Problem 7.9.13

(1)

Heat flow = $\rho\left(4\pi r^2 \frac{dr}{dt}\right)\Delta H_f$

(2)

Resistance of a hollow sphere, $R = \frac{(r_o - r_i)}{4\pi r_o r_i k}$

Heat flow = $\frac{T_m - T_p}{R}$

(3) Equating (1) & (2)

$$\frac{T_m - T_p}{(r - r_i)/4\pi r r_i k} = \rho 4\pi r^2 \frac{dr}{dt}\Delta H_f$$

$$\frac{(T_m - T_p) r_i k}{\rho \Delta H_f} \quad dt = (r - r_i) r \, dr$$

(4) Integrating from r_i to r

$$\frac{(T_m - T_p) r_i k}{\rho \Delta H_f} \quad \int_o^t dt = \int_{r_i}^{r} (r^2 - r_i r)\, dr$$

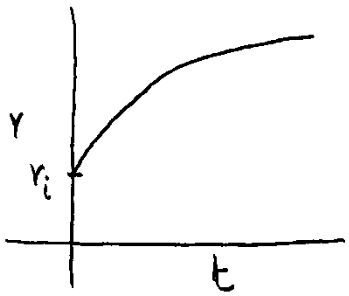
$$\frac{(T_m - T_p) r_i k t}{\rho \Delta H_f} \quad = \left(\frac{r^3 - r_i^3}{3}\right) - r_i\left(\frac{r^2 - r_i^2}{2}\right)$$

$$\frac{(T_m - T_p) r_i k t}{\rho \Delta H_f} \quad = \frac{r^3}{3} + \frac{r_i^3}{6} - \frac{r_i r^2}{2}$$

$$t \quad = \frac{\rho \Delta H_f}{(T_m - T_p) k r_i}\left(\frac{r^3}{3} + \frac{r_i^3}{6} - \frac{r_i r^2}{2}\right)$$

Problem 7.9.13

(5)



Problem 7.9.16

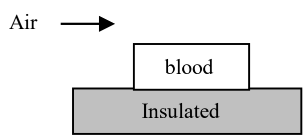
KNOWN: Thickness of package, Freezing temperature of blood (initial temperature), air temperature, heat transfer coefficient, thermal conductivity, density

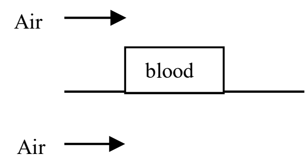
FIND:

4) Thawing time for insulated condition
5) Thawing time when convection occurs on two surfaces

SCHEMATIC AND GIVEN DATA

k = 0.55 W/m-K
x = 2.5 cm





T_{air} = 15 °C; T_{blood} = -0.5 C; h = 20 W/m^2-K

STRATEGY:
This is straightforward application of freezing time calculation for a 1D slab. Although the solution was derived for freezing from both sides (right figure), the bottom boundary condition for the left figure can be seen as equivalent to the boundary condition at the symmetry line for the right figure.

ASSUMPTIONS:

1-D heat conduction

SOLUTION:

a) Time for thawing is given as t:

$$t = \frac{\lambda\rho}{\Delta T}\left(\frac{x^2}{2k} + \frac{x}{h}\right)$$

Where λ = Latent heat of fusion

$$t = \frac{335 \times 10^3 \times 950}{15.5}\left(\frac{\left(2.5 \times 10^{-2}\right)^2}{2 \times 0.55} + \frac{2.5 \times 10^{-2}}{20}\right)$$

t = 37331.38 secs
= 10.37 hrs

b) When natural convection occurs at the top and bottom surfaces of the blood, the thickness taken into consideration is halved.

$$t = \frac{\lambda\rho}{\Delta T}\left(\frac{x^2}{2k} + \frac{x}{h}\right)$$

$$t = \frac{335 \times 10^3 \times 950}{15.5}\left(\frac{\left(1.25 \times 10^{-2}\right)^2}{2 \times 0.55} + \frac{1.25 \times 10^{-2}}{20}\right)$$

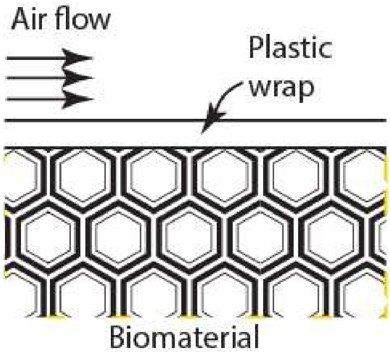
= 15749.18 secs.

= 4.37 hrs

COMMENTS:

Problem 7.9.22

Known:
All required parameters.
Schematic:



Find:
Depth of freezing as function of time.
Strategy:
In these simplified analysis for freezing time, as was done in class for a slab geometry, we equate the rate of heat transfer through the already frozen layer to the rate at which heat needs to be removed to freeze an additional layer. The analysis here is exactly same as that done for a slab, with the only difference being the resistance here is not just of that of the frozen layer but also the plastic wrap in series with the frozen layer.
Solution:
1)

$$\frac{T_m - T_\infty}{\dfrac{L_{plastic}}{k_{plastic}A} + \dfrac{x}{k_{frozen}A} + \dfrac{1}{hA}} = \Delta H_f \rho A \frac{dx}{dt}$$

$$\frac{T_m - T_\infty}{\dfrac{L_{plastic}}{k_{plastic}} + \dfrac{x}{k_{frozen}} + \dfrac{1}{h}} = \Delta H_f \rho \frac{dx}{dt}$$

$$\frac{T_m - T_\infty}{\Delta H_f \rho} = \left(\frac{L_{plastic}}{k_{plastic}} + \frac{x}{k_{frozen}} + \frac{1}{h}\right)\frac{dx}{dt}$$

$$\int_0^t \frac{T_m - T_\infty}{\Delta H_f \rho} dt = \int_0^{L_{bio}} \left(\frac{L_{plastic}}{k_{plastic}} + \frac{x}{k_{frozen}} + \frac{1}{h}\right) dx$$

$$\frac{T_m - T_\infty}{\Delta H_f \rho} t = \frac{L_{plastic}}{k_{plastic}} x + \frac{x^2}{2k_{frozen}} + \frac{x}{h}$$

$$\frac{T_m - T_\infty}{\Delta H_f \rho} t = \frac{L_{plastic}}{k_{plastic}} L_{bio} + \frac{L_{bio}^2}{2k_{frozen}} + \frac{L_{bio}}{h}$$

$$t_{freeze} = \frac{\Delta H_f \rho}{T_m - T_\infty}\left(\frac{L_{plastic}}{k_{plastic}} L_{bio} + \frac{L_{bio}^2}{2k_{frozen}} + \frac{L_{bio}}{h}\right)$$

2)
It will add an extra layer of conductive resistance that increases the time to freeze.