

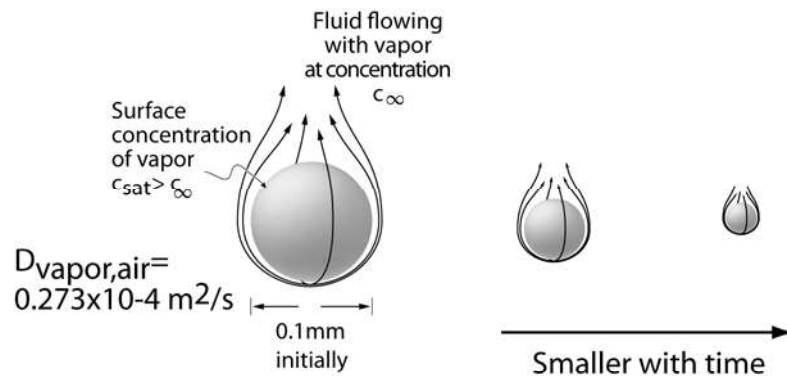
### Problem 14.9.11

**KNOWN:** Water evaporates at the droplet surface. There is a convective transfer of water vapor from the droplet surface into the air

**FIND:**

- 1) Equate the reduction in mass of a droplet to the convective loss of water vapor
- 2) Simplify the equation in 1) and solve for the time to completely evaporate a droplet with radius  $r$
- 3) Calculate the time it takes for a 0.1mm radius drop to completely evaporate

**SCHEMATIC AND GIVEN DATA**



**STRATEGY:** As the droplet size changes, the rate of convective water transfer from the droplet surface changes ( $h_m$  is a function of radius). At any given time, the rate of water loss from a droplet surface can be calculated using the convective mass transfer coefficient formula. This amount can be equated to the change in the mass of the droplet. This would relate the changing convective mass transfer from the surface to the droplet size, leading to a differential equation for radius as a function of time.

**ASSUMPTIONS:**

- 1) Droplet is very small, so is suspended in air, no relative velocity
- 2)  $Re_D = 0$

**SOLUTION:**

- 1) Let  $r$  be the radius of a droplet at time  $t$ . The mass of this droplet is  $m$

$$\frac{dm}{dt} = h_m (c_{sat} - c_\infty) 4\pi r^2 \quad (1)$$

where  $c_{sat}$  is the vapor concentration at the surface.

$p_{\text{vap,surf}}$  at  $40^\circ\text{C} = 0.07375 \cdot 10^5 \text{ N/m}^2$  (Table C.10 on p. 342)

**Problem 14.9.11**

$$c_{sat} = \frac{P_{vap,surf}}{Rt} = \frac{0.07375 \cdot 10^5 \text{ N/m}^2}{\left(8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}}\right)(273 + 40) \text{ K}} = 2.83 \text{ mol/m}^3 = 0.051 \text{ kg/m}^3$$

$c_\infty = 0$  is the vapor concentration in dry air

$$\text{Also, } dm = \rho_{water} (4\pi r^2 dr) \quad (2)$$

Compare (1) and (2), we have

$$-\rho_{water} dr = h_m c_{sat} dt \quad (3)$$

Convective transfer of water vapor at the surface, for no relative motion between the water drop and the surrounding fluid, amounts to diffusion of water vapor into the air. The formula can be obtained by using the equation for forced convection and plugging in a zero value for Reynolds number (corresponding to zero velocity): (Note: The same formula can also be obtained by solving the diffusion equation in spherical coordinate, as was done in class)

$Nu_D = 2$  for heat transfer will be rewritten for mass transfer as

$$Sh_D = 2$$

$$h_m = Sh \frac{D_{AB}}{d} = 2 \frac{D_{AB}}{2r} = \frac{D_{AB}}{r}$$

$$\dot{m} = \frac{dm}{dt} = \frac{D_{AB}}{r} c_{sat} 4\pi r^2 = 4\pi D_{AB} c_{sat} r = 1.75 \times 10^{-5} r \text{ kg/s} \quad (4)$$

2) From Eq. (3) & (4)

$$-\rho_{water} dr = \frac{D_{AB}}{r} c_{sat} dt$$

Integrating

$$\begin{aligned} \int_0^t dt &= -\frac{\rho_{water}}{D_{AB} c_{sat}} \int_{r_0}^r r dr \\ t &= -\frac{\rho_{water}}{D_{AB} c_{sat}} \left( \frac{r^2 - r_0^2}{2} \right) \\ &= 3.59 \times 10^8 (r_0^2 - r^2) \end{aligned}$$

**Problem 14.9.11**

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3) For  $r_o = 10^{-4} m$ ,  $r = 0$ .

$$\begin{aligned} t &= 3.59 \times 10^8 \times \left[ (10^{-4})^2 - 0 \right] \\ &= 3.59 \text{ s} \end{aligned}$$

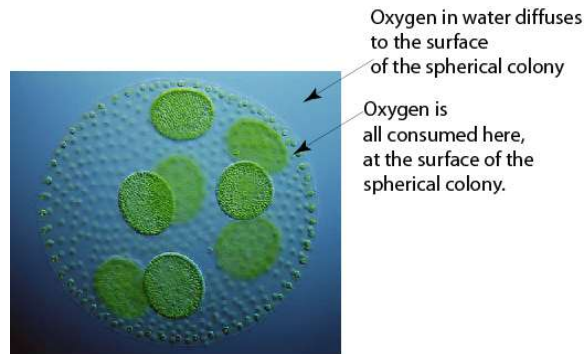
**COMMENTS:** Note that  $\frac{dr}{dt} = -\frac{D_{AB}c_{sat}}{\rho_{water}} \frac{1}{r}$  means that the radius  $r$  does not change uniformly with time. The radius changes faster as it gets smaller.

### Problem 14.9.13

**KNOWN:** Oxygen concentration in bulk water and at the surface of a spherical colony of microorganisms

**FIND:** Calculate the rate of oxygen transport from the bulk water to the colony surface

**SCHEMATIC AND GIVEN DATA:**



<http://www.microscopy-uk.org.uk/mag/imgsmall/volvox2.jpg>

**STRATEGY:** Assuming there is no relative movement between the sphere and the bulk water, oxygen moves by diffusion only, i.e., no flow needs to be considered. Thus, one can get the concentration profile of oxygen in water and, from the gradient of the concentration profile at the surface, flux at the surface can be calculated. Note that our domain of calculation is from the outer radius of the colony to infinity, i.e., not inside the colony.

**ASSUMPTIONS:**

**SOLUTION:** Considering only the radial variation, governing equation for  $O_2$  transfer through water becomes:

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial c}{\partial r} \right) = 0$$

The solution to which is

$$c = \frac{A}{r} + B$$

Where A and B are integration constants which are obtained using the BCs

$$c = 0 \text{ at } r = R$$

$$c = c_0 \text{ at } r \rightarrow \infty$$

Where  $c_0$  = concentration of  $O_2$  in bulk water, obtained using Henry's Law

**Problem 14.9.13**

Plugging in the second boundary condition,  $B = c_o$ . Using the first boundary condition,  $A = BR$ . Using these in the equation for  $c$ ,

$$c = \frac{-BR}{r} + B = B \left( 1 - \frac{R}{r} \right) = c_o \left( 1 - \frac{R}{r} \right)$$

which is the required concentration profile as function of radius.

To calculate  $c_o$  use Henry's Law

$$p = H \cdot c'_o$$

where  $p = 0.21$  P = 0.21 atm

$$\begin{aligned} c'_o &= \frac{0.21}{4.4 \times 10^4} \\ &= 4.77 \times 10^{-6} \text{ mole fraction} \\ &\approx 4.77 \times 10^{-6} \frac{\text{moles of O}_2}{\text{moles of H}_2\text{O}} \end{aligned}$$

We can calculate  $c_o$  from this as

$$\begin{aligned} c_o \left[ \frac{\text{moles of O}_2}{\text{m}^3} \right] &= \frac{\text{moles of O}_2}{\text{moles of H}_2\text{O}} \times \frac{\text{moles of H}_2\text{O}}{\text{grams of H}_2\text{O}} \times \frac{\text{grams of H}_2\text{O}}{\text{m}^3 \text{H}_2\text{O}} \\ &= 4.77 \times 10^{-6} \times 1 \text{ mol} / 0.018 \text{ kg} \times 1000 \text{ kg} / \text{m}^3 \\ &= 0.265 \text{ mol} / \text{m}^3 \end{aligned}$$

Oxygen consumption by the colony is everything that is coming into it, which is the flux of oxygen at its surface,  $r=R$ . We calculate this flux on the water side where we know how concentration  $c$  varies with  $r$ . Thus, flux is given by

$$= -D_{\text{O}_2, \text{H}_2\text{O}} \left. \frac{dc}{dr} \right|_{r=R} = -D_{\text{O}_2, \text{H}_2\text{O}} \frac{c_o}{R}$$

For  $D_{\text{O}_2, \text{H}_2\text{O}} = 3.25 \times 10^{-9} \text{ m}^2/\text{s}$ , Rate of oxygen consumption per unit surface area is given by

$$\begin{aligned} &= \frac{D c_o}{R} \\ &= 1.72 \times 10^{-5} \frac{\text{mol}}{\text{m}^2 \text{sec}} \end{aligned}$$

**COMMENTS:** This problem can also be thought of the limiting case of zero natural convection. Thus, from the formula for natural convection over a sphere, when densities are the same,  $h_m(2R) / D_{\text{O}_2, \text{H}_2\text{O}} = 2 + 0$ , which will also give a flux of  $h_m(c_o - 0) = (D_{\text{O}_2, \text{H}_2\text{O}} / R)c_o$

**Problem 14.9.15**

1)

Governing equation

$$\frac{\partial c^*}{\partial t} + u \frac{\partial c^*}{\partial z} = E \frac{\partial^2 c^*}{\partial z^2}$$

Boundary and initial conditions

$$c^*(z \rightarrow \infty) = c_i$$

$$c^*(z = 0) = 0$$

$$c^*(t = 0) = c_i$$

Governing equation

$$\frac{\partial c^{**}}{\partial t} + u \frac{\partial c^{**}}{\partial z} = E \frac{\partial^2 c^{**}}{\partial z^2}$$

Boundary and initial conditions

$$c^{**}(z \rightarrow \infty) = c_i$$

$$c^{**}(z = z_o) = 0$$

$$c^{**}(t = 0) = c_i$$

2)

The  $ut$  part is subtracted from  $z$  to give the distance traveled due to dispersion. The total distance traveled is  $z$  and this is due to both convection and dispersion. The distance traveled due to convection is the convection velocity multiplied by time. So the distance due to dispersion is  $z - ut$

3)

90

240

4)

The peak gets smaller because the concentration is dropping due to dispersion. Some of the pollutant advances faster and slower than average and so the material spreads out, reducing the concentration.

5)

No. We require the bulk movement of the soil water through the porous media to disperse the pollutant. The dispersion is caused by differential flow velocities due to the many differing paths through the porous media.

6)

**Problem 14.9.15**

6)

The downward flux with just convection would be,  $uc$ .

But, there is a dispersion component, which would be just like Fick's Law formulation:

$$E \frac{dc}{dz} \quad [E \text{ in dispersion coefficient instead of } D]$$

Then flux would be:  $\text{flux} = uc - E \frac{dc}{dz}$

Now, at 50 cm for 90 days, from fig. 14.11, we have:

$$c = 0.6 \mu\text{g}/\text{cm}^3; \quad u = 0.5 \frac{\text{cm}}{\text{day}}; \quad E = 2 \text{cm}^2/\text{day}$$

$$\left. \frac{dc}{dz} \right|_{z=50\text{cm}} = 0 \quad [\text{point of maximum}]$$

Hence,

$$\begin{aligned} \text{flux} &= 0.5 \frac{\text{cm}}{\text{day}} \times 0.6 \frac{\mu\text{g}}{\text{cm}^3} - 0 \\ &= 0.3 \frac{\mu\text{g}}{\text{cm}^2 \text{day}} \quad (\text{Answer}) \end{aligned}$$

7)

What would be the approximate flux at 30 cm depth at 90 days?

Equation:

$$\text{flux} = uc - E \frac{dc}{dz}$$

from the curve:

$$c|_{30\text{cm}} = 0.4 \mu\text{g}/\text{cm}^3$$

If we choose two points as 0.2 and 0.5  $\mu\text{g}/\text{cm}^3$  [of the horizontal axis], we can see the curve is almost a straight line.

Hence, the slope

$$\frac{dc}{dz} = \frac{\Delta c}{\Delta z} = \frac{0.5 - 0.2}{38 - 25} = .023 \mu\text{g}/\text{cm}^4$$

hence

$$\begin{aligned} \text{flux} &= 0.5 \frac{\text{cm}}{\text{day}} \times 0.4 \frac{\mu\text{g}}{\text{cm}^3} - 2 \frac{\text{cm}^2}{\text{day}} \times 0.023 \frac{\mu\text{g}}{\text{cm}^4} \\ &= 0.154 \frac{\mu\text{g}}{\text{cm}^2 \text{day}} \quad (\text{Answer}) \end{aligned}$$

**Problem 14.9.21**

$c_\infty$  can be assumed to be negligible.

$$P_A = Hx_A$$

$$c_s = \frac{P_A}{RT}$$

$$= \frac{Hx_A}{RT}$$

$$= \frac{6 \times 10^{-6} \times 10^6 \text{ Pa m}^3/\text{mol} \times 25 \text{ mg/L} \times \frac{10^{-3} \text{ g/mg}}{10^{-3} \text{ m}^3/\text{L}} \times \frac{1 \text{ mol}}{17 \text{ g}}}{8.314 \text{ J/mol K} \times 293 \text{ K}}$$

$$= \frac{6 \times 25}{8.314 \times 17 \times 293} \text{ mol/m}^3$$

$$= 3.62 \times 10^{-3} \text{ mol/m}^3$$

$$\text{Flux} = 0.002 \text{ m/s} (3.62 \times 10^{-3} - 0) \text{ mol/m}^3$$

$$= 7.28 \times 10^{-6} \text{ mol/m}^2\text{s}$$

Without using molecular weight:



**Problem 14.9.21**

$$c_s = \frac{Hx_A}{RT}$$

$$= \frac{6 \times 10^{-6} \times 10^6 \text{ Pa m}^3/\text{mol} \times 25 \text{ mg/L} \times \frac{10^{-3} \text{ g/mg}}{10^{-3} \text{ m}^3/\text{L}}}{8.315 \text{ J/mol K} \times 293 \text{ K}}$$

$$= 6.158 \times 10^{-2} \text{ g/m}^3$$

$$\text{Flux} = 0.002 \text{ m/s} \times (6.158 \times 10^{-2} - 0) \text{ g/m}^3$$

$$= 1.24 \times 10^{-4} \text{ g/m}^2\text{s}$$

3.  $h_m \propto Sh_L$

$$Sh_L = 0.664 \text{ Re}^{1/2} Sc^{1/2}$$

$$Sh_L \propto u^{1/2}$$

$$h_m \propto u^{1/2}$$

$$\frac{h_{m0.5 \text{ m/s}}}{h_{m0.1 \text{ m/s}}} = \sqrt{\frac{0.5}{0.1}}$$

$$= \sqrt{5}$$

$$\approx 2.236$$

4. Rates (experimental) =  $\frac{1.02}{0.84}$

$$= 1.214$$

The theoretical ratio and the experimental ratio are not close to each other.