $D_H$
- Leisure arrives before business and demands $D_L$ and $D_H$ are independent.

- What is booking limit $X$?
Littlewood’s 2-Fare Model

Suppose we sold x leisure (L) tickets so far. Should we sell another L ticket?

- Sell a ticket at price \( p_L \) if the marginal benefit is nonnegative. Marginal benefit from selling one more L ticket is:

\[
p_L - p_H \cdot \text{Prob}(D_H > C - x)
\]

Littlewood’s rule:

Set \( b^* \) to be \( 1 - F_H(C - b^*) = p_L/p_H \)

\( b^* \) is the booking limit for leisure class

\( C - b^* \) is the protection level for business class

A proof

- There are two prices \( p_L \) and \( p_H \):
  - \( p_L < p_H \)
  - Full fare customers book later.
- Suppose current protection level is \( Q+1 \). We sold all discount capacity but have demand for one more room at the discount price. Should we drop protection level to \( Q \)?
- \( F \) is the distribution of full fare demand:

\[
F(x) = P(\text{full fare demand} \leq x),
\]

Solution: set the protection level \( Q \) to be the minimal such that:

\[
F(Q) \geq \frac{p_H - p_L}{p_H}
\]
Newsvendor Model

• Motivation:
  • At the start of each day, a newspaper vendor must decide on the number of papers to purchase and sell.
  • Daily sales cannot be predicted exactly, and are represented by the random variable, $D$.

• Relevant costs:
  • $c_o =$ unit cost of overage (not enough demand)
  • $c_u =$ unit cost of underage (too much demand)

• Example: Fashion
  • Retail: Stanford Shopping Center, Web Sites
  • Overage: Gilroy Outlet Mall

Intuition

\[ C_o P(D \leq Q^*) = C_u P(D \geq Q^*) \]

Expected loss – cost of having $Q^*$ unit in inventory

\[ C_o P(D \leq Q^*) \]

Expected gain – profit from selling $Q^*$ unit

\[ C_u P(D \geq Q^*) \]

$Q^*$: Order amount where Expected loss = Expected gain

\[ F(Q^*) = \frac{c_u}{c_u + c_o} \]
Determining Booking Limit is a Newsvendor Problem

- Newsvendor tradeoff!
- Overage cost = cost of reserving too many business seats
  \[ C_O = p_L \]
- Underage cost = cost of reserving too few business seats
  \[ C_u = p_H - p_L \]
- “Inventory” for business seats = \( C - X \)

Determining Booking Limit

- Apply Newsvendor Formula:
- Set \( X \) such that

\[
\text{Prob}(D_H \leq C - X) = \frac{C_u}{C_u + C_o} = \frac{p_H - p_L}{p_H}
\]

Note that this is exactly Littlewood’s rule
Example – Hotel Room reservation

An organization needs to reserve rooms for the conference. Rooms can be reserved at a cost of $50 per room. Demand for rooms is normally distributed with mean 5000 and standard deviation 2000. If the number of rooms required exceeds the number of rooms reserved, extra rooms will have to be found at neighboring hotels at a cost of $80 per room. Inconvenience of staying at another hotel is estimated at $10. How many rooms should be reserved to minimize the expected cost?

Newsvendor Model:
Example – Hotel Room reservation

- $D$ = number of rooms actually required
- $Q$ = number of rooms reserved
- What are $c_o$ and $c_u$?

$c_o = ?; c_u = ?$ What is $Q^*$?
Newsvendor Model:  
Example – Hotel Room reservation

- Recall: \( D \sim \text{Normal}(5000,2000) \)
- We have lookup tables for Standard Normal distribution (\( \mu = 0, \sigma = 1 \)) \( \rightarrow \) Convert to Standard Normal

\[
F(Q) = P(D \leq Q) = \frac{4}{9} = 0.444
\]

\[
P(D \leq Q) = P(D - \mu \leq Q - \mu) = \Phi\left(\frac{D - \mu}{\sigma} \leq \frac{Q - \mu}{\sigma}\right) = P(Z \leq z) = \Phi(z)
\]

- Find \( z \) from the lookup table (alternatively, use NormsInv(.) function in Excel)
- \( Q = \sigma z + \mu \)

\[
\Phi(z) = 0.444
\]
Newsvendor Model: Example – Hotel Room reservation

- Why is $Q^* < 5000$, i.e., less than the expected demand?

Generalize to multiple fare classes

- Assumptions
  - $n$ fare classes, each with an associated fare. Fare classes are numbered in descending order: $p_1 > p_2 > \ldots > p_n$
  - Demand in each class $i$: $D_i$
    - $F_i(x)$: probability the class $i$ demand is less than or equal to $x$
  - Capacity is $C$
  - Lowest-fare customers (i.e., those paying $p_n$) book first, and the highest-fare customers (i.e., those paying $p_1$) book last
  - Demands are independent.
Booking multiple fare classes – nested allocation

Suppose we have 3 fare classes

\[ C_3 : \text{total available seats} \]

\[ b_3 = \text{Booking limit for class 3} \]

\[ Q_1 = \text{Protection level for class 1} \]

\[ Q_2 = \text{Protection level for classes 1 and 2} \]

\[ C_2 = \text{Remaining capacity} \]

\[ b_2 = \text{Booking limit for class 2} \]

\[ Q_1 = \text{Protection level for class 1} \]

Similar logic but we now have to consider displacing two classes of customers:

opportunity cost \(= p_1 - p_3 \) or \( p_2 - p_3 \)

\( \Rightarrow \) With a large number of classes, this becomes computationally heavy

Heuristic approach: EMSR-b

- Suppose you want to determine booking limit of class \( i+1 \)
- Idea
  - Create an “artificial class” by aggregating the demands of classes \( i \) and higher:
  - Suppose that class \( i \) demand \( D_i \) is normally distributed with the mean \( \mu_i \) and standard deviation \( \sigma_i \)
    \[ D_1 + D_2 + \ldots + D_i \sim \text{Normal}(\sum \mu_i, \sum \sigma_i^2) \]
  - Price of the artificial class:
    \[ \frac{p_1 \mu_1 + p_2 \mu_2 + \ldots + p_i \mu_i}{\mu_1 + \mu_2 + \ldots + \mu_i} \]
Multiple class example

- Number of coach seats on the plane = 220
- Three fares
  - Class 1 fare = $100
  - Class 2 fare = $75
  - Class 3 fare = $50
- Demand forecasts for normally distributed
  - $\mu_1 = 150$, $\sigma_1 = 20$
  - $\mu_2 = 50$, $\sigma_2 = 15$
  - $\mu_3 = 120$, $\sigma_3 = 25$

Step 1: Set protection level for Class 1

- Class 1
  - $p_1 = $100
  - $D_1 \sim N(150, 20^2)$
- Class 2
  - $p_2 = $75
  - $D_2 \sim N(50, 15^2)$
- Littlewood's rule:
  - $1 - F_1(Q_1^*) = p_2/p_1$
- Reserve 136 seats for Class 1
Step 2: “Artificial class” - Aggregate Class 1 and Class 2

• Effective demand
  • Recall: $\mu_1 = 150$, $\sigma_1 = 20$ and $\mu_2 = 50$, $\sigma_2 = 15$
  • $D_{1+2} \sim \text{Normal with mean and variance}$
    $\mu_{1+2} = \mu_1 + \mu_2 = 150 + 50 = 200$
    $\sigma_{1+2}^2 = \sigma_1^2 + \sigma_2^2 = 20^2 + 15^2 = 625 \rightarrow \sigma_{1+2} = 25$

• Effective price
  • Recall: $p_1 = $100, $p_2 = $75
  $$p_{1+2} = \frac{\mu_1 p_1 + \mu_2 p_2}{\mu_1 + \mu_2} = \frac{150 \times 100 + 50 \times 75}{150 + 50} = $93.75$$

Step 3: Set protection level for Classes 1+2

• Classes 1+2
  - $p_{1+2} = $93.75
  - $D_{1+2} \sim \text{N}(200, 25^2)$
• Class 3
  - $p_3 = $50
  - $D_3 \sim \text{N}(120, 15^2)$

• Littlewood’s rule:
  - $1- F_{1+2}(Q_{1+2}) = p_3/p_{1+2}$
• Reserve 198 seats for Class 1 and Class 2 combined
  - 136 for Class 1 (previous slide)
  - 198 for Class 1 and Class 2
  - This leaves 22 seats for Class 3 (note that expected demand for Class 3 is 120).