An industry dynamic model with price controls

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We describe a dynamic model of entry, exit and product choice in an industry that is subject to a price floor regulation. The model illustrates the role of price controls in distorting the structure of retail markets. We are particularly interested in characterizing the potential effects of regulation on the number of and type of competitors and the resulting prices.

1 Model

The model has two components: (i) a static pricing game, and (ii) a dynamic entry and product choice game. We first describe the pricing game and then the industry dynamic equilibrium. Finally, we compare the equilibria with and without price regulation using numerical examples.

1.1 Demand and prices

In the short-run the structure of the market is constant and firms post prices simultaneously. There are two types of firms, large and small (i.e. l and s). These two types differ with respect to their quality ($\delta_l \geq \delta_s$) and their constant marginal cost ($c_l \leq c_s$). Firms face a passive outside competitor that provides a good with net value ν. The market structure is therefore described by the number of active firms of each type and the value of the outside option. The state of the industry is further characterized by the number n_s, n_l of active firms and an exogenous price floor $p_f \geq 0$ set by the government. We group these state variables in a vector $\omega = \{n_s, n_l, \nu, p_f\}$. For simplicity we assume that the state space is finite.

To characterize demand at each store, we use a logit model with congestion similar to the model proposed by Ackerberg and Rysman (2005), who suggest the addition of congestion to the logit model of product differentiation to mimic the localized nature of competition in retail markets. Since in the logit model differentiation is measured by the variance of consumers' idiosyncratic tastes for products, we let it depend on the number of competitors as follows:

$$\sigma(n) = \frac{1}{\mu} n^{-\mu}, \tag{1}$$

where $0 < \mu < 1$.

Demand for product $j \in \{s, l\}$ is therefore given by:

$$D_j\left(p|\omega\right) = M \frac{e^{(\delta_j - p_j)/\sigma(n)}}{e^{\nu/\sigma(n)} + \sum_k e^{(\delta_k - p_k)/\sigma(n)}}. \tag{2}$$

Without a price floor constraint, a symmetric price equilibrium is described by two prices $p(\omega) = \{p_s(\omega), p_l(\omega)\}$ solving the following FOCs:

$$D_j\left(p(\omega)\right) - M\frac{1}{\sigma(n)}\left(p_j(\omega) - c_j\right)s_j\left(1 - s_j\right) = 0, \quad j \in \{s, l\} \tag{3}$$

where $s_j = \frac{e^{\left(\delta_j - p_j\right)/\sigma(n)}}{e^{\nu/\sigma(n)} + \sum_k e^{\left(\delta_k - p_k\right)/\sigma(n)}}$.

When firms are constrained by a price floor $p_f > 0$ the equilibrium is characterized by four Kuhn-Tucker conditions:

$$D_j\left(p(\omega)\right) - M\frac{1}{\sigma(n)}\left(p_j(\omega) - c_j\right)s_j\left(1 - s_j\right) + \lambda_j = 0 \tag{4}$$

$$\lambda_j\left(p_j\left(\omega\right) - p_f\right) = 0 \tag{5}$$

for each $j \in \{s, l\}$ and $\lambda_j \geq 0$. Throughout the paper we focus on cases that satisfy the following two properties:[1]

If $n_j > 0$ for all j: $p_l(\omega) \leq p_s(\omega)$,
If $n = 1$: $p_l(\omega) \leq p_s(\omega)$.

These conditions simply mean that the large firm always charges a weakly lower price in equilibrium, both in oligopoly and monopoly setups. As a result the price floor will generate three possible outcomes: (i) no prices are constrained ($\lambda_l = \lambda_s = 0$), (ii) both prices are constrained ($p_s = p_l = p_f$), or (iii) only the large firm is constrained ($\lambda_s = 0$ and $p_l = p_f$). We assume that a symmetric Nash equilibrium satisfying these conditions exists and is unique. Let $\pi_j(\omega)$ denote the static equilibrium profit of type j firm.

1.2 Entry and exit

In the long-run incumbents are able to adjust their configurations and new firms can enter the market. Conditional on their current type, the state of the industry and a vector of privately observed profitability shocks $\epsilon = \{\epsilon_o, \epsilon_s, \epsilon_l\}$, firms simultaneously choose between three options: (i) small configuration, (ii) large configuration, (iii) exit/stay-out. As in Rust (1987) we assume that the private information component of the state is iid across players and time, and distributed according to a double-exponential distribution (i.e. multinomial logit).

The timing of actions is as follows. At the beginning of the period firms observe the state of the industry ω and their private information shock ϵ. Firms then simultaneously commit to a price and a configuration choice, and profits are realized. Entry/exit and reconfiguration actions are taken at the end of the period.

Potential entrants face the same problem but live for only one period as in Pakes and McGuire (1994). For tractability we assume that only one firm can enter every period, and that no more than $\bar{n}$ firms can be active.

We follow Aguirregabiria and Mira (2007) and Doraszelski and Satterthwaite (2009) in defining a Markov-perfect Bayesian equilibrium. Because actions are stochastic (prior to observing ϵ),

[1] These conditions are implicit constraints on the cost and quality parameters. In particular, the cost efficiency of the large firm dominates the effect of quality on prices.

players use choice probabilities $\sigma_j(a|\omega)$ to form beliefs about their opponents' actions. We focus on symmetric Markovian strategies. Therefore these probabilities are symmetric and depend only on the current observed state vector ω.

Given beliefs σ, an incumbent's problem is described by the following Bellman equation:

$$V_j^\sigma(\omega, \epsilon) = \max_{a \in \{o,s,l\}} \pi_j(\omega) - F_j - K(j,a) + \epsilon_a + \beta \sum_{\omega'} EV_a^\sigma(\omega') F^\sigma(\omega'|\omega, a) \tag{6}$$

where $EV_j^\sigma(\omega') = \int V_j^\sigma(\omega', \epsilon') f(\epsilon') d\epsilon'$. The adjustment cost function $K(j,a)$ incorporates both the cost of entering and reconfiguring an existing store, as well as the cleaning cost of leaving the market. We parametrize the function as follows:

$$K(j,a) = \begin{cases} 0 & \text{if } j \neq a \\ \kappa & \text{if } j = o \text{ and } a \neq o \\ \kappa + x & \text{if } j \neq o \text{ and } a \neq j \\ x & \text{if } j \neq o \text{ and } a = o \end{cases} \tag{7}$$

Once firms leave the market, they receive a payoff ϵ_o and are not able to re-enter. The expected continuation value of being out of the market is therefore zero. The problem of potential entrants is described by:

$$V_o^\sigma(\omega, \epsilon) = \max \left\{ \epsilon_o, \max_{a \in \{s,l\}} -\kappa + \epsilon_a + \beta \sum_{\omega'} EV_a^\sigma(\omega') F^\sigma(\omega'|\omega, a) \right\}. \tag{8}$$

The transition probability matrix $F^\sigma(\omega'|\omega, a)$ is constructed from the beliefs probabilities σ and the transition matrix for the outside option valuation $G(\nu_0'|\nu)$. Let $A_{-j}(\omega', \omega, a)$ be the set of possible actions that player j's opponents can take in state ω in order to reach state ω'. The transition probability function is then given by:

$$F^\sigma(\omega'|\omega, a) = \Pr\left[\{n_s', n_l', \nu', p_f\}|\omega, a, \sigma\right] = \sum_{a_{-j} \in A_{-j}(\omega', \omega, a)} \prod_i \sigma_i(a_i|\omega) G(\nu'|\nu). \tag{9}$$

Given beliefs σ and the assumed distribution of the unobserved private information, the best-response choice probability of a firm of type j takes a multinomial logit form. A Markov-perfect Bayesian Nash equilibrium is defined as a fixed point in the belief choice probabilities:

$$\sigma_j(a|\omega) = \frac{\exp\left(v_j^\sigma(a|\omega)\right)}{\sum_{k \in \{o,s,l\}} \exp\left(v_j^\sigma(k|\omega)\right)}, \tag{10}$$

where $v_j^\sigma(a|\omega) = \pi_j(\omega) \times 1\,(a \neq o) - K(j,a) + \beta EV^\sigma(\omega') F^\sigma(\omega'|\omega, a)$ is the choice specific value function prior to observe ϵ_a.

1.3 Numerical examples

In this section we discuss several numerical examples to illustrate the properties of the dynamic game and the impact of price floor constraints on the dynamics of the industry. Table 1 presents

Table 1: Parameter values for the numerical simulations

	Specification 1	**Specification 2**
Demand parameters		
M	5	5
δ_s	0	0
δ_l	0	0
ν	1.5	1.5
μ	$3/4$	$3/4$
Marginal costs		
c_s	$1/2$	$1/2$
c_l	0	0
Fixed costs		
F_s	1.2	1.25
F_l	2.7	2.85
Entry/exit costs		
κ	5	5
x	5	5

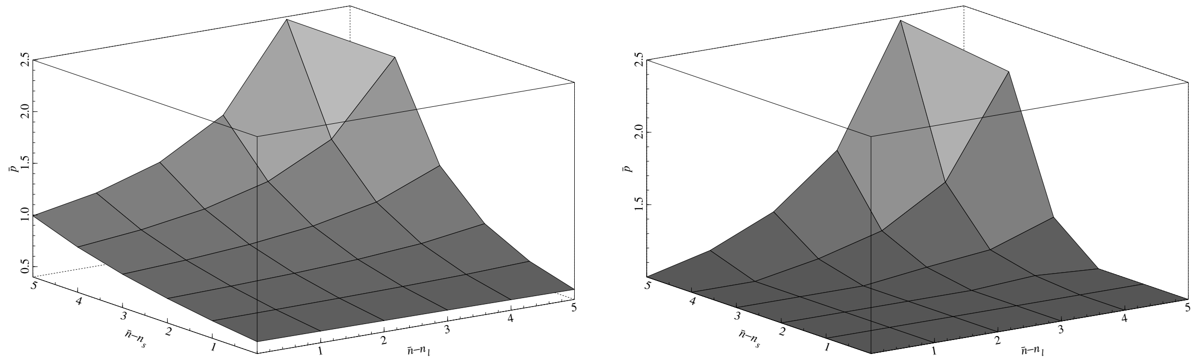
the two sets of parameters that are used. The first one illustrates an industry with moderate fixed-costs and little turnover, and the second one has larger fixed-costs and a higher turnover rate. We set the maximum number of firms to 5. Without large firms the long run average number of firms is always smaller than 5, so this constraint does not affect the results. Note also that in all examples we fixed the value of the outside option to -1.5. Therefore, only entry, exit and reconfiguration decisions generate randomness in the industry. Moreover, the adjustment cost parameters κ and x are set to relatively high levels which creates infrequent turnover in the industry.

In all specifications we set the price floor to 1. At this level, the price floor does not bind in equilibrium unless a large firm is active in the industry. Figures 1(a) and 1(b) graph the average market prices with and without a price floor. Because of the congestion term, prices fall sharply with the number of firms. In the unconstrained case, large firms set their prices very close to the marginal cost of the small firms when there are more than four firms active. The price floor regulation therefore protects the small type and restores the price that would be observed if large firms were not present in the industry (see top left corner of Figure 1(a)).

To analyze the long-run effect of a price floor constraint we simulate the evolution of the industry for $10,000$ periods and calculate the proportion of time that the industry spends in each discrete state. In order to compare across specifications we use exactly the same sequence of random numbers. The first panel of Table 2 illustrates the simulation results for the first specification. In this specification, the fixed costs of both types are relatively small and the steady state level of competition is important. Without the regulation (left panel), the most likely industry structure has one small and two large firms. Because the large firm is significantly more efficient, the average price in that state is 1.

The right panel shows that a price floor regulation can significantly distort the structure of the industry. Since fixed costs are relatively small, the equilibrium number of firms is large and competition is intense. This implies that the price floor binds for at least one firm in about 40% of the simulated periods. This offers a lot of protection for the small types.

This protection implies that in the most likely industry structure, no large firm enters the mar-



(a) Average prices ($p_f = 0$) (b) Average prices ($p_f = 1$)

Figure 1: Equilibrium average price and market structure under specification 1

ket. This is because the efficient technology has large fixed costs and is not profitable unless two or three small firms leave the market. Since the regulation protects the small types, firms correctly anticipate that they would not be profitable by entering with the most efficient technology.

Because competition is intense in both cases and the floor blocks entry of the more efficient technology, the regulation tends to raise prices on average. The bottom lines of the Table show that the average and median pries are about 10% higher with the price floor regulation. Therefore, in this example the regulation creates large inefficiencies both because consumers pay higher prices on average and because too many firms are active in the industry.

The simulation results from the second specification are reproduced in the lower panel of Table 2. In this example the fixed costs of both types are increased by 0.05, which leads to fewer large firms in equilibrium. In the unregulated market the more likely industry structure is the one with two small firms and one big firm, which leads to higher prices on average than in the first specification. With the price floor regulation, the distribution of equilibrium states in the long-run is similar to the previous example. In nearly 100% of the simulation periods, the industry has three or four small firm, but no large firm. As a result, the regulation completely blocks the adoption of the large technology.

In terms of average prices however, the two examples differ. The mean simulated prices are almost the same with or without the regulation, but the median and the mode are smaller **with** the price floor. As before two opposite forces are in action. On the one hand the presence of the price floor makes the market relatively more competitive because it blocks the entry of large firms. On the other hand, firms tend to be more efficient in the unregulated market. In the first example, the efficiency gains were dominant and prices were higher with the price floor. The second example shows that when the fixed cost of the efficient type is large enough, the competitive effect can dominate and prices can be higher without the floor.

Another interesting feature of the second example is that the price floor binds only in 2% of the simulated periods. Therefore, the price floor can significantly distort the structure of markets without actually binding in equilibrium. This is an important feature of the model that we observe in the gasoline example.

Table 2: Long run distribution of industry structure (top four industry states)

Specification 1

Unconstrained equilibrium				Constrained equilibrium ($p_f = 1$)			
Market structure	Freq.	Prices	Welfare	Market structure	Freq.	Prices	Welfare
$(N_s = 1, N_l = 2)$	0.73	1	2.11	$(N_s = 4, N_l = 0)$	0.59	1.11	2.32
$(N_s = 2, N_l = 1)$	0.1	1.16	1.88	$(N_s = 3, N_l = 1)$	0.16	1.08	2.38
$(N_s = 0, N_l = 2)$	0.06	1.25	1.32	$(N_s = 2, N_l = 1)$	0.16	1.2	1.82
$(N_s = 0, N_l = 3)$	0.06	0.84	2.33	$(N_s = 3, N_l = 0)$	0.04	1.3	1.65
Mean price		1.063	1.951			1.127	2.219
Median price		1.002	2.113			1.109	2.320

Specification 2

Unconstrained equilibrium				Constrained equilibrium ($p_f = 1$)			
Market structure	Freq.	Prices	Welfare	Market structure	Freq.	Prices	Welfare
$(N_s = 2, N_l = 1)$	0.86	1.16	1.88	$(N_s = 4, N_l = 0)$	0.71	1.11	2.32
$(N_s = 3, N_l = 1)$	0.06	1	2.58	$(N_s = 3, N_l = 0)$	0.27	1.3	1.65
$(N_s = 4, N_l = 0)$	0.03	1.11	2.32	$(N_s = 5, N_l = 0)$	0.01	1	2.92
$(N_s = 3, N_l = 0)$	0.02	1.3	1.65	$(N_s = 3, N_l = 1)$	0.01	1.08	2.38
Mean price		1.115	2.073			1.158	2.151
Median price		1.158	1.880			1.109	2.320

The effect of the policy on consumer welfare is not trivial, however. Consumer welfare in the second example is likely to be higher with the policy since prices are typically lower. In addition to this price tradeoff, the welfare impact of the policy depends on consumers' valuation for variety. In all examples the number of products is larger on average with the policy. Since consumer valuation for variety is important in logit models, the average consumer welfare is larger with the policy in all cases.

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