Problem 10.9.8

KNOWN:
Diffusivity increases by a factor of two due to increase in temperature

FIND:
What must be the increase in temperature?

SCHEMATIC AND GIVEN DATA

STRATEGY:
Equations are available for diffusivities in liquid and gas phase as function of temperature. It is a straightforward application of those equations.

ASSUMPTIONS:
The same assumptions that go into the equations for diffusivities

SOLUTION:

1.) For Liquid $D = \dfrac{\kappa T}{6\pi\mu r}$ or we can say $D \propto T$.

Therefore, $\dfrac{D_1}{D_2} = \dfrac{T_1}{T_2}$.

To increase diffusivity by a factor of 2

$$\text{Final Temp. } (T_2) = T_1 \times \frac{D_2}{D_1} = (273+10)\times 2$$
$$= 566K = 293°C$$

2.) For Gas $D_{AB} = \dfrac{0.001858\, T^{3/2}\left(1/M_A + 1/M_B\right)^{1/2}}{p\sigma^2_{AB}\Omega_{D,AB}}$

or we can say $D \propto T^{3/2}$

Therefore, $\dfrac{D_1}{D_2} = \dfrac{T_1^{3/2}}{T_2^{3/2}}$

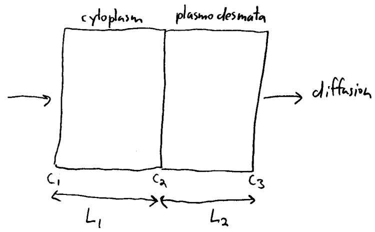
To increase diffusivity by a factor of 2,

$$\text{Final Temp. } (T_2) = T_1 \times \left(\frac{D_2}{D_1}\right)^{2/3} = 283\times(2)^{2/3} = 449.23K = 176.23°C$$

3.) Diffusion in gases is more easier than in liquids and increases more rapidly.

COMMENTS:

Problem 10.9.10



1)
Flux through cytoplasm = flux through plasmodesmata

$$j_{sugar,L_1+L_2} = -D_{sugar,\,cytoplasm} \times \frac{c_1 - c_2}{L_1}$$

$$= D_{sugar,\,plasmodesmata} \times \frac{c_2 - c_3}{L_2}$$

$$c_1 - c_2 = -j_{sugar,L_1+L_2} \frac{L_1}{D_{sugar,\,cytoplasm}} \quad \text{--- (1)}$$

$$c_2 - c_3 = -j_{sugar,L_1+L_2} \frac{L_2}{D_{sugar,\,plasmodesmata}} \quad \text{--- (2)}$$

Adding Eqns. 1 and 2,

$$c_1 - c_3 = -j_{sugar,L_1+L_2} \times \left[\frac{L_1}{D_{sugar,\,cytoplasm}} + \frac{L_2}{D_{sugar,\,plasmodesmata}} \right] \qquad (3)$$

For an effective diffusivity giving the same flux due to the same concentration gradient

$$j_{sugar,L_1+L_2} = -D_{sugar,\,effective} \frac{dc}{dx}$$

$$= -D_{sugar,\,effective} \frac{c_1 - c_3}{L_1 + L_2}$$

$$\Rightarrow c_1 - c_3 = -j_{sugar,L_1+L_2} \times \left[\frac{L_1 + L_2}{D_{sugar,\,effective}} \right] \quad \text{--- (4)}$$

Equating (3) and (4)

Problem 10.9.10

$$\frac{L_1 + L_2}{D_{sugar,\,effective}} = \frac{L_1}{D_{sugar,\,cytoplasm}} + \frac{L_2}{D_{sugar,\,plasmodesmata}}$$

$$\frac{1}{D_{sugar,\,effective}} = \frac{\frac{L_1}{L_1 + L_2}}{D_{sugar,\,cytoplasm}} + \frac{\frac{L_2}{L_1 + L_2}}{D_{sugar,\,plasmodesmata}}$$

$$= \frac{f}{D_{sugar,\,cytoplasm}} + \frac{1 - f}{D_{sugar,\,plasmodesmata}}$$

$$\frac{1}{D_{sugar,\,effective}} = \frac{0.5}{5.6 \times 10^{-10}\, m^2 / s} + \frac{1 - 0.5}{7.8 \times 10^{-12}\, m^2 / s}$$

$$= 6.5 \times 10^{10}\, \frac{s}{m^2}$$

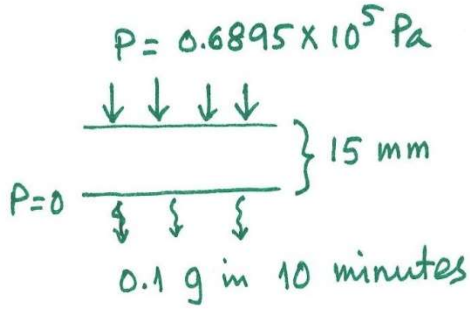
$$D_{sugar,\,effective} = 1.54 \times 10^{-11}\, m^2 / s$$

Problem 10. 9.12

KNOWN:
Rate of flow and pressure gradient

FIND:
Intrinsic permeability

SCHEMATIC AND GIVEN DATA



STRATEGY:
This is a straightforward application of Darcy's law that relates pressure gradient to the rate of flow.

ASSUMPTIONS:
1D flow; muscle dimension does not change as pressure is applied and water flows through it.

SOLUTION:
Using Darcy's Law, rewriting in terms of permeability

j^v = volumetric flux

$$= -K\frac{\partial h}{\partial s}$$

$$= -\frac{k\,\rho g}{\mu}\frac{\partial h}{\partial s}$$

$$= -\frac{k}{\mu}\frac{\partial(\rho g h)}{\partial s}$$

$$= -\frac{k}{\mu}\frac{\partial p}{\partial s}$$

$$= -\frac{k}{\mu}\frac{\Delta p}{\Delta s}$$

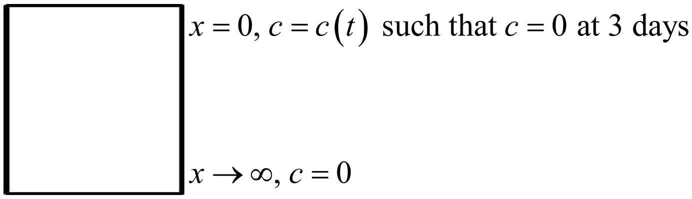
Q (volume collected) = Volumetric Flux x Area x time

$$\frac{0.1\times10^{-3}\,\text{kg}}{10^{3}\,\text{kg/m}^{3}} = \frac{k\left[\text{m}^2\right]}{0.001\dfrac{\text{N}\cdot\text{s}}{\text{m}^2}}\frac{0.6895\times10^{5}\,\text{N/m}^2}{15\times10^{-3}\,\text{m}}\cdot\frac{\pi}{4}\left(40\times10^{-3}\right)^2\text{m}^2\left(600\text{s}\right)$$

$$k = 2.89\times10^{-17}\,\text{m}^2$$

Problem 11.8.7

SCHEMATIC:



Governing equation:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2} - r$$

$u = 0;$ no convection

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} - r$$

Boundary conditions:

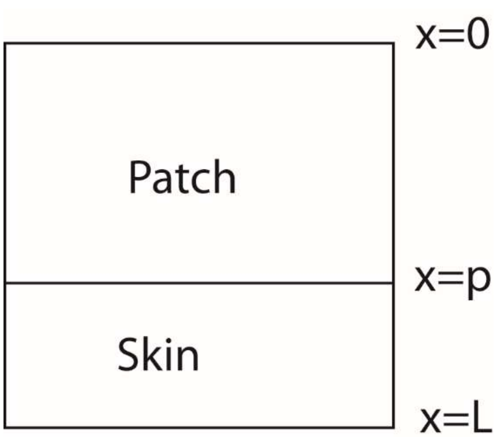
$$c(x = 0) = c(t)$$

$$c(x \to \infty) = 0 \text{ or } -D\left.\frac{\partial c}{\partial x}\right|_{x\to\infty} = 0$$

Initial conditions:

$$c(t = 0) = 0$$

Alternate formulation



Governing equation:

$$\frac{\partial c_p}{\partial t} = D\frac{\partial^2 c_p}{\partial x^2} \quad \text{In the patch}$$

$$\frac{\partial c_s}{\partial t} = D\frac{\partial^2 c_s}{\partial x^2} - r \quad \text{In the skin}$$

Boundary conditions:

$$-D_{\text{in patch}} \left. \frac{\partial c_p}{\partial x} \right|_{x=0} = 0$$

$$-D_{\text{in patch}} \left. \frac{\partial c_p}{\partial x} \right|_{x=p} = -D_{\text{in skin}} \left. \frac{\partial c_s}{\partial x} \right|_{x=p}$$

$$c_{\text{p, x=p}} = c_{\text{s, x=p}} \quad \text{assuming } \mathrm{K}^* = 1$$

$$c_s \left(x = L \right) = 0$$

Initial conditions:

$c_p \left(t = 0 \right) = c_i$ in the patch

$c_s \left(t = 0 \right) = 0$ in the skin

Problem 12.6.5

KNOWN:

FIND:

SCHEMATIC AND GIVEN DATA

G.E. $\cancel{\frac{\partial c}{\partial t}}^{o} + \mu \cancel{\frac{\partial c}{\partial r}}^{o} = D\frac{1}{r}\frac{d}{dr}\left(r\frac{dc}{dr}\right) - k''$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dc}{dr}\right) = \frac{k''}{D}$$

B.C.

$r = r_i \qquad c = c_i \qquad (1)$

$r = r_o \qquad \frac{dc}{dr} = o \qquad (2)$

STRATEGY:

ASSUMPTIONS:

SOLUTION:

$$\frac{d}{dr}\left(r\frac{dc}{dr}\right) = \frac{k''}{D}r$$

$$r\frac{dc}{dr} = \frac{k''}{D}\frac{r^2}{2} + C_1$$

Using (2): $\quad 0 = \frac{k''}{D}\frac{r_o^2}{2} + C_1 \quad \Rightarrow C_1 = -\frac{k''}{D}\frac{r_o^2}{2}$

Integrating after rearranging

$$\frac{dc}{dr} = \frac{k''}{D}\frac{r}{2} - \frac{k''}{D}\frac{r_o^2}{2r}$$

$$c = \frac{k''}{2D}\frac{r^2}{2} - \frac{k''}{2D}r_o^2 \ln\ r + C_2$$

Using (1):

$$C_2 = c_i - \frac{k''}{2D}\frac{r_i^2}{2} + \frac{k''}{2D}r_o^2 \ln\ r_i$$

$$c - c_i = \frac{k''}{4D}\left(r^2 - r_i^2\right) - \frac{k''}{2D}r_o^2 \ln\ r/r_i$$

Problem 12.6.5

At steady state, oxygen consumption is equal to oxygen inflow at the inner surface

$$= -D \left. \frac{dc}{dr} \right|_{r=r_i}$$

Flux $$= -\cancel{D}\left[\frac{k''}{4\cancel{D}} \cdot 2r_i - \frac{k''}{2\cancel{D}} r_o^2 \cdot \frac{1}{r_i} \right]$$

$$= -\frac{k''}{2}\left[r_i - \frac{r_o^2}{r_i} \right]$$

Flow = Flux × Area = $2\pi r_i L$

Flow/length = Flux x $2\pi r_i$

$$= -\frac{k''}{2}\left[r_i - \frac{r_o^2}{r_i} \right] 2\pi r_i$$

$$= -k''\pi\left(r_i^2 - r_o^2\right)$$

$$= k''\pi\left(r_o^2 - r_i^2\right)\left[\frac{\text{Flow}}{\text{m of vessel}}\right]$$

COMMENTS: