1 Morning class: Week 2 day 1: The van der Waals gas.

1. Introduction to the van der Waals gas.

   (a) In a van der Waals gas, $(p_{\text{measured}} + d)(V_{\text{measured}} - c) = nRT$, where $c$ and $d$ are both positive. More simply, $(p + d)(V - c) = nRT$. Based on what we learned last time, discuss with your neighbor the differences between an ideal gas and a van der Waals gas and how the equation $pV = nRT$ changes as a result of these differences.

   (b) The van der Waals gas equation is

   $$(p + a \frac{n^2}{V^2})(V - nb) = nRT.$$ 

   Both $a$ and $b$ are positive constants, with each gas, e.g., steam, air and argon having its own $a$ and $b$ constants. Discuss with your neighbor what kind of analysis will be needed to derive this equation.

   (c) We explain the harder term first: $a \frac{n^2}{V^2}$. This term is due to the attraction between neighboring gas molecules. Just like people, two molecules must come into close contact for an attraction to come about. There will be two effects. First, we need to determine the proportionality relations controlling the number of molecules which come near to the wall of the gas balloon. Second, we need to determine the proportionality relations controlling the number of close contacts each individual molecule feels. We then combine the two effects.

   (d) Look at the picture below. Only gas molecules within a small fixed volume next to the surface of the balloon can impinge on the balloon in the next small instant of time, that is only the gas molecules within the small fixed volume can hit the wall and affect the pressure. (Please note that the small volume’s size is determined by $v$.) How is the number of gas molecules contained in this small volume controlled by $n$ and $V$?

   (e) Now to the second factor: what are the proportionality relations between the number of close attractions a single gas molecule feels and $n$ and $V$ for the gas?

   (f) Explain to your neighbor why the $p$ in $pV = nRT$ has been replaced by $p + a \frac{n^2}{V^2}$ in the van der Waals equation.

   (g) Last lecture we found the term $V$ in the ideal gas law $pV = nRT$ has been replaced for van der Waals gases by the equation by $(V - c)$. $c$ is the volume occupied by the van der Waals gas molecules themselves. What is the proportionality relation between $c$ and $n$?

   (h) Explain why $V$ in $pV = nRT$ has been replaced in the van der Waals equation by $(V - nb)$, where $b$ is a constant. What does $b$ measure?
(i) Review what you have just learned. From scratch, explain to a fellow student why for a van der Waals gas
\[(p + a \frac{n^2}{V^2})(V - nb) = nRT.\]

1.1 Working exam problems

2. The absent minded professor
An absent-minded professor records data on three different ideal gas samples. Sample A contains 3 moles of ideal gas and was held at a constant temperature of 400 K. Sample B contains 5 moles and was held at a constant temperature of 300 K. Sample C contains 4 moles of ideal gas and was held at a constant temperature of 350 K.

She records the data as shown above. Unfortunately the professor forgets to record which of the three curves goes with which of the three samples. Which curve goes with which sample?

3. A pair of balloons
We have two balloons both at STP. The one contains 4.0 grams of H\(_2\) while the other contains 64.0 g of O\(_2\). Does the hydrogen balloon look bigger, the oxygen balloon look bigger or do the two balloons look the same size?

4. A piston
A piston is shown on the right of the figure below. The piston has a moveable upper wall allowing the volume to change. It also has a valve at the point X which can be opened and closed. The chamber inside the piston contains an ideal gas. Over the course of an experiment the piston is manipulated so that its \(p\) and \(T\) travels from point A to B to C and finally to D as plotted on the \(p\) vs. \(T\) plot shown below on the left.

(a) Assuming the valve X was closed, state an experimental procedure which would have resulted in going from A to B. In your procedure was the gas was heated (for heating \(E_{\text{trans}}\) increases) or cooled (for cooling \(E_{\text{trans}}\) decreases) and did the chamber volume increase, decrease, or was it kept fixed at a constant value?

(b) If in going from B to C, the valve X was closed, is the only possible procedure one where temperature was constant temperature and the volume was increased?

(c) If in going from C to D, the valve X was open slightly, and gas escaped slowly, what could have been a procedure which would have \textbf{always} produced the desired result?