## Problem 1 (15 points): Dating of fossils

Many fossils are assumed to take up trace elements by diffusion and the extent of this diffusion can be used to predict the age of the fossil. Measurement of trace element on a certain deer-like animal tooth (see Figure 1) showed that the distance from the surface, at which the concentration is one-hundredth that of surface concentration, is 70 $\mu$m. Diffusivity (in both enamel and dentin) of the trace element can be assumed to be the same, at $10^{-20}\text{m}^2/\text{s}$. Also, assume there is no trace element initially in the fossil. We suspect the trace element enters only a small distance from the surface.

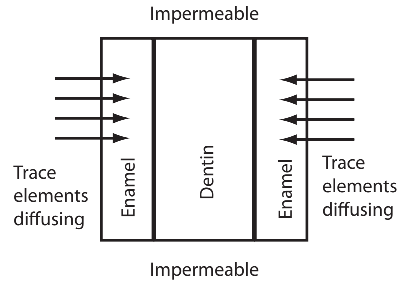


Figure 1: Schematic of a fossil tooth with trace elements diffusing

1) What is the age of the fossil in years? ◄ *8 points* ►

2) Hypothetically, if the trace element were also involved in reactions where it would be used up, is your estimate of age likely to be lower or higher than the true age? Provide reasoning. ◄ *2 points* ►

3) What is the ***total amount*** of trace element uptake per unit area from one side over a time period of $t$ for a surface concentration of $c_s$? ◄ *5 points* ►

## Problem 2 (24 points):Nutrient supply in engineered cartilage

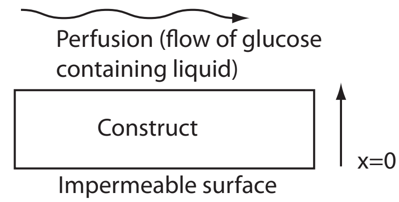


Figure 2: Cartilage construct with glucose containing fluid flowing over the top surface

Since tissue-engineered cartilage is without blood vessels, nutrient (such as glucose) supply relies on diffusion. In a greatly simplified situation shown in Figure 2, consider steady-state 1D transport of glucose into the cartilage construct where flow of glucose containing liquid maintains a concentration $c_s$ mol/ml of glucose at the top surface. The bottom surface rests on an impermeable surface, i.e., glucose cannot go through this surface. Diffusivity of glucose is $D$, the constant rate of consumption of glucose is $R$ mol of glucose per million cell/second and the cell density is $N$ (million cells/ml).

1) Write the governing equation for this problem. ◀ *4 points* ▶

2) Write the boundary conditions for the problem. ◀ *2 points* ▶

3) Solve for concentration of glucose as a function of position into the construct. ◀ *6 points* ▶

4) Assuming cell density is decided by the glucose concentration, at what location is the minimum cell density? ◀ *2 points* ▶

5) ***(Be careful with units)*** In a 2 mm construct, assuming cells need a minimum glucose concentration of $0.2 \times 10^{-6}$ mol/ml, what minimum cell density (in million cell/ml) can be maintained? Use surface concentration of $c_s = 5.56 \times 10^{-6}$ mol/ml, glucose diffusivity of $7.78 \times 10^{-10} \text{m}^2/\text{s}$ and consumption rate of $3.61 \times 10^{-11}$ mol of glucose per million cell/second. ◀ *5 points* ▶

6) If we want to maintain a minimum cell density (million cell/ml) that is twice as large as what you calculated in step 5, for the same thickness of construct, same diffusivity and same rate of consumption, what parameter would you change and what would be its new value? ◀ *5 points* ▶

## Problem 3 (23 points): Radioactive water leak in Fukushima

Effect of radioactive water leakage from the Fukushima plant in Japan (Figure 3) on fish and eventually humans has been an ongoing concern since we started the semester. This problem is not at all 1D but we will do a somewhat related problem hoping to get some qualitative understanding of the problem. Consider radioactive water coming to a flowing stream that can be treated as 1D. Also, the process reaches a steady state, unlike the real one where the leak has since stopped. Assume Iodine-131 is the primary radioactive component that is of concern—it has a half life of 8 days, following a first order reaction (Hint: half life for first order reaction is given by $0.5c_0 = c_0 e^{-k'' t_{1/2}}$). Seven tons (7000 kg) of radioactive water is being let into the river continuously every hour and it contains 300,000 becquerels per $cm^3$. The average velocity of water in the river can be approximated as 0.833 m/s (3 km/hr) and the volumetric flow rate of the stream is $280 m^3/s$ (one fourth of the Susquehanna river in New York). The turbulence in the current leads to a dispersion coefficient of 10 $m^2/s$. Density of water is $1000 kg/m^3$.

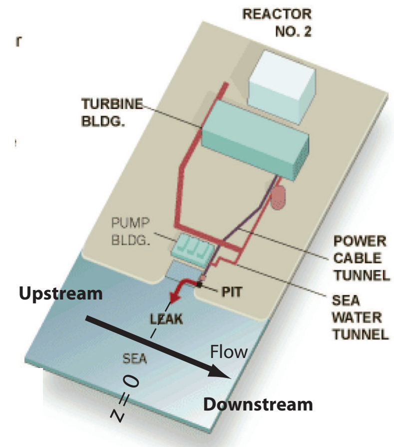


Figure 3: Radioactive water leaked into ocean from the Fukushima power plant

***Caution: Maintain several significant digits in your calculations until the end***

1) Up to what distance downstream would the water be considered contaminated, given the legal limit for radiation in water is 0.04 becquerels per $cm^3$? ◀ *7 points* ▶

2) Up to what distance upstream would the water be considered contaminated, given the legal limit for radiation in water is 0.04 becquerels per $cm^3$? ◀ *4 points* ▶

3) What is the concentration in the river at the point where the radioactive water is being dumped, in becquerels per $cm^3$? ◀ *3 points* ▶

4) What transport mode contributes to contamination upstream? ◀ *1 point* ▶

5) At the point where water is being dumped, but on its downstream side, what are the dispersive and convective fluxes of the radioactive water? ◀ *6 points* ▶

6) How do you expect the distances calculated here with a 1D model will change for the real situation where the Fukushima plant let out water into the Pacific Ocean? Provide reasoning for your answer but no calculations needed. ◀ *2 points* ▶

## Problem 4 (20 points): Contaminated wine?

Aerial contamination of vegetation is potentially an important source of human exposure to some toxic chemicals. We are interested in estimating toluene absorption from the atmosphere in grapes on a vine. A grape can be assumed spherical, with a typical radius of 0.0125 m, and containing no toluene initially. Diffusivity of dissolved toluene in a grape can be approximated for all layers to be that of the pulp, at $1.15 \times 10^{-7}$ m$^2$/s. The partition coefficient is $K^* = 3.39 \times 10^{-3}$; here $K^*$ is used exactly as defined in Chapter 13 for convective boundary condition, i.e., $K^* = c^{\text{air}}_{\text{tolune}}/c^{\text{grape}}_{\text{tolune}}$. The average windspeed is 0.254 m/s. The toluene concentration in the atmosphere is $2.18 \times 10^{-3}\text{g}/\text{m}^3$ (which is well below the legal limit, just so that you know). Initially, the grape can be considered toluene free. The viscosity and density of air are $1.98 \times 10^{-5}$ kg/m·s and $1.24\ \text{kg}/\text{m}^3$, respectively, and the diffusivity of toluene in air is $8.11 \times 10^{-6}\text{m}^2/\text{s}$.

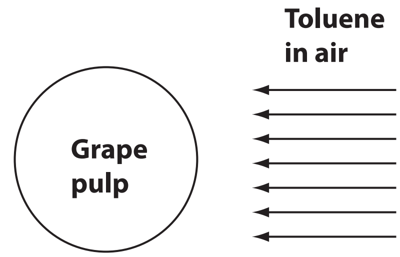


Figure 4: A spherical grape surrounded by flowing air with toluene in it

1) Calculate the mass transfer coefficient at the grape's surface? ◄ *8 points* ►

2) What is the exposure time at which the concentration in the center would reach 80% of the final value? ◄ *6 points* ►

3) For a different air flow situation, when the mass transfer coefficient at the surface is effectively infinite, what is the *surface* concentration of toluene *in the grape*? ◄ *2 points* ►

4) As you know, for a very short time after exposure, the grape can be considered very thick. With this assumption, and for the surface condition equal to a given concentration, how does the *rate at which toluene enters grape* depend on the diffusivity of the grape's tissue? Discuss using appropriate formula and symbols, no numerical calculations necessary. ◄ *4 points* ►

## Short Ques. (18 pts/3 pts each): Provide reasons when asked

1. You can have a mass transfer coefficient over a surface when there is no flow

   - Yes
   - No

   *Reasons:*

2. If there were no velocity boundary layer (velocity on the solid surface is the same as bulk velocity due to slip on the surface), mass transfer coefficient would be

   - Very small
   - Infinite

   *Reasons:*

3. When the size of a material increases, the time required for the same amount of mass (or heat) transfer increases as

   - Square of the characteristic length
   - Square root of the characteristic length
   - Proportional to (i.e, linear with) the characteristic length

   *Reasons:*

4. *Be as specific as possible.* What are the mechanisms of dispersion in 1) radioactive gases released by the accident in Japan; 2) drug transport in a liver; 3) pesticide movement in soil saturated with water

5. Show that although the units for heat transfer coefficient ($W/m^2K$) looks quite different from that for mass transfer coefficient (m/s), they are still completely analogous to each other. Just writing the convective transport equations will not give you points but can give you a hint on how to approach, by looking at the units of individual terms.

6. Provide an example *outside of heat and mass transfer* of the concept:

   Flow = (Driving force)/Resistance