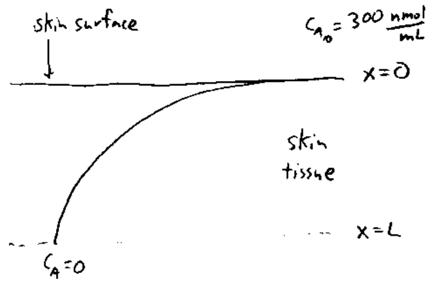
Problem 12.6.11

KNOWN:

FIND:

SCHEMATIC AND GIVEN DATA



STRATEGY:

ASSUMPTIONS:

SOLUTION:

1)

$$\frac{\partial c_A}{\partial t} + u\frac{\partial c_A}{\partial x} = D\frac{\partial^2 c_A}{\partial x^2} - k'' c_A$$

Steady state $\Rightarrow \quad \frac{\partial c_A}{\partial t} = 0$

No Convection $\Rightarrow \quad u\frac{\partial c_A}{\partial x} = 0$

Therefore, $\frac{d^2 c_A}{dx^2} - \frac{k''}{D} c_A = O$

2)

Constant surface concentration

$$c_A(x=0) = c_{A,0} = 300\ nmol/mL$$

Substrate used up at significant depth

$$c_A(x=\infty) = 0$$

3)

$$\frac{c_A}{c_{Ao}} = e^{-mx}$$

$$c_A = c_{A,0} e^{-\sqrt{\frac{k''}{D_{AB}}}x}$$

$$= 300\frac{nmol}{mL} e^{-\sqrt{\frac{10^3 \frac{1}{\min}\times\frac{1\min}{60s}}{2\times10^{-10} m^2/s}}x} = 300\left[\frac{nmol}{mL}\right] e^{-2.89\cdot10^5\left[\frac{1}{m}\right]x[m]}$$

4)

$$\frac{c_A}{c_{A,o}} = 0.1$$

$$0.1 = e^{-\sqrt{\frac{10^3 \frac{1}{\min} \times \frac{1\min}{60s}}{2\times10^{-10} m^2/s}}x}$$

$$x = 7.98\times10^{-6} m = 7.98\ \mu m$$

5)

$$n = -D_{AB} \left.\frac{dc}{dx}\right|_{x=0}$$

$$\frac{dc}{dx} = -c_{A,0}\sqrt{\frac{k^{"}}{D_{AB}}} e^{-\sqrt{\frac{k^{"}}{D_{AB}}}x}$$

$$\left.\frac{dc}{dx}\right|_{x=0} = -c_{A,0}\sqrt{\frac{k^{"}}{D_{AB}}} \times 1$$

$$n = -D_{AB} \times -c_{A,0}\sqrt{\frac{k^{"}}{D_{AB}}}$$

$$= 2\times10^{-10} m^2/s \times 300 \frac{n\ mol}{mL} \times \sqrt{\frac{10^3 \frac{1}{\min} \times \frac{1\min}{60s}}{2\times10^{-10} m^2/s}}$$

$$= 0.0173 \frac{m}{s}\frac{n\ mol}{mL}$$

$$= 1.73\times10^4 \frac{n\ mol}{m^2 \cdot s}$$

6)

$$0.1 = e^{-\sqrt{\frac{2\times10^3 \frac{1}{\min} \times \frac{1\min}{60s}}{2\times10^{-10} m^2/s}}x}$$

$$x = 5.64\times10^{-6} m$$

$$= 5.64\ \mu m$$

7)

You could change the surface concentration $c_{A,0}$

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$$(0.1)\left(300\frac{n\ mol}{mL}\right)=c_{A0}e^{-\sqrt{\frac{2\times10^{3}\frac{1}{\min}\times\frac{1\min}{60s}}{2\times10^{-10}m^{2}/s}}\left(7.98\times10^{-6}m\right)}$$

$$c_{A,0}=779\frac{n\ mol}{mL}$$

or you could change the diffusivity D

$$(0.1)\left(300\frac{n\ mol}{mL}\right)=300\frac{n\ mol}{mL}e^{-\sqrt{\frac{2\times10^{3}\frac{1}{\min}\times\frac{1\min}{60s}}{D}}\left(7.98\times10^{-6}m\right)}$$

$$D=4\times10^{-10}m^{2}/s$$

COMMENTS:

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1. $$\frac{D}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial c}{\partial r}\right)=\frac{\partial c}{\partial t}$$

2. At steady state,

$$\frac{D}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial c}{\partial r}\right)=0$$

$$r^2\frac{\partial c}{\partial r}=A$$

$$dc=\frac{A}{r^2}dr$$

$$c=-\frac{A}{r}+B$$

Using the boundary conditions,

$$c_o=B-\frac{A}{r_o}$$

$$c_i=B-\frac{A}{r_i}$$

$$c_i-c_o=A\left(\frac{1}{r_o}-\frac{1}{r_i}\right)$$

$$A=\frac{-r_or_i}{(r_o-r_i)}(c_i-c_o)$$

$$B=\frac{c_or_o-c_ir_i}{r_o-r_i}$$

$$c=\frac{r_or_i}{r(r_o-r_i)}(c_i-c_o)+\frac{c_or_o-c_ir_i}{r_o-r_i}$$

3. Flow = Flux ×Area

$$=\left(-D\frac{\partial c}{\partial r}\right)\times 4\pi r^2$$

$$=-D\frac{A}{r^2}\times 4\pi r^2$$

$$=4\pi D\frac{r_or_i}{r_o-r_i}(c_i-c_o)$$

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4.

$$\text{Flow} = \frac{c_i - c_o}{\left\{\dfrac{r_o - r_i}{4\pi D r_o r_i}\right\}}$$

$$\text{Resistance} = \frac{r_o - r_i}{4\pi D r_o r_i} = \frac{1}{4\pi D}\left(\frac{1}{r_i} - \frac{1}{r_o}\right)$$

5. Instantaneous flow through the outer surface

$$= \frac{c_i - c_o}{\dfrac{1}{4\pi}\sum\left(\dfrac{1}{D}\left(\dfrac{1}{r_i} - \dfrac{1}{r_o}\right)\right)}$$

$$= \frac{1.062 \text{ mg/mm}^3}{\dfrac{1}{4\pi \times 10^{-2}}\left\{\dfrac{1}{9.7}\left(\dfrac{1}{4.57} - \dfrac{1}{5.46}\right) + \dfrac{1}{0.15}\left(\dfrac{1}{5.46} - \dfrac{1}{6.22}\right) + \dfrac{1}{0.24}\left(\dfrac{1}{6.22} - \dfrac{1}{6.35}\right)\right\}\dfrac{\text{s}}{\text{mm}^3}}$$

$$= 0.8011 \text{ mg/s}$$

Problem 12.6.20

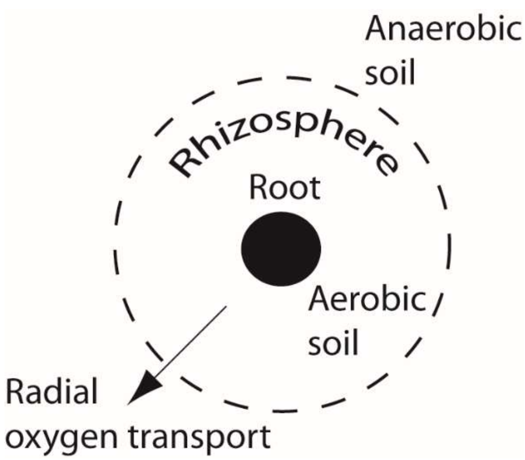
Known:

Property data

Find:

Governing equation, boundary conditions, concentration as a function of r, and the distance at which c equals 0.

Schematic and Given Data:



$$D = 2.41 \times 10^{-9} \frac{m^2}{s}$$

$$k'' = 5.1 \frac{mg}{L \cdot h} = 1.42 \frac{mg}{m^3 \cdot s}$$

$$c\big|_{r=R} = 6.8 \frac{mg}{L} = 6800 \frac{mg}{m^3}$$

$$L = 0.1m$$

Strategy:

This is a problem of steady-state with reaction. It is in cylindrical coordinate, with zeroth order reaction and with the two boundary conditions of concentration and flux given, respectively. Since this situation does not match with what was covered in class (slab, first-order reaction, boundary condition of concentration given), the solution needs to be obtained starting from the governing equation.

Assumptions:

None

Solution:

1)

$$D\frac{1}{r}\frac{d}{dr}\left(r\frac{dc}{dr}\right)-k''=0$$

2)

R = root radius and S = aerobic/anaerobic radius mean

$$c\Big|_{r=R}=c_i$$

$$\left.\frac{dc}{dr}\right|_{r=S}=0$$

3)

$$\frac{d}{dr}\left(r\frac{dc}{dr}\right)=\frac{k''r}{D}$$

$$r\frac{dc}{dr}=k''\frac{r^2}{2D}+k_1$$

$$-\frac{k''S^2}{2D}=k_1$$

$$r\frac{dc}{dr}=\frac{k''r^2}{2D}-\frac{k''S^2}{2D}$$

$$\frac{dc}{dr}=\frac{k''}{2D}\left(r-\frac{S^2}{r}\right)$$

$$c=\frac{k''}{2D}\left(\frac{r^2}{2}-S^2\ln r\right)+k_2$$

$$c_i=\frac{k''}{2D}\left(\frac{R^2}{2}-S^2\ln R\right)+k_2$$

$$k_2=c_i-\frac{k''}{2D}\left(\frac{R^2}{2}-S^2\ln R\right)$$

$$c=\frac{k''}{2D}\left(\frac{r^2-R^2}{2}+S^2\ln\frac{R}{r}\right)+c_i$$

4)

$$\frac{6800}{100} = \frac{1.42}{2\left(2.41\times10^{-9}\right)}\left(\frac{r^2-\left(1\times10^{-4}\right)^2}{2} + S^2 \ln\frac{1\times10^{-4}}{r}\right) + 6800$$

$$-2.3\times10^{-5} = \left(\frac{r^2}{2} + 0.003^2 \ln\frac{1\times10^{-4}}{r}\right)$$

$$r = 1.41mm$$

5)

$$-D\frac{dc}{dr}\bigg|_{r=R}(2\pi RL) = -D\frac{k^{"}}{2D}\left(R-\frac{S^2}{R}\right)(2\pi RL) = -\frac{k^{"}}{2}\left(R-\frac{S^2}{R}\right)(2\pi RL)$$

$$= -\frac{1.42}{2}\left[\frac{\text{mg}}{\text{m}^3\text{s}}\right]\left(1\times10^{-4} - \frac{0.003^2}{1\times10^{-4}}\right)[\text{m}]2\pi\left(1\times10^{-4}\right)0.1\left[\text{m}^2\right] = 4.01\times10^{-6}\,\frac{\text{mg}}{\text{s}}$$