Problem 1

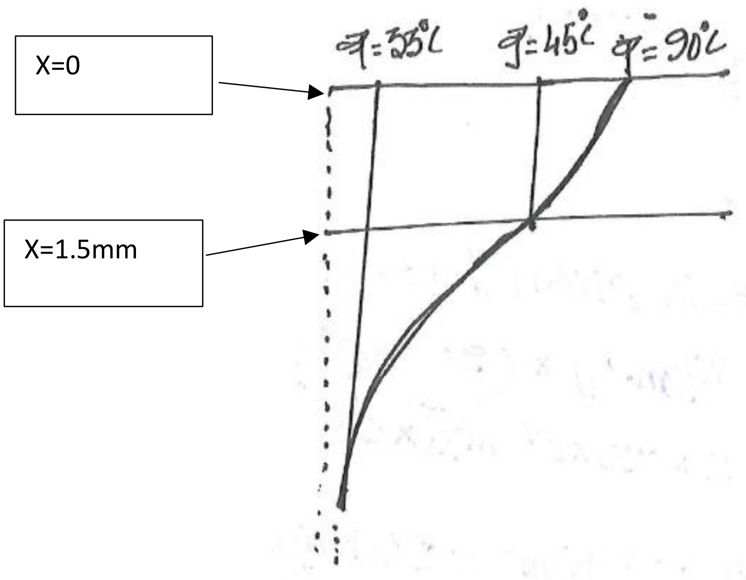
Known

Skin tissue initial temperature and surface temperature at $t = 0^{+}s$.

Find

1. Time require to start heat pain.
2. Heat flux at the surface at t=time at which heat pain starts, and at half of that time.

Schematic Plots



Strategy

Since we are not given enough information to define two boundaries, we can safely use semi-infinite solution.

Assumption

Temperature of the potato doesn't change appreciably over the time we are interested in.

Solution

1. The appropriate solution to the given problem statement is

$$\frac{T-T_i}{T_s-T_i} = 1 - erf\left[\frac{x}{2\sqrt{\alpha t}}\right]$$

$$\Rightarrow erf(\emptyset) = 1 - \frac{45° - 33°}{90° - 33°} = \left[where, \emptyset = \frac{x}{2\sqrt{\alpha t}}\right]$$

From the relevant chart, $\emptyset = 0.885$

So $\frac{x}{2\sqrt{\alpha t}} = 0.885$

$$\Rightarrow \frac{1.5x10^{-3}m}{2\sqrt{0.15x10^{-6}\,m^2/s^{x}\,ts}} = 0.885$$

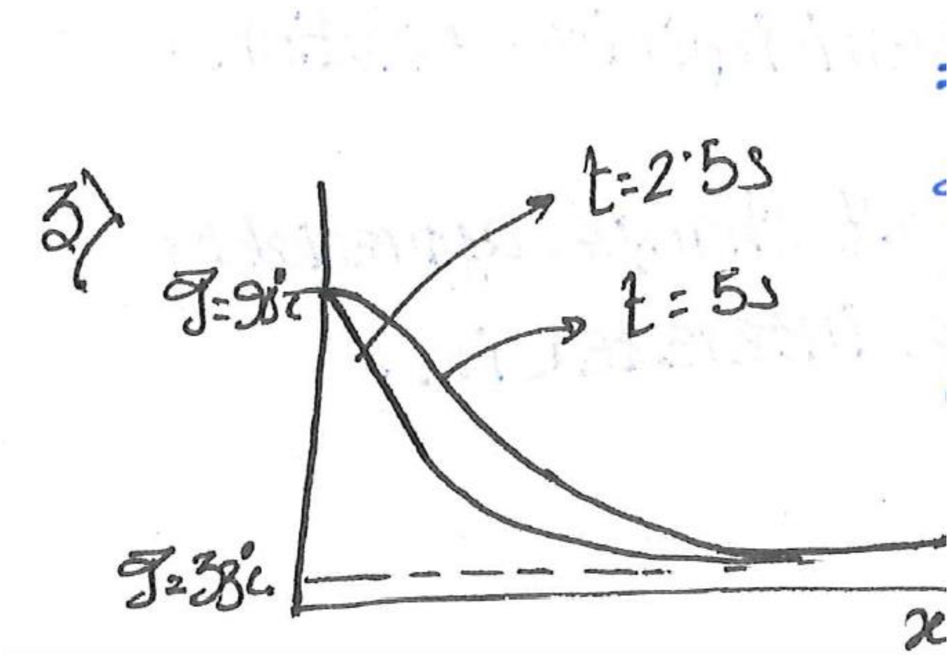
$$\Rightarrow t = 4.79 \approx 5s.$$

2. Heat flux at the skin surface is given by:

$$q_s'' = \frac{k\left(T_s - T_i\right)}{\sqrt{\pi\alpha t}} = \frac{0.35W/(m\cdot c)\,x\,(90-33)°C}{\sqrt{\pi x 0.15x10^{-6}\,m^2/s\,x5s}}\left(\text{ for } t = 5s\right)$$

$$= 12996\cdot 83W/m^2 \approx 13KW/m^2\left(\text{ for } t = 5s\right)$$

$$= 18380 \cdot 29\frac{W}{m^2} \simeq 18.4\frac{KW}{m^2}(for\ t = 2.5s)$$



The reason for change in heat flux is really evident from the temperature profile. At $t = 2\cdot 5s$ the temperature gradient at surface is much higher than the same at $t = 5s$.

PROBLEM 2

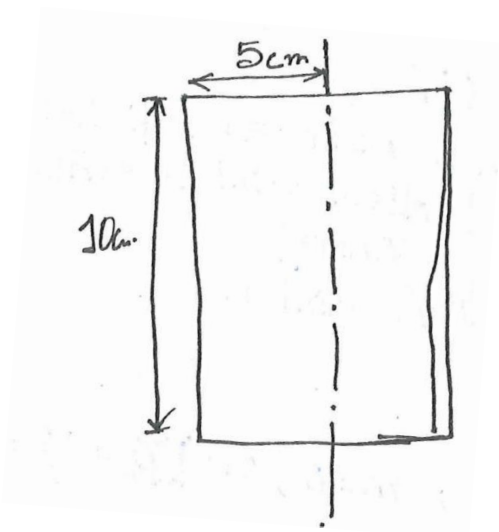
Known

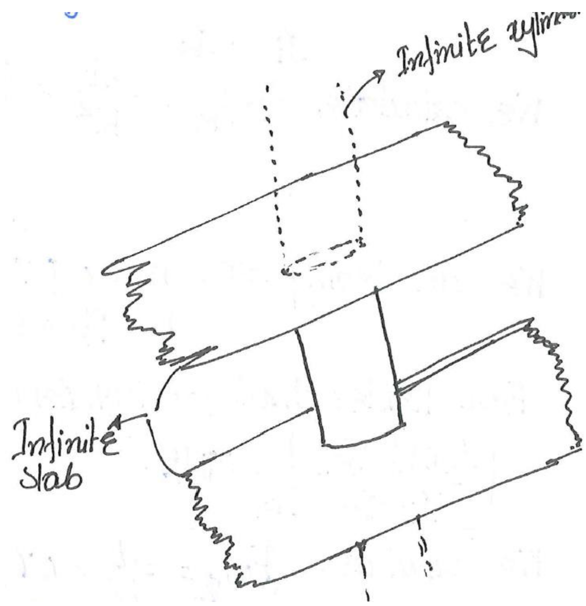
Dimensions, i.e., radius and height of a can. With the temperature at the boundary and the uniform initial temperature.

Find

1. Temperature after 168 minutes at the coldest point of the can in problem.
2. Temperature after same time period at the coldest point assuming can to be an infinite cylinder.

Schematic and Given Data





Strategy

Noting the numerical values of radius and height, we can conclude that both of the dimensions are important and we cannot consider the problem to be a 1-D problem. From Fournier's no. calculation, we can conclude that we are trying to find a long-term solution.

Assumption

We assume that thickness of the canning material is low enough and the thermal conductivity of the material is high enough such that it offers negligible resistance.

Solution

Since the problem is 2-D and we are looking for a long-term solution, we refer to the concept of multidimensional Heisler chart.

$$\frac{T(r,z,t)-T_0}{T_i-T_0}=\left(\frac{T(r,t)-T_0}{T_i-T_0}\right)_{\text{inf } inite \text{ cylinder}} \qquad \left(\frac{T(r,t)-T_0}{T_i-T_0}\right)_{\text{inf } inite \ slab}$$

We calculate $(F_0)_R=\frac{\alpha t}{R^2}=\frac{1\cdot 48\times 10^{-1}\left(m^2/s\right)\times 168\,\text{min}\times 60\left(s/\text{min}\right)}{\left(5\times 10^{-2}m\right)^2}=0\cdot 6$

We can clearly see $m=0$[$\therefore$ h is considered infinitely high]

$n={}^{r}\!/_{R}=0$ [coldest point is evidently at center]

From Heisler chart for infinitely long cylinder.

$$\left.\left(\frac{T(r,t)-T_0}{T_i-T_0}\right)\right|_{r=0}=0\cdot 075.$$

We calculate $(F_0)_L=\frac{\alpha t}{L^2}=0\cdot 6; m=0, n=\frac{x}{L}=0$

From Heisler chart for an infinite slab,

$$\left.\left(\frac{T(z,t)-T_0}{T_i-T_0}\right)\right|_{z=0}=0\cdot 25.$$

So, $\left.\left(\frac{T(r,z,t)-T_0}{T_i-T_0}\right)\right|_{r=0,z=0}=\left.\left(\frac{T(r,t)-T_0}{T_i-T_0}\right)\right|_{r=0}\left.\left(\frac{T(z,t)-T_0}{T_i-T_0}\right)\right|_{z=0}=0\cdot 075\times 0\cdot 25=0\cdot 01875.$

So, $T(r=0,z=0,t=168,\text{min})=118\cdot 7°C$

So, the sterilization is achieved.

(2) We have already calculated the ratio of $\left.\left(\frac{T(r,t)-T_0}{T_i-T_0}\right)\right|_{r=0}$ from Heisler chart for infinitely long cylinder.

So, $\left.\left(\frac{T(r,t)-T_0}{T_i-T_0}\right)\right|_{r=0}=0\cdot 075$

$T(r=0,t=168\,\text{min})=114\cdot 75.$

So, calculation considering infinite cylinder leads to a result, which shows the sterilization is not achieved.

(3) We can see that the temperature rise at the coldest point to lesser in case of 1-D assumption compared to actual 2-D problem. This is due to lower heat transfer area available per unit volume of the material for infinitely long cylinder compared to the 2-D problem.

PROBLEM 3

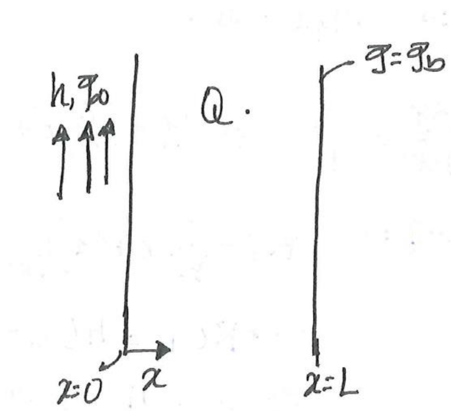
Known

The tissue layer is considered to be a slab with one relevant dimension. The heat generation in the tissue due to absorption of radiation is uniform $w \cdot r \cdot t$ space.

Find

(1) Governing equation and boundary condition for the problem.
(2) Steady-state temperature as a function of spatial variable.
(3) Spatial location of the maximum temperature.
(4) Maximum temperature rise over the core body temperature.
(5) Temperature profile.

Schematic and Given Data



Assumption

Dimension except x-direction considered very long to contribute to heat transfer.

Strategy

Since there is heat generation in the tissue, we can't go for resistance analysis. Also, since the boundary condition isn't symmetric, we cannot use conventional solution. So, we go for solving the problem from scratch with proper $G \cdot E$ and BC.

Solution

(1) The geometry clearly suggests, Cartesian coordinate. So, we know $G \cdot E$ in Cartesian coordinate.

$$\rho C_p \frac{\partial T}{\partial t} + \rho C_p u \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + Q$$

(1) Steady state $\Rightarrow \frac{\partial T}{\partial t} = 0$; (2) no bulk flows $\Rightarrow \frac{\partial T}{\partial x} = 0$.

So appropriate governing equation $\Rightarrow K \frac{d^2 T}{dx^2} + Q = 0$

→ Note: heat flux is a vector so direction is important. So it is not $-k\left.\frac{dT}{dx}\right|_{x=0} = h\left(T|_{x=0} - T_\infty\right)$.

<u>B.C→</u>1: $q''|_{x=0} - k\left.\frac{dT}{dx}\right|_{x=0} = h\left(T_\infty - T|_{x=0}\right)$

<u>B.C→</u>2: $T|_{x=L} = T_\infty$.

(2) So, $\frac{d^2T}{dx^2} = -\frac{Q}{K} \Rightarrow T = -\frac{Q}{2K}x^2 + C_1 x + C_2$.

From <u>B.C→</u>1: $-K\left.\left(-\frac{Q}{K}x + C_1\right)\right|_{x=0} = h\left(T_\infty - C_2\right)$

$\Rightarrow KC_1 = h\left(C_2 - T_\infty\right)$

$\Rightarrow C_1 = \frac{h}{K}\left(C_2 - T_\infty\right) - (1)$

<u>B.C→</u>2: $T_b = -\frac{Q}{2K}L^2 + C_1 L + C_2 - (2)$

$(1)\times L + (2) \Rightarrow$

$$T_b = \frac{hL}{K}\left(C_2 - T_\infty\right) + C_2 - \frac{Q}{2K}L^2$$

$$C_2 = \frac{T_b + \frac{hL}{K}T_\infty + \frac{Q}{2K}L^2}{\left(\frac{hL}{K}+1\right)} = \frac{LT_\infty + \frac{K}{h}T_b + \frac{Q}{2h}L^2}{\left(L + \frac{K}{h}\right)}$$

$$C_1 = \frac{h}{K}\left(C_2 - T_\infty\right)$$

$$= \frac{h}{K}\left(\frac{T_b + \frac{hL}{K}T_\infty + \frac{Q}{2K}L^2}{\left(\frac{hL}{K}+1\right)} - T_\infty\right)$$

$$= \frac{h}{K}\left(\frac{\left(T_b - T_\infty\right) + \frac{Q}{2K}L^2}{\left(\frac{hL}{K}+1\right)}\right)$$

$$= \frac{\left(T_b - T_\infty\right) + \frac{Q}{2K}L^2}{\left(L + \frac{K}{h}\right)}$$

So, $T = -\frac{Q}{2K}x^2 + \left(\frac{(T_b - T_\infty) + \frac{Q}{2K}L^2}{\left(L + \frac{K}{h}\right)} \right) x + \left(\frac{LT_\infty + \frac{K}{h}T_b + \frac{Q}{2h}L^2}{\left(L + \frac{K}{h}\right)} \right)$

(3) For maximum temperature, we need to set $\frac{dT}{dx} = 0.$

$$\frac{dT}{dx} = -\frac{Q}{K}x + C_1 = 0$$

$$\Rightarrow x = C_1 \times \frac{K}{Q} = \left(\frac{(T_b - T_\infty) + \frac{Q}{2K}L^2}{L + \frac{K}{h}} \right) \times \frac{K}{Q}.$$

So, $x_{\text{max.}} = \left(\frac{(37 - 30) + \frac{1200}{2 \times 0.27} \times 0.04^2}{0.04 + \frac{0.27}{3}} \right) \times \frac{0.27}{1200}$

=0.018m.

(4) $T_{\max} = T\big|_{x = x\max} = 38.05°C.$

So, rise in temperature $\Rightarrow T_{\max} - T_{body}$

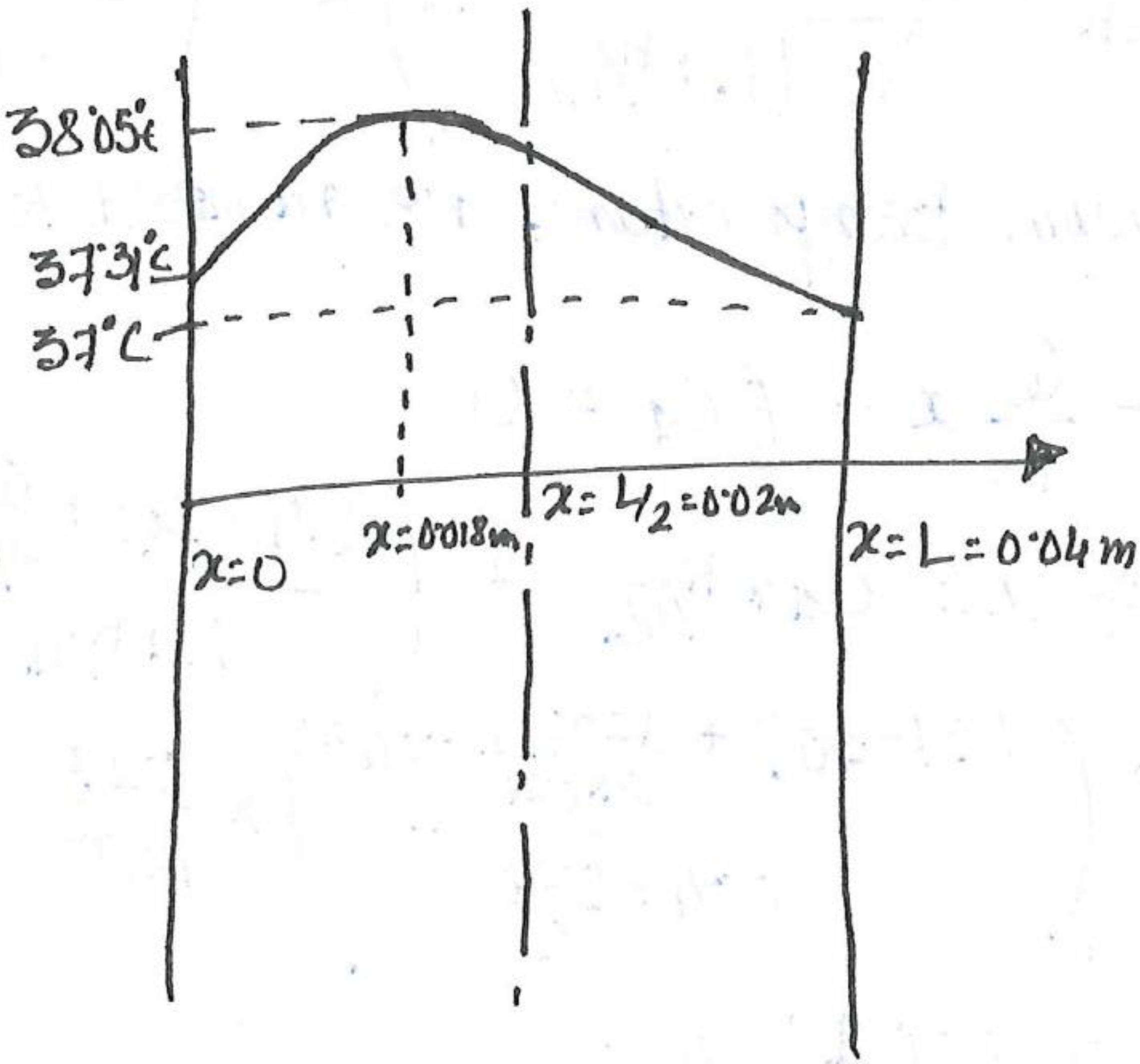
=1.05°C.

(5) Clearly there exists a local maxima at an intermediate point. So, to plot this function, we need the value of the function at three points that are the two end points and the point of local maxima.

So, $T\big|_{x=0} = C_2 = 37.31°C$

$T\big|_{x=L} = T_b = 37°C$

$T\big|_{x=x\max} = T_{\max} = 38.05°C.$



PROBLEM 4

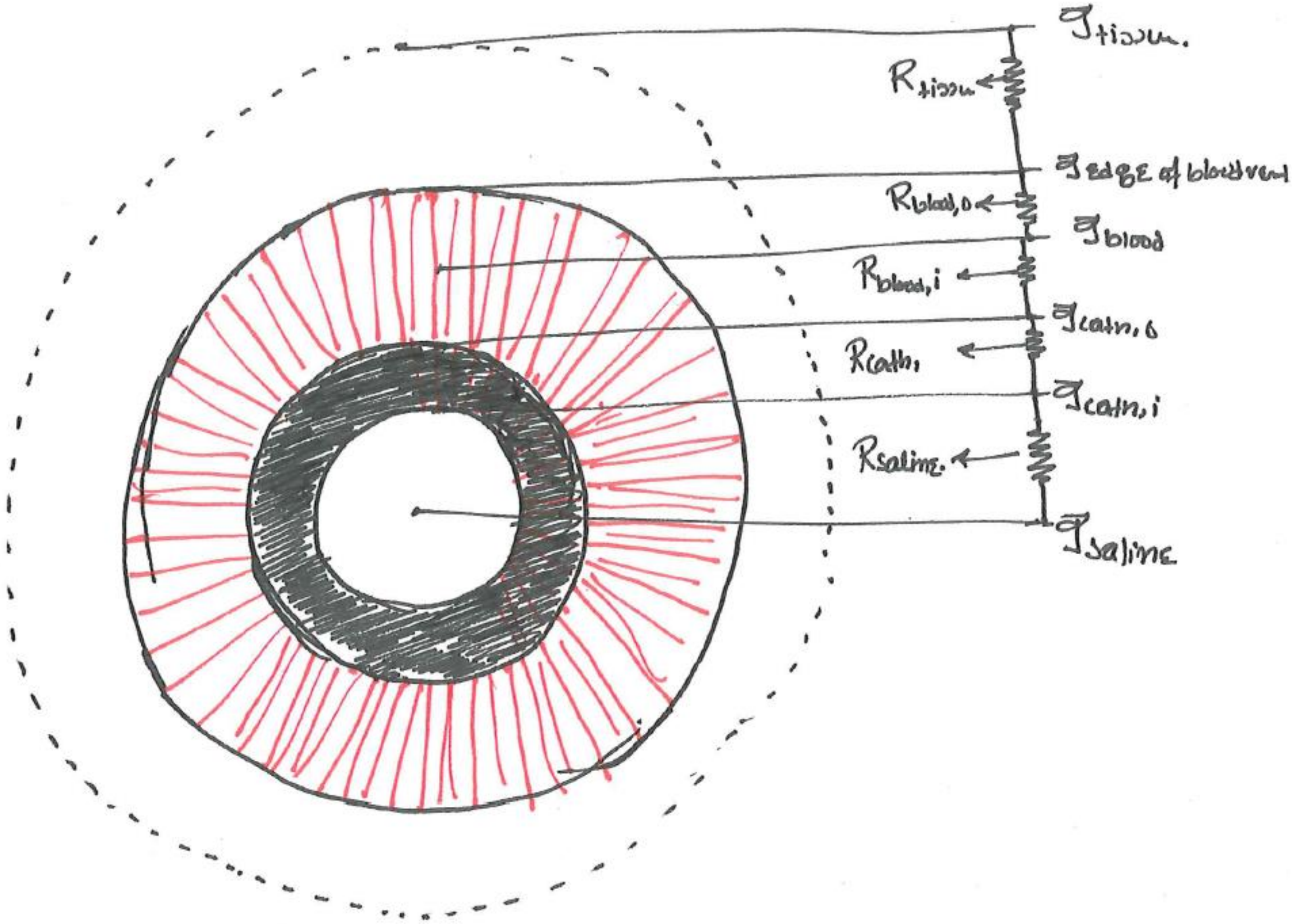
Known

Steady state heat transfer through layers of concentric cylinders. The thermal conductivity values and heat transfer coefficients for each layer are also given in the problem.

Find

Temperature of the cold saline water such that the temperature at the edge of blood vessel at 25°C.

Schematic and Given Data



Strategy

The heat transfer takes place across a number of layers. As the temperature is constant with time, the system is in steady state. Also, since there is no heat generation in any of the layers, we can opt for resistance analysis.

Assumption

Temperature doesn't change in the direction perpendicular to the plane of paper. Also, the blood vessel is considered to be thin enough to neglect its resistance.

Solution

As the thermal resistances are arranged in series, we can say the heat flow across each resistance is constant.

So, $q = \dfrac{T_{edge\ \text{of}\ blood\ \ vessel} - T_{saline}}{R_{saline} + R_{cath} + R_{blood,i} + R_{blood,s}} = \dfrac{\Delta T_1}{\sum R}$

$$q = \frac{T_{tissue} - T_{saline}}{\sum R + R_{tissue}} = \frac{\Delta T_2}{\sum R + R_{tissue}}$$

$$R_{saline} = \frac{1}{h_i A_{cath,i}} = \frac{1}{h_i (2\pi r_{cath,i}) L}$$

From $\dfrac{h_i D}{k_{saline}} = 4.36 \Rightarrow h_i = 1264.4 W/m^2 \cdot C.$

So $R_{saline} = \dfrac{0.126}{L} K/W$

$$R_{cath} = \frac{1n\left(\dfrac{r_{cath,0}}{r_{cath,i}}\right)}{2\pi L K_{cath}} = \frac{0.104}{L} K/W$$

$$R_{blood,i} = \frac{1}{h_0 A_{cath,0}} = \frac{1}{h_0 \left(2\pi r_{cath,0}\right) L} = \frac{0.215}{L} K/W$$

$$R_{blood,0} = \frac{1}{h_0 A_{edge}} = \frac{1}{h_0 \left(2\pi r_{edge}\right) L} = \frac{0.089}{L} K/W$$

So $\sum R = \dfrac{0.534}{L} K/W$

$$R_{tissue} = \frac{1n\left(\frac{r_{tissue}}{r_{edge}}\right)}{2\pi K_{tissue} L} = \frac{0.281}{L} K/W$$

So $\sum R + R_{tissue} = \frac{0.815}{L} K/W$

So $\frac{T_{tissue} - T_{saline}}{\frac{0.815}{L}} = \frac{T_{edge} - T_{saline}}{\frac{0.534}{L}}$

So $0.534 \times T_{tissue} - 0.815 T_{edge} = (0.534 - 0.815) T_{saline}$

$$\Rightarrow T_{saline} = \frac{0.815 T_{edge} - 0.534 T_{tissue}}{0.815 - 0.534}$$

$\square\ 2°C.$