Problem 8.12.12

KNOWN: Diameter and emissivity of a horizontal thermocouple (TC) sheath located in a large room. Air and wall temperatures.

Table A-4, Air (assume T_s = 30 °C, $T_f = (T_s + T_\infty)/2 \approx 300$ K, 1 atm:

FIND: (a) Temperature indicated by the TC, (b) Measurement error.

SCHEMATIC AND GIVEN DATA

STRATEGY:

ASSUMPTIONS:

(1) Room walls approximate isothermal, large surroundings
(2) Room air is quiescent
(3) TC approximates horizontal cylinder
(4) No conduction losses
(5) TC surface is opaque, diffuse and gray.

SOLUTION:

(a) Perform an energy balance on the thermocouple considering convection and radiation processes. On a unit area basis, with $q''_{conv} = \bar{h}\left(T_s - T_\infty\right)$,

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = 0$$
$$\alpha G - \varepsilon E_b\left(T_s\right) - \bar{h}\left(T_s - T_\infty\right) = 0. \tag{1}$$

Since the surroundings are isothermal and large compared to the thermocouple, G = E_b (T_{sur}). For the gray-diffuse surface, α = ε. Using the Stefan-Boltzman law, $E_b = \sigma T^4$, Eq. (1) becomes

$$\varepsilon\sigma\left(T_{sur}^4 - T_s^4\right) - \bar{h}\left(T_s - T_\infty\right) = 0. \tag{2}$$

Using the Churchill-Chu correlation for a horizontal cylinder, estimate $\bar{h}$ due to free convection.

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + \left(0.559/\Pr\right)^{9/16}\right]^{8/27}}\right\}^2 \tag{3}$$

$$Ra_D = \frac{g\beta\Delta T D^3}{\nu\alpha} = Gr \cdot \Pr \tag{4}$$

$$\text{Where Gr} = \frac{\beta g p^2 L^3 \Delta T}{\mu^2}$$

Problem 8.12.12

$$Gr = \frac{\left(\frac{1}{300K}\right)\left(9.8\,\frac{m}{s^2}\right)\left(1.1769\,\frac{kg}{m^3}\right)^2 (0.004m)^3(30-25)K}{\left(1.845.\,10^{-5}\,\frac{kg}{m\cdot s}\right)} = 42.5$$

Pr=0.708

To evaluate Ra_D and $\bar{N}u_D$, $T_f = 300$ K, giving

$$Ra_D = \frac{9.8\ \text{m/s}^2\left(1/300\ \text{K}\right)\left(30-25\right)\text{K}\left(0.004\ \text{m}\right)^3}{\left(\dfrac{1.8465\times10^{-5}\,\text{kg/m.s}}{1.1769\text{kg/m}^3}\right)\times 2.2156\times10^{-5}\,\text{m}^2/\text{s}} = \left(42.5\right)\left(0.708\right) = 30.07$$

$$\bar{h} = \frac{0.02624\ \text{W/m}\cdot\text{K}}{0.004\ \text{m}}\left\{0.60 + \frac{0.387\left(30.07\right)^{1/6}}{\left[1+\left(0.559/0.708\right)^{9/16}\right]^{8/27}}\right\}^2 = 8.93\ \text{W/m}^2\cdot\text{K}.$$

(5)

With ε = 0.5, the energy balance, Eq. (2), becomes

$$0.5\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left[\left(35+273\right)^4 - T_s^4\right]\text{K}^4 - 8.93\ \text{W/m}^2\cdot\text{K}\left[T_s - \left(25+273\right)\right]\text{K} = 0 \qquad (6)$$

By trial-and-error, find

$T_s = 27.64$°C

(b) The thermocouple measurement error is defined as $\Delta T = T_s - T_\infty$ and is a consequence of radiation exchange with the surroundings. The measurement error is 2.64 °C.

Problem 8.12.18

Global energy balance

Earth and atmosphere system
$\varepsilon = \alpha$

$$F_s \cdot \pi R^2 \cdot (1-\rho) - \sigma\, T_o^4 \cdot 4\pi R^2 \cdot (1-\alpha) - \sigma T_1^4 \cdot 4\pi R^2 \cdot \alpha = 0$$

$$\frac{F_s(1-\rho)}{4} - \sigma\, T_o^4 (1-\alpha) - \sigma T_1^4 \alpha = 0 \qquad (1)$$

Atmosphere only

$$\sigma\, T_o^4 \cdot 4\pi R^2 \cdot \alpha = \sigma T_1^4 \cdot 4\pi R^2 \alpha (2)$$

$$T_o^4 = 2\, T_1^4 \qquad (2)$$

From (1), Substituting (2)

$$\frac{F_s(1-\rho)}{4} - \sigma\, T_o^4 (1-\alpha) - \frac{1}{2}\sigma T_o^4 \alpha = 0$$

$$\sigma\, T_o^4 \left[(1-\alpha) + \frac{\alpha}{2}\right] = \frac{F_s(1-\rho)}{4}$$

$$T_o = \left[\frac{F_s(1-\rho)}{4\sigma\left(1-\frac{\alpha}{2}\right)}\right]^{1/4}$$

$$T_o = \left[\frac{1353(1-0.28)}{4 \times 5.67 \times 10^{-8}\left(1-\frac{0.75}{2}\right)}\right]^{1/4} \text{K} = 287.9 \text{ K}$$

For observed T_o = 288K, we get α = 0.77, not inconsistent with the value assumed here.

As α increases, T_0 increases.

Problem 8.12.25

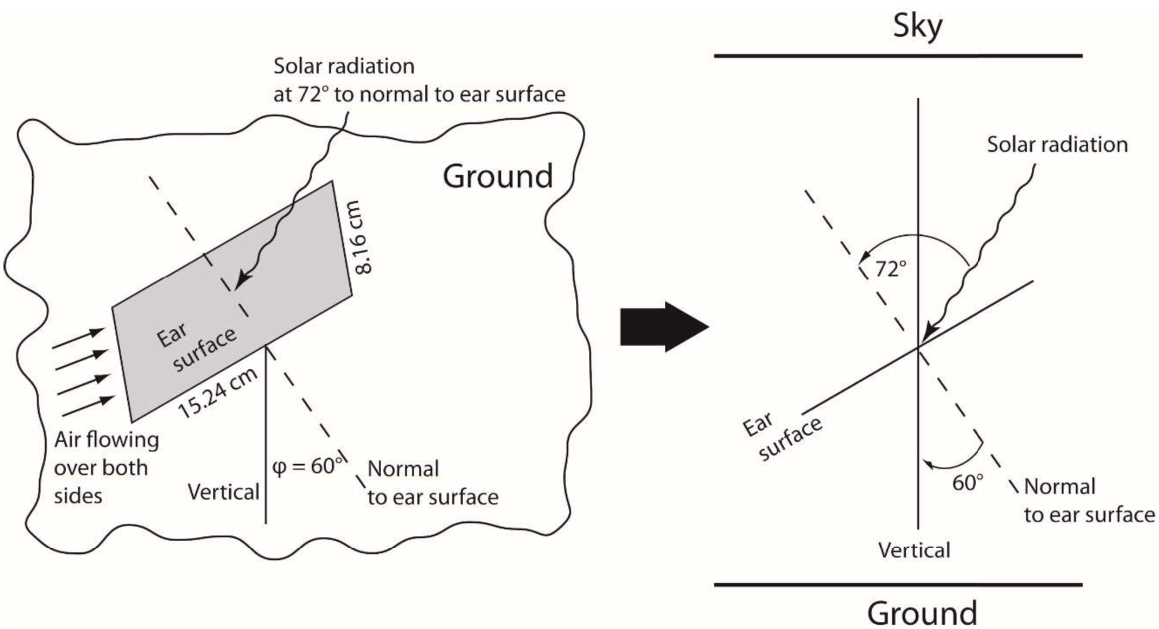
KNOWN:

Ear dimensions. Solar radiation and how it impacts the ear. Heat exchange with ground and sky. Heat exchange with air. Metabolic heat generation. The system is at steady state.

FIND:

The convective heat transfer coefficient using the equation for forced convection over a flat plate. Then, we are looking for an energy balance and then temperature at steady state. If only a fraction of the sun's energy hit the jackrabbit's ear, how much would the temperature change.

SCHEMATIC AND GIVEN DATA



$$Ear = 0.0816m \times 0.1524m \times 0.005m$$

$$v = 1\frac{m}{s}$$

$$T_{air} = 21^{\circ}C$$

$$T_{sky} = 24^{\circ}C$$

$$T_{gr} = 40^{\circ}C$$

$$Q = 1015 \frac{\text{W}}{\text{m}^3}$$

$$I = 1000 \frac{\text{W}}{\text{m}^2}$$

$$\theta = 72^\circ$$

STRATEGY:

Use the formula from the text for forced convection over a plate to calculate the average heat transfer coefficient over the ear. Set up the energy balance for the ear and solve for its T, the only unknown in the energy balance equation. We need to be careful to consider all radiative fluxes and the convective fluxes. For the fourth part, recalculate the energy from incidence and solve for T.

ASSUMPTIONS:

We will calculate *h* at the bulk air properties. Ideally, we would like to use the arithmetic average of the surface and the bulk air but we do not know the surface temperature. It would also be acceptable to guess a reasonable temperature 35°C and hence calculate the air properties at 28°C to get a better first answer. After calculating a surface temperature, *h* should be recalculated at the average of the calculated surface temperature and bulk air temperature until the surface temperature does not change from the previously calculated.

SOLUTION:

1)

Properties of air at $21^\circ C$

$$\rho = 1.20 \frac{kg}{m^3}$$

$$\mu = 1.82 \times 10^{-5} \frac{kg}{m \cdot s}$$

$$\Pr = 0.709$$

$$k = 0.0258 \frac{W}{m \cdot K}$$

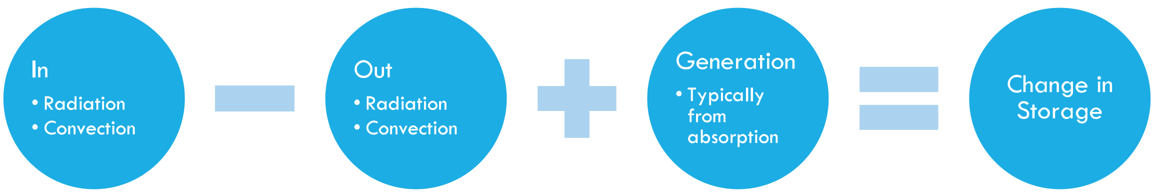
Calculating the Reynolds number we see that the flow is laminar so we use the correct formula from the text.

$$Re_L = \frac{Lv\rho}{\mu} = \frac{0.1524m \times 1\frac{m}{s} \times 1.20\frac{kg}{m^3}}{1.82 \times 10^{-5}\frac{kg}{ms}} = 10048$$

$$Nu = 0.664 Re^{1/2} Pr^{1/3} = 59.4 = \frac{hL}{k}$$

$$h = \frac{59.4k}{L} = \frac{59.4 \times 0.0258}{0.1524} = 10.06\frac{W}{m^2 \cdot K}$$

2)



$$\underbrace{I_o \cos\theta\, A}_{\text{In, solar}} - \underbrace{h\left(T - (273 + T_{air})\right)2A}_{\text{Out, convection}}$$
$$- \underbrace{\varepsilon\sigma F_{TE-sky} A\left(T^4 - (T_{sky} + 273)^4\right)}_{\text{out (top surface to sky), radiation}} - \underbrace{\varepsilon\sigma F_{TE-gr} A\left(T^4 - (T_{gr} + 273)^4\right)}_{\text{Out (top surface to ground), radiation}}$$
$$- \underbrace{\varepsilon\sigma F_{BE-sky} A\left(T^4 - (T_{sky} + 273)^4\right)}_{\text{Out (bottom surface to sky), radiation}} - \underbrace{\varepsilon\sigma F_{BE-gr} A\left(T^4 - (T_{gr} + 273)^4\right)}_{\text{Out (top surface to ground), radiation}} + \underbrace{QA\Delta x}_{\text{Generation, metabolic}} = \underbrace{0}_{\text{steady state}}$$

$where:$

$$F_{TE-sky} = \frac{1+\cos\phi}{2} \quad F_{BE-sky} = \frac{1-\cos\phi}{2} \quad F_{TE-gr} = \frac{1-\cos\phi}{2} \quad F_{BE-gr} = \frac{1+\cos\phi}{2}$$

$$\varepsilon = 1$$

3)

$$0 = I_o \cos 72 + Q\Delta x - 2h\left(T - (273 + 21)\right) -$$
$$\sigma\left[F_{TE-sky}\left(T^4 - 297^4\right) - F_{TE-gr}\left(T^4 - 313^4\right) - F_{BE-sky}\left(T^4 - 297^4\right) - F_{BE-gr}\left(T^4 - 313^4\right)\right]$$
$$0 = I_o \cos 72 + Q\Delta x - 2h\left(T - 294\right) -$$
$$\frac{\sigma}{2}\left[\left(1+\cos\varphi\right)\left(T^4 - 297^4\right) + \left(1-\cos\varphi\right)\left(T^4 - 313^4\right) + \left(1-\cos\varphi\right)\left(T^4 - 297^4\right) + \left(1+\cos\varphi\right)\left(T^4 - 313^4\right)\right]$$
$$0 = I_o \cos 72 + Q\Delta x - 2h\left(T - 294\right) - \sigma\left[\left(T^4 - 297^4\right) + \left(T^4 - 313^4\right)\right]$$
$$0 = 1000 \times 0.309 + 1015 \times 0.005 - 2 \times 10.06\left(T - 294\right) - 5.67 \times 10^{-8}\left[\left(T^4 - 297^4\right) + \left(T^4 - 313^4\right)\right]$$
$$T = 307.9\,\text{K} = 34.9^\circ\text{C}$$

4)

The temperature would be higher since there is more heat flux coming in from the sun.

$$0 = 1000\cos\left(10^{\circ}\right) + 1015\Delta x - (2)(10.06)\left(T - 294\right) - 5.67\times10^{-8}\left[\left(T^4 - 297^4\right) + \left(T^4 - 313^4\right)\right]$$

$$0 = 1000\times0.985 + 1015\times0.005 - (2)(10.06)\left(T - 294\right) - 5.67\times10^{-8}\left[\left(T^4 - 297^4\right) + \left(T^4 - 313^4\right)\right]$$

$T = 327.41\,\text{K} = 54.4^{\circ}\text{C}$

5)

Metabolic heat generation would decrease to compensate for the increase energy being absorbed.