Measuring Substitution Patterns in Differentiated Product Industries

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Motivation

- **The Beauty of BLP:** Flexible estimation of substitution patterns with *many* products, aggregate data, and unobserved attributes.
- Achieving this flexibility can be difficult in practice...
  - **Precision:** Often rely on external restrictions (e.g. supply, survey, etc.)
  - **Numerical:** Multiple solutions and/or poor convergence properties

What explains the difficulties in practice?

- Is the variation in data simply too weak?
- Or is it weakness of the instruments (IVs)?
  - i.e., Are we using the variation in the data in the optimal way?
- **Our paper** argue that many *empiricists'* problems could be caused by **weak** IVs
- Show how to construct **strong** IVs using a new approximation to the optimal instruments of the model.
  - Building on insights from BLP (1995) [Especially Sections 5.1-5.2!]

Key Takeaways

- **Differentiation IV:** Capture the relative position of each product in the characteristic space
  - Approximate optimal IV without requiring initial estimates
  - Simple to construct and test
- Powerful in practice:
  - 10+ improvement in precision
  - Fast convergence + Numerically stable
  - Flexible substitution: Multiple dimensions + Correlated heterogeneity
- Related work:
  - **Weak IV in BLP:** BLP (1999), Conlon (2013), Reynaert & Verboven (2013), Metaxoglou and Knittel (2014)
  - **Differentiation IV:** *Nested-Logit* (e.g. Berry (1994), Verboven (1996), Bresnahan et al. (1997)), and *Spatial Differentiation* (e.g. Davis (2006), Thomadsen (2005), Manuszak (2012), Houde (2012))

A Motivating Example

- Linear random-coefficient model with **exogenous** characteristics:

$$\sigma_j\left(\boldsymbol{\delta}_t, \boldsymbol{x}_t^{(2)}; \sigma_x\right) = \int \frac{\exp\left(\delta_{jt} + \sigma_x v_i x_{jt}^{(2)}\right)}{1 + \sum_{j'=1}^{n_t} \exp\left(\delta_{j't} + \sigma_x v_i x_{j't}^{(2)}\right)} \phi(v_i) dv_i$$

where $\delta_{jt} = \beta_0 + \beta_1 x_{jt}^{(1)} + \beta_2 x_{jt}^{(2)} + \xi_{jt}$.

- Invert demand system to obtain model residual:

$$\xi_{jt}(\boldsymbol{\theta}) = \sigma_j^{-1}\left(\boldsymbol{s}_t, \boldsymbol{x}_t^{(2)}; \sigma_x\right) - \beta_0 - \beta_1 x_{jt}^{(1)} - \beta_2 x_{jt}^{(2)}$$

- **Berry-Haile:** There exists a simultaneity problem associated with σ_x due to **endogenous shares**, <u>even</u> without prices.

Identification and Estimation

- Conditional moment restriction

$$E\left[\xi_{jt}(\boldsymbol{\theta})|\boldsymbol{x}_t\right] = 0 \quad \text{iff} \quad \boldsymbol{\theta} = \boldsymbol{\theta}^0 \text{ (i.e. true).}$$

- Non-linear IV estimation:

$$E\left[\xi_{jt}\left(\boldsymbol{\theta}^0\right) \times A_j(\boldsymbol{x}_t)\right] = 0.$$

For any $A_j(\cdot)$ with

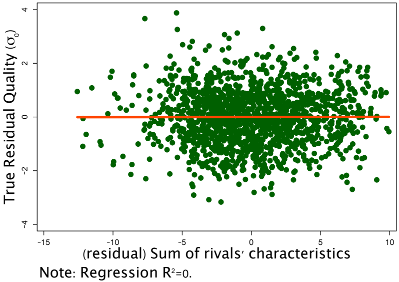
$$\dim\left(A_j\left(\boldsymbol{x}_t\right)\right) \geq \dim\left(\boldsymbol{\theta}\right)$$

- Does the choice of instruments $A_j(\boldsymbol{x}_t)$ matter?

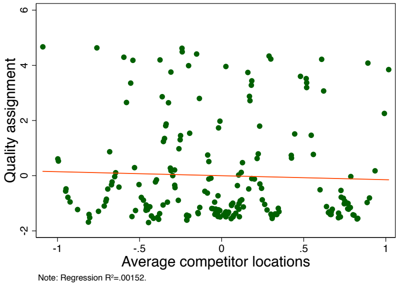
Identification in a Picture

Market IV: Sum of rivals' characteristics

(A) Residual quality at $\sigma_x = \sigma_x^0$



(B) Residual quality at $\sigma_x = 0$



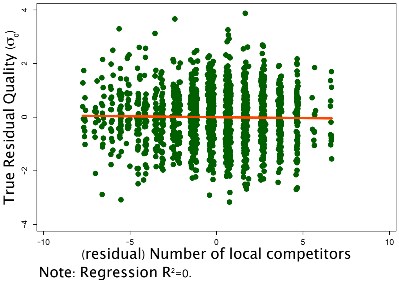
- **Moment restriction:** Independence of ξ_{jt} and the *sum of rivals characteristics* **cannot** distinguish (A) from (B)
- **Weak identification:** The moment restrictions are "almost" satisfied at $\sigma_x \neq \sigma_x^0$ (Stock and Wright, 2000).

▸ Data-set: 15 products × 100 markets.

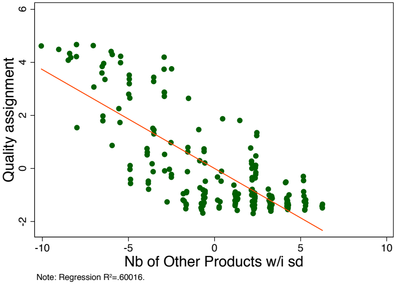
Identification in a Picture

Differentiation IV: Number of competitors located within 1 std-dev of $x_{jt}^{(2)}$

(A) Residual quality at $\sigma_x = \sigma_x^0$



(B) Residual quality at $\sigma_x = 0$



- **Moment restriction:** Independence of ξ_{jt} and the *number of close competitors* rules out (B).
  - **Testable implication:** *Strong* instruments must reject $H_0 : \hat{\sigma}_x = 0$
- Let's formalize this intuition for the general model:

$$\boldsymbol{\beta}_i^{(2)} = \boldsymbol{\beta}^{(2)} + \boldsymbol{v}_i, \quad \Pr(\boldsymbol{v}_i) = F_v(\boldsymbol{v}_i; \boldsymbol{\Sigma})$$

Optimal IV

- The "optimal" IV for $\boldsymbol{\Sigma}$ is (Amemyia (1977), Chamberlain (1987)):

$$D_{j,k}\left(\boldsymbol{x}_t;\boldsymbol{\Sigma}^0\right) = E\left[\frac{\partial\sigma_j^{-1}\left(\boldsymbol{s}_t,\boldsymbol{x}_t^{(2)};\boldsymbol{\Sigma}^0\right)}{\partial\boldsymbol{\Sigma}_k}\middle|\boldsymbol{x}_t\right]$$

- **How?** Approximate the optimal IV by estimating a non-parametric first-stage regression (Newey, 1990)

- **Curse of Dimensionality:** $D_{j,k}\left(\boldsymbol{x}_t\right)$ is a *j-specific* function of the entire menu of characteristics.
  - First order: $D_{j,k}(\boldsymbol{x}_t;\boldsymbol{\Sigma}^0) \approx \sum_{j=1}^{n_t}\boldsymbol{x}_{jt}\boldsymbol{a}_j$
  - Impossible: BLP (1995)

- **Bottom line:** If we can't estimate the first-stage, how can we construct powerful instruments?

What does the characteristic structure imply for the reduced-form of the model?

- Market-structure facing product j (dropping t):

$$(\boldsymbol{w}_j, \boldsymbol{w}_{-j}) \equiv \left(\left(\delta_j, \boldsymbol{x}_j^{(2)}\right), \left(\boldsymbol{\delta}_{-j}, \boldsymbol{x}_{-j}^{(2)}\right)\right)$$

- Properties of the linear-in-characteristics model:
  - Symmetry:

$$\sigma_j\left(\boldsymbol{w}_j, \boldsymbol{w}_{-j}\right) = \sigma_k\left(\boldsymbol{w}_j, \boldsymbol{w}_{-j}\right) \quad \forall k \neq j$$

  - Anonymity:

$$\sigma\left(\boldsymbol{w}_j, \boldsymbol{w}_{-j}\right) = \sigma\left(\boldsymbol{w}_j, \boldsymbol{w}_{\rho(-j)}\right) \forall \rho$$

  - Translation invariant: for any $\boldsymbol{c} \in \mathbb{R}^K$

$$\sigma\left(\boldsymbol{w}_j + (0, \boldsymbol{c}), \boldsymbol{w}_{-j} + (0, \boldsymbol{c})\right) = \sigma\left(\boldsymbol{w}_j, \boldsymbol{w}_{-j}\right)$$

Re-Express the Demand System

- Express the "state" of the market in *differences* relative to j and treat the outside option just like any other product.
  - Characteristic differences:

$$\boldsymbol{d}_{j,k}^{(2)} = \boldsymbol{x}_k^{(2)} - \boldsymbol{x}_j^{(2)}$$

  - New normalization:

$$\tau_j = \frac{\exp(\delta_j)}{1 + \sum_{j'} \exp(\delta_{j'})}, \forall j = 0, \ldots, n.$$

  - Product k attributes: $\boldsymbol{\omega}_{j,k} = \left(\tau_k, \boldsymbol{d}_{jt,k}^{(2)}\right)$
- Demand for product j is a fully exchangeable function of $\boldsymbol{\omega}_j$:

$$\sigma\left(\boldsymbol{w}_j, \boldsymbol{w}_{-j}\right) = \mathcal{D}(\boldsymbol{\omega}_j)$$

where $\boldsymbol{\omega}_j = \{\omega_{j,0}, \ldots, \omega_{j,j-1}, \omega_{j,j+1}, \ldots, \omega_{j,n}\}$.

Main Theory Result

- Define the *exogenous* state of the market facing product j:

$$\begin{aligned}\boldsymbol{d}_{j,k} &= \boldsymbol{x}_k - \boldsymbol{x}_j \\ \boldsymbol{d}_j &= \left(\boldsymbol{d}_{j,0}, \ldots, \boldsymbol{d}_{j,j-1}, \boldsymbol{d}_{j,j+1}, \ldots, \boldsymbol{d}_{j,n}\right)\end{aligned}$$

Theorem

If the distribution of $\{\xi_j\}_{j=1,\ldots,n}$ is exchangeable, then the reduced form becomes

$$D_{j,k}\left(\boldsymbol{x}; \boldsymbol{\Sigma}^0\right) = E\left[\left.\frac{\partial \sigma_j^{-1}\left(\boldsymbol{s}, \boldsymbol{x}^{(2)}; \boldsymbol{\Sigma}^0\right)}{\partial \Sigma_k}\right| \boldsymbol{x}\right] = h_k\left(\boldsymbol{d}_j\right)$$

where h_k is a **symmetric** *function of the state vector.*

- **Implication:** h_k is a *vector symmetric function* (see Briand 2009)

Why is it useful?

1. **Curse of dimensionality:** In any fixed order approximation the number of basis functions is *independent* of the number of products.
   - A vector generalization of Pakes (1994), Pakes and McGuire (1994).

2. **Examples:**
   - First-order (i.e. market):

$$\sum_{j' \neq j} \boldsymbol{d}_{jt,j'} \left(\equiv \sum_{j' \neq j} \boldsymbol{x}_{jt} \right)$$

   - Second-order (i.e. distance):

$$\sum_{j' \neq j} \left(\boldsymbol{d}_{jt,j'}^{k} \right)^2, \quad \forall k = 1, \ldots, K$$

   - Histogram basis (i.e. nested-logit or hotelling):

$$\sum_{j' \neq j} 1(|d_{jt,j'}^{k}| < \kappa_k) \boldsymbol{d}_{jt,j'}, \quad \forall k = 1, \ldots, K$$

Closing the loop

- Let $A_j(\boldsymbol{x}_t)$ be an L vector of basis functions summarizing the empirical distribution of characteristic differences: $\{\boldsymbol{d}_{jt,k}\}_{k=0,\dots,n_t}$.
- **Differentiation IV:** These functions are moments describing the relative isolation of each product in characteristic space.
- **Donald, Imbens, and Newey (2003):** Using basis functions directly as IVs, is asymptotically equivalent to approximating the optimal IV.
  - Recommended practice is to use low-order basis functions (Donald, Imbens, and Newey 2008).
- Alternatively use Lasso methods with higher order polynomials (e.g. Belloni, Chernozhukov, and Hansen (2012))

Experiment 1: Exogenous Characteristics

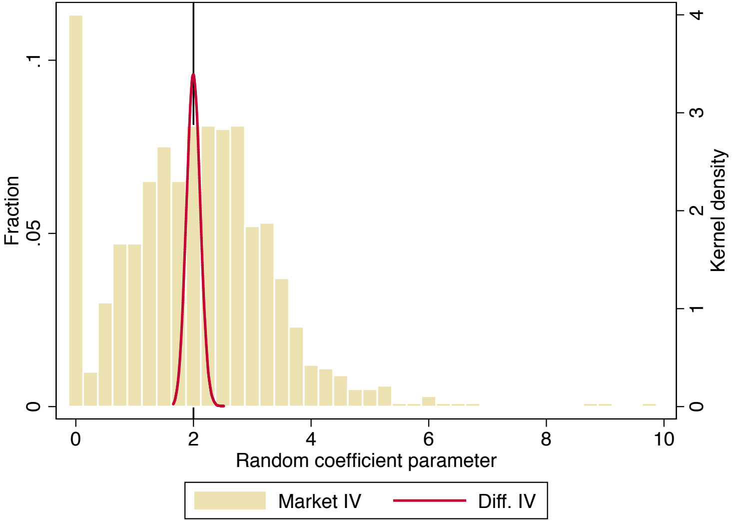
- Random coefficient model:

$$u_{ijt} = \delta_{jt} + \sum_{k=1}^{K} v_{ik} x_{jt,k}^{(2)} + \varepsilon_{ijt}, \quad v_i \sim N(0, \sigma_x^2 \boldsymbol{I}).$$

- Data:
  - Panel structure: 100 markets $\times$ 15 products
  - Characteristics: $(\xi_{jt}, \boldsymbol{x}_{jt}) \sim N(0, \mathbf{I})$.
  - Dimension: $|\boldsymbol{x}_{jt}| = K + 1$
  - Monte-Carlo replications = 1,000
- Instruments $(K+1)$:
  - Market IVs (first-order): $A_j(\boldsymbol{x}_t) = \sum_{l \neq j}^{n_t} x_{lt,k}, \forall k = 1, \ldots, K$
  - Diff. IVs (2nd-order): $A_j(\boldsymbol{x}_t) = \sum_{j'=1}^{n_t} \left(d_{jt,j'}^{k} \right)^2, \forall k = 1, \ldots, K$

Result 1: Weak IV Problem

Specification: One dimension of heterogeneity



Result 2: Weak IVs with Multiple Dimensions

VARIABLES	Dimensions of consumer heterogeneity			
	1	2	3	4
σ_1	2.012	1.931	1.919	2.055
	(1.3)	(1.423)	(1.383)	(1.467)
σ_2		1.978	1.934	2.055
		(1.302)	(1.313)	(1.412)
σ_3			1.992	2.088
			(1.353)	(1.564)
σ_4				2.106
				(1.494)
Nb. IVs	2	3	4	5

Parenthesis: RMSE. Starting at true parameter = 2.

Result 3: Differentiation IVs with Multiple Dimensions

VARIABLES	Dimensions of consumer heterogeneity				
	1	2	3	4	6
σ_1	2.002	1.998	1.994	1.994	1.991
	(.107)	(.109)	(.105)	(.111)	(.115)
σ_2		1.995	1.996	1.992	1.993
		(.108)	(.105)	(.111)	(.115)
σ_3			1.995	1.994	1.991
			(.109)	(.113)	(.116)
σ_4				1.995	1.990
				(.11)	(.115)
σ_5					1.997
					(.118)
σ_6					1.993
					(.118)
Nb. IVs	2	3	4	5	7

Parenthesis: RMSE. Starting at true parameter = 2.

Result 4: Weak IVs and Numerical Problems

Simulation: 1,000 samples and 10 random starting values each

	Differentiation IV		Market IV	
	Est.	RMSE	Est.	RMSE
σ_1	1.994	0.111	2.107	1.536
σ_2	1.992	0.111	2.053	1.477
σ_3	1.994	0.113	2.088	1.618
σ_4	1.995	0.110	2.131	1.640
Algorithm	Simplex		Simplex	
Nb. IVs	5		5	
CPU time (sec)	0.718		33	
Local optima (fraction)	0.002		0.53	
Global min/Local min	46.6		53	

Result 4: Weak IVs and Numerical Problems

Simulation: 1,000 samples and 10 random starting values each

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	Est.	RMSE	Est.	RMSE
σ_1	1.994	0.111	2.107	1.536
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Algorithm	Simplex		Simplex	
Nb. IVs	5		5	
CPU time (sec)	0.718		33	
Local optima (fraction)	0.002		0.53	
Global min/Local min	46.6		53	

Experiment 2: Correlated Random Coefficients

- Consumer heterogeneity:

$$\beta_i^{(2)} \sim N(\beta^{(2)}, \mathbf{\Sigma})$$

  Note: 4 dimensions $\Rightarrow$ 10 non-linear parameters (choleski)

- Panel structure:

$$100 \text{ markets} \times 50 \text{ products}$$

- **Differentiation IVs:** Second-order polynomials (with interactions):

$$\sum_{j'=1}^{n_t} \left(d_{jt,j'}^k\right)^2 \text{ and } \sum_{j'=1}^{n_t} \left(d_{jt,j'}^k \times d_{jt,j'}^l\right)$$

  for all characteristics k and l.

Simulation Results: Unrestricted Covariances

	$\Sigma_{.1}$	$\Sigma_{.2}$	$\Sigma_{.3}$	$\Sigma_{.4}$
Estimated parameters:				
$\Sigma_{1.}$	4.066			
	(.214)			
$\Sigma_{2.}$	-2.080	4.268		
	(.132)	(.242)		
$\Sigma_{3.}$	1.993	0.697	4.502	
	(.143)	(.154)	(.212)	
$\Sigma_{4.}$	-1.147	-0.405	-0.458	3.050
	(.123)	(.124)	(.135)	(.204)
True parameters:				
$\Sigma_{1.}$	4.064			
$\Sigma_{2.}$	-2.083	4.270		
$\Sigma_{3.}$	1.995	0.689	4.505	
$\Sigma_{4.}$	-1.152	-0.398	-0.462	3.071
CPU time (sec)	3.151			

Parenthesis: RMSE. Starting values = true. Algorithm: Gauss-Newton.

Weak IV results

How to incorporate endogenous prices?

- Vertical differentiation example:

$$u_{ijt} = \delta_{jt} - \alpha_i p_{jt} + \varepsilon_{ijt}$$

  where $\alpha_i = \sigma_p y_i^{-1}$, and $\log(y_i) \sim \text{N}(\mu_y, \sigma_y)$ (known).

- Price instruments: $\boldsymbol{w}_t = \{w_{jt}\}_{j=1,\dots,n_t}$
- New reduced-form:

$$D_j(\boldsymbol{x}_t, \boldsymbol{w}_t; \sigma_p^0) = E\left(\frac{\partial \sigma_j^{-1}(\boldsymbol{s}_t, \boldsymbol{x}_t, \boldsymbol{p}_t | \sigma_p^0)}{\partial \sigma_p} \middle| \boldsymbol{x}_t, \boldsymbol{w}_t \right) \neq h(\boldsymbol{d}_j^x, \boldsymbol{d}_j^p)$$

- **Solution:** Use insights from BLP (1999) and Reynaert & Verboven (2013) to take expectation for price *inside*:

$$D_j(\boldsymbol{x}_t, \boldsymbol{w}_t; \sigma_p^0) \approx h\left(\boldsymbol{d}_j^x, \hat{\boldsymbol{d}}_j^p\right)$$

  I.e., $d_{jt,k}^p = p_{kt} - p_{jt}$ is replaced by $\hat{d}_{jt,k}^p = E(p_{kt}|\boldsymbol{w}_{kt}) - E(p_{jt}|\boldsymbol{w}_{jt})$.

Conclusion

- What did we do:
  - Show how that the characteristic model can be used to construct **relevant** instruments to identify substitution patterns
  - And, eliminate the weak IV problem that is present in applied work
  - *Differentiation IV's*: Capture the relative position of each product in the characteristic space.
- Extensions (in paper):
  - **Endogenous prices**
  - **Natural experiments**
  - **Demographic variation**
  - **Weak IV tests**
- What's next?
  - Higher-order basis: Lasso
  - Conduct tests
  - Non-parametric estimation

DGP: Exogenous Characteristics

- Random coefficient model:

$$u_{ijt} = \beta_0 + \beta_1 x_{jt}^{(1)} + \beta_2 x_{jt}^{(2)} + \xi_{jt} + \sigma_x v_i x_{jt}^{(2)} + \varepsilon_{ijt}.$$

- Unobserved heterogeneity:
  - $v_i \sim N(0, 1)$
  - Numerical integration: 10 points Gauss-Hermite quadrature.
- Data:
  - Panel structure: 100 markets $\times$ 15 products
  - Characteristics: $(\xi_{jt}, x_{jt}) \sim N(0, \mathbf{I})$.
  - Dimension: $|x_{jt}| = K + 1$

Return

Weak IV: BLP (1999) with **random** starting values

	$\Sigma_{.1}$	$\Sigma_{.2}$	$\Sigma_{.3}$	$\Sigma_{.4}$
Estimated parameters:				
$\Sigma_{1.}$	4.491			
	(3.136)			
$\Sigma_{2.}$	-2.047	4.394		
	(1.379)	(1.57)		
$\Sigma_{3.}$	2.059	0.747	4.558	
	(.735)	(.41)	(.355)	
$\Sigma_{4.}$	-1.243	-0.389	-0.484	3.089
	(.497)	(.307)	(.264)	(.256)
True parameters:				
$\Sigma_{1.}$	4.064			
$\Sigma_{2.}$	-2.083	4.270		
$\Sigma_{3.}$	1.995	0.689	4.505	
$\Sigma_{4.}$	-1.152	-0.398	-0.462	3.071
CPU time (sec)	191.690			
Singular matrix (fraction)	0.24			

Return

Extension: Endogenous Prices

- **Demand:** Mixed-logit with vertical differentiation
  - Indirect utility

$$u_{ijt} = \delta_{jt} - \alpha_i p_{jt} + \varepsilon_{ijt}$$

where $\alpha_i = \sigma_p y_i^{-1}$, and $\log(y_i) \sim \mathrm{N}(\mu_y, \sigma_y^2)$ (known).

- **Supply:** Bertrand-Nash with multi-product firms
  - Marginal cost: $\log(\mathrm{mc}_{jt}) = \gamma_0 + \gamma_x x_{jt} + \gamma_\xi \xi_{jt} + \omega_{jt}$

Differentiation IVs with Endogenous Prices

- How to incorporate endogenous prices into Differentiation IV's?
- **Solution:** Use the exogenous component of prices
  - Let w_{jt} denote a valid/relevant price IV
  - Exogenous differences:

    $$\hat{d}^{p}_{jt,j'} = \hat{p}_{j't} - \hat{p}_{jt}$$

    where $\hat{p}_{jt} = E(p_{jt}|x_{jt}, \omega_{jt})$
  - Similar approach in Reynaert and Verboven (2013)
- Examples:

$$\sum_{j' \in \mathcal{F}_{ft}} \left(\hat{d}^{p}_{jt,j'}\right)^2 \text{ and } \sum_{j' \notin \mathcal{F}_{ft}} \left(\hat{d}^{p}_{jt,j'}\right)^2$$

The weak IV problem is very severe without cost-shifters

Panel structure: $\bar{J} = 50$ and $T = 10$

		IV: Sum of charact.		IV: Local competitors	
		w/o cost	w/ cost	w/o cost	w/ cost
β_p					
	Average	0.46	1.08	1.00	1.02
	RMSE	2.19	1.32	0.22	0.18
σ_p					
	Average	13.24	17.47	19.10	19.68
	RMSE	10.84	7.95	3.93	1.51

Simulation Results: Optimal IV Approximation

- **Optimal IV:** Jacobian at $\xi_{jt} = 0$ (Berry et al. (AER, 1999), and Reynaert & Verboven (JE, 2013)) :

$$E\left(\frac{\partial \sigma_j^{-1}(s_t, x_t, p_t|\theta^0)}{\partial \theta}\bigg| x_t\right) \approx \frac{\partial \sigma_j^{-1}(s_t, x_t, \hat{p}_t|\hat{\theta})}{\partial \theta}\bigg|_{\xi_t=0}$$

Where $\hat{\theta}$ is a **first-stage** estimate using IVs.

		IV: Sum of charact.		IV: Local competitors	
		BLP (1995)	Opt. IV	Diff. IV	Opt. IV
β_p					
	Average	0.46	1.29	1.00	1.16
	RMSE	2.19	0.93	0.22	0.45
σ_p					
	Average	13.24	16.61	19.10	17.28
	RMSE	10.84	28.23	3.93	19.07

Return

Extension: Demographic Panel

- In many settings, product characteristics are fixed across markets, but the distribution of consumer types vary (e.g. Nevo 2001).
- Focus on a single characteristics x_j
- **Assumption:** Consumer heterogeneity η_{it} follows distribution of a demographic can be decomposed as

$$\eta_{it} = \mu_t + \sigma_t \nu_i$$

  where (μ_t, σ_t) are known, and $\Pr(\nu_i < x) = F(x)$ is common across t.
- Preferences

$$\begin{aligned} u_{ijt} &= \delta_{jt} + \theta \eta_{it} x_j + \varepsilon_{ijt} \\ &= \tilde{\delta}_{jt} + \theta \nu_i \tilde{x}_{jt} + \varepsilon_{ijt} \end{aligned}$$

  where $\tilde{x}_{jt} = \sigma_t x_j$, and $\delta_{jt} = x_j(\beta + \theta \mu_t) + \xi_{jt} = x_j \beta_t + \xi_{jt}$.
- Differentiation IVs can be constructed as before: $d^x_{jt,k} = \tilde{x}_{kt} - \tilde{x}_{jt}$.

Return

Extension: Evaluating the Relevance of Instruments

- **Ex-post:** First-stage test
  - First-stage of the non-linear GMM problem at $\hat{\theta}^{gmm}$:

$$J_{jt,k}(\hat{\theta}^{gmm}) = \pi_x x_{jt} + \pi_z z_{jt} + e_{jt}$$

    where $J_{jt,k} = \partial\sigma^{-1}/\partial\theta_k$.
  - Standard tests for weak instruments in linear models can be used to test the relevance of z_{jt} (Wright, 2003).

- **Ex-ante:** IIA test
  - A **strong** instrument for Σ is able to reject the wrong model (Stock and Wright, 2000)
  - Under $H_0 : \Sigma = 0$, the inverse demand equation is independent of x_{-j}:

$$\sigma_{jt}^{-1}(s_t, x_t; \theta = 0) = \ln s_{jt}/s_{0t} = x_{jt}\beta + z_{jt}\gamma + \xi_{jt}$$

  - Standard test statistics for $H_0 : \gamma = 0$, can be used to test null hypothesis of IIA preferences

Return

Weak IV Tests: Endogenous prices experiment

		Market (1)	Neighborhood (2)	Second order (3)
No cost shifter				
IIA test	χ^2	2.31	70.28	99.15
	p-value	0.47	0.00	0.00
Weak IV (θ^0)	χ^2	2.21	355.52	957.38
	p-value	0.48	0.00	0.00
Degree of freedom		0	0	6

Return

Extension: Natural Experiments

- **Hotelling example:** Exogenous entry of a new product ($x' = 5$)
- Three-way panel: product j, market m, and time ($t = 0, 1$).
- Treatment variable:

$$D_{jm} = 1\left(|x_{jm} - 5| < \text{Cutoff}\right)$$

- First-stage difference-in-difference regression:

$$J_{jmt}(\theta) = \hat{\mu}_{jm} + \hat{\tau}_t + \hat{\gamma} D_{jm} \times 1(t = 1) + \hat{e}_{jmt}$$

- GMM Problem:
  - Linear characteristics: $x^{(1)}_{jmt}$ = Market/Product FE + After Dummy
  - Differentiation IV: $z_{jmt} = D_{jm} \times 1(t = 1)$
  - $\hat{\theta}^{gmm}$ is identified from the DiD variation in z_{jmt}.

Return