Problem 13.8.18

KNOWN:
Diffusion in a semi-infinite region

FIND:
Concentration at a location for two different diffusivities

SCHEMATIC AND GIVEN DATA

Co

↓ ↓ ↓ Ci=0

− − − − − − − − − ← 2 mm

STRATEGY:
If we assume a semi-infinite region (see assumptions below), the concentrations can be found using available solution and concentrations for two different diffusivities can be compared.

ASSUMPTIONS:
The problem description does not make it clear that it is not for a semi-infinite region, thus semi-infinite region is an assumption. Such an assumption cannot be arbitrarily and depends on the physical situation.

SOLUTION:

Governing Eqn, $\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$

$$\frac{c-c_i}{c_s-c_i} = 1-\mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

With enhancer

$$\begin{aligned}
\frac{c-0}{c_s-0} &= 1-\mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right) \\
&= 1-\mathrm{erf}\left(\frac{2\times10^{-3}}{2\times\sqrt{7.7\times10^{-11}\times150\times60}}\right) \\
&= 1-\mathrm{erf}\left(1.201\right) \\
&= 1-0.91
\end{aligned}$$

$$\frac{\mathrm{c}}{\mathrm{c_s}} = 1-0.91 = 0.09$$

$$\mathrm{c} = 0.09\ \mathrm{c_s}$$

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Without Enhancer

$$\frac{c-0}{c_s-0} = \text{erf}\left(\frac{2\times10^{-3}}{2\times\sqrt{2.9\times10^{-11}\times150\times60}}\right)$$
$$= \text{erf}(1.957)$$
$$= 0.9944$$

So,

$$\frac{\text{c}}{\text{c}_\text{s}} = 1-0.9944 = 0.0056$$
$$\text{c} = 0.0056\ \text{c}_\text{s}$$

Percent increase in concentration from without enhancer to with enhancer

$$= \frac{0.09c_s - 0.0056c_s}{0.0056c_s}\times100\%$$
$$= 1507\%$$

COMMENTS:

Problem 13.8.20

1)

Diffusive mass flux $n_s = \sqrt{\dfrac{D}{\pi t}}\left(c_s - c_i\right)$ (formula 66 in booklet)

Amount of drug released over time t is the integral of the flux multiplied by area (note there are two surfaces for the slab)

$$= \left(\int_0^t \sqrt{\frac{D}{\pi t}}\left(c_s - c_i\right)dt\right)2A$$

$$= 2A\sqrt{\frac{D}{\pi}}\left(c_s - c_i\right)\int_0^t t^{-1/2}dt$$

$$= 2A\sqrt{\frac{D}{\pi}}\left(c_s - c_i\right)\left[\frac{1}{1/2}t^{1/2}\right]_0^t$$

$$= 2A\sqrt{\frac{D}{\pi}}\left(c_s - c_i\right)\frac{t^{1/2}}{1/2}$$

$$= \sqrt{\frac{Dt}{\pi}}\left(c_s - c_i\right)4A$$

If $A = \dfrac{V}{2L}$ ($2L$ since L is half thickness)

$$= 4\left(c_s - c_i\right)\sqrt{\frac{Dt}{\pi}}\frac{V}{2L}$$

$$= \left(c_s - c_i\right)\sqrt{\frac{Dt}{\pi}}\quad\frac{2V}{L}$$

The average concentration in the slab as a function of time at long times is:

$$\frac{c_{av} - c_s}{c_i - c_s} = \frac{8}{\pi^2}\exp\left(-D\left(\frac{\pi}{2L}\right)^2 t\right)$$

$$c_{av} = \frac{8}{\pi^2}\exp\left(-D\left(\frac{\pi}{2L}\right)^2 t\right)\left(c_i - c_s\right) + c_s$$

Amount of drug released

$$= c_i V - c_{av} V$$

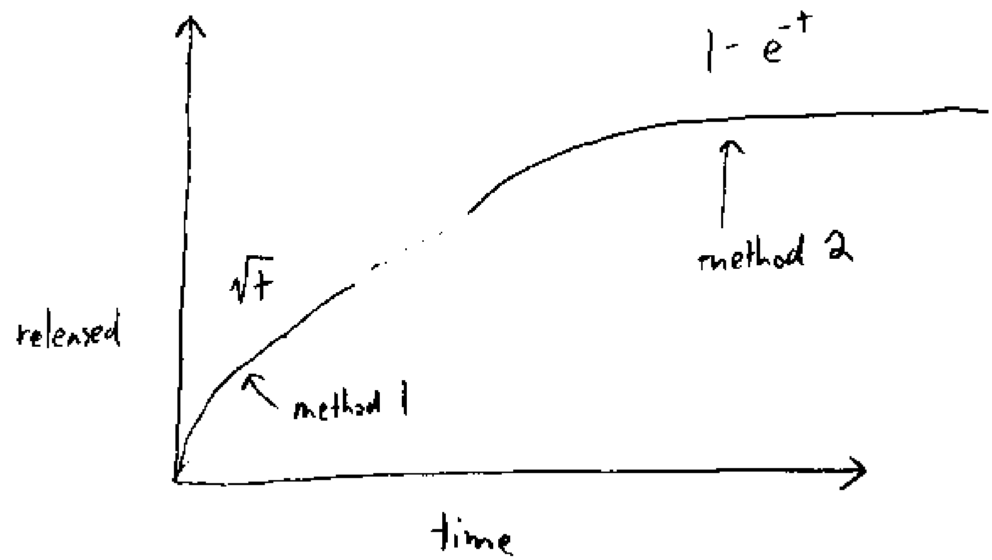
$$= \left[c_i - \left(\frac{8}{\pi^2}\exp\left(-D\left(\frac{\pi}{2L}\right)^2 t\right)\left(c_i - c_s\right) + c_s\right)\right]V$$

$$= \left[c_i - c_s - \frac{8}{\pi^2}\exp\left(-D\left(\frac{\pi}{2L}\right)^2 t\right)\left(c_i - c_s\right)\right]V$$

$$= \left(c_i - c_s\right)\left[1 - \frac{8}{\pi^2}\exp\left(-\frac{\pi^2 Dt}{4L^2}\right)\right]V$$

Problem 13.8.20

3)



Problem 13.8.30

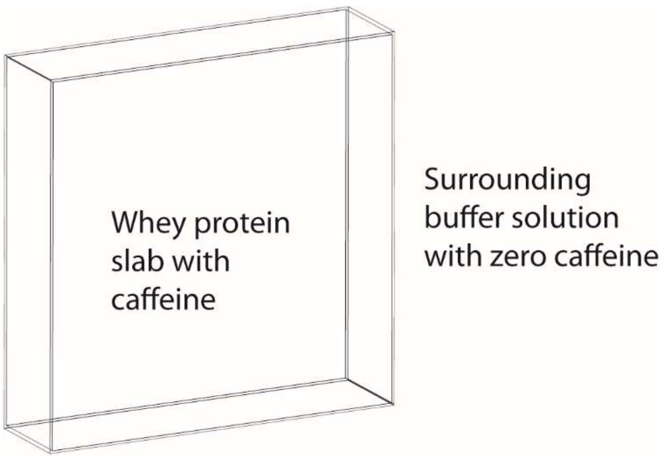
Known:

Geometry

Find:

Diffusivity

Schematic and Given Data:



$L = 0.005m$

$c_s = 0$

75% is released after 1 hour.

Strategy:

The amount released can be found by subtracting the amount in the slab from the initial amount. The amount in the slab is related to the average concentration. This is transient mass transfer in a slab so we have a choice of series solution or its equivalent Heisler chart. Heisler chart will not work because it does not give us the average. Thus, the choice is to use the series solution. We already have an expression for average concentration using the series solution—we just need to use this equation. This series solution (with one term) is for $Fo > 0.2$.

Assumptions:

None

Solution:

1)

$$c_{av} = c_s + (c_i - c_s)\frac{8}{\pi^2}\exp\left(-D\left(\frac{\pi}{2L}\right)^2 t\right)$$

$$c_s = 0$$

$$c_{av} = \frac{8c_i}{\pi^2}\exp\left(-D\left(\frac{\pi}{2L}\right)^2 t\right)$$

Amount released = $(c_i - c_{av})V = Vc_i\left(1 - \frac{8}{\pi^2}\exp\left(-D\left(\frac{\pi}{2L}\right)^2 t\right)\right)$

2)

$$0.75Vc_i = Vc_i\left(1 - \frac{8}{\pi^2}\exp\left(-D\left(\frac{\pi}{2L}\right)^2 t\right)\right)$$

$$0.75 = 1 - \frac{8}{\pi^2}\exp\left(-D\left(\frac{\pi}{2L}\right)^2 t\right)$$

$$D = -\frac{4L^2}{\pi^2 t}\ln\left[\frac{\pi^2}{8}(1-0.75)\right] = -\frac{4(0.005^2)}{\pi^2 3600}\ln\left[\frac{\pi^2}{8}(1-0.75)\right] = 0.33\times 10^{-8}\,\frac{m^2}{s}$$

3)

$$0.2 < \frac{Dt}{L^2} = \frac{0.33\times 10^{-8} t}{0.005^2}$$

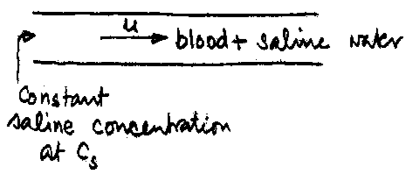
$$t > 1515s$$

Problem 14.9.2

KNOWN: Transport of saline solution in a vein

FIND: Concentration profile

SCHEMATIC AND GIVEN DATA



STRATEGY: It is a 1D convection-dispersion problem. Such a 1D convection dispersion problem in a semi-infinite domain (right side has no end) has been solved in Section 14.2 and the solution can be used readily. Note that here the boundary condition is not a pulse, i.e., injection is continuous.

ASSUMPTIONS: Transport in the other dimensions are ignored.

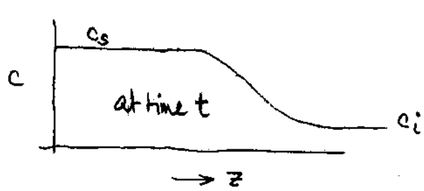
SOLUTION:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial z} = D\frac{\partial^2 c}{\partial z^2} + \cancel{r_A}^{\,0}$$

c = c_i everywhere at t = 0
c = c_s at z = 0
c $\rightarrow$ 0 at z $\rightarrow \infty$

(see Section on convection-dispersion in the text) By transforming the governing equation we can obtain the solution as

$$\frac{c_s - c}{c_s - c_i} = \text{erf}\left[\frac{z - ut}{2\sqrt{Dt}}\right]$$



COMMENTS: