Problem 6.10.4

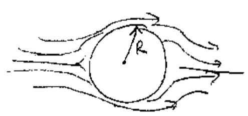
KNOWN: A heated sphere of radius R is suspended in a large, motionless body of fluid. It is desired to study the heat conduction in the fluid surrounding the sphere.

FIND:

1. Set up the differential equation describing the temperature T in the surrounding fluid as a function of r, the distance from the center of the sphere. The thermal conductivity of the fluid k is constant.
2. Integrate the differential equation and use the boundary conditions.
   $B.C.1$: at $r = R \quad T = T_s$
   $B.C.2$: $\quad r = \infty \quad T = T_\infty$
   to determine the constants of integration.
3. From the temperature profile, obtain an expression for the heat flux at the surface. Equate this result to the heat flux written as "Newton's Law of Cooling" and show that a dimensionless heat transfer coefficient (Nusselt Number) is given by $Nu = \frac{hD}{k} = 2$
   where D = sphere diameter.

   NOTE: This is a well-known result, which is the limiting value of Nu for heat transfer from spheres at low Reynolds or Grashof numbers e.g. for small spheres.

SCHEMATIC AND GIVEN DATA



STRATEGY:

ASSUMPTIONS:

1. There are no free convective effects.

SOLUTION:

The fundamental equation for transport in the surrounding fluid is: Rate of Storage = rate of diffusive flux change and generation.

Under steady state conditions, the storage term is zero.

Generation term is zero as there are no heat sources and heat sinks in the surrounding fluid.

The center of the sphere is located at the origin.

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The diffusive flux change for radial variations is spherical co-ordinates is

$$\text{flux} = k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right)$$

- therefore, $k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right) = 0$ (A)
- Using the two boundary conditions, the above equation (A) can be integrated resulting in $T = \frac{A}{r} + B$ where A & B are constants

Using $B.C.$ 2, we find $B = T_\infty$

Using $B.C.$ 1, we find
$$T_s = \frac{A}{R} + B$$
$$A = \left(T_s - T_\infty\right)R$$

Therefore, the temperature profile in the surrounding fluid is: $T = \frac{\left(T_s - T_\infty\right)R}{r} + T_\infty$ (B)

The heat flux at the surface $= h\left(T_S - T_\infty\right) == -k\left.\frac{dT}{dr}\right|_{r=R}$

(C)

where h = heat transfer coefficient

If we differentiate Eq. (B) we get, $\left.\frac{dT}{dr}\right|_{r=R} = -\left(\frac{T_s - T_\infty}{R}\right)$ (D)

The Nusselt number, Nu, is defined as $Nu = \frac{hD}{k}$

where $D = 2R$, sphere diameter

Substitute Eq. (D) into (C) to find h

$$h = \frac{-k}{\left(T_s - T_\infty\right)}\left(-\frac{T_s - T_\infty}{R}\right) = \frac{k}{R}$$

$$Nu = \frac{\left(k/R\right)\left(2R\right)}{k} = 2$$

COMMENTS:

If you look at the correlations for h, for a sphere ∞ the $Nu = 2 + f(\text{Re}\bullet\text{Pr})$ (Eq. 6.52, Pg. 111). There are two contributions to h, namely due to conduction and due to convection. The convection term will depend on the velocity and hence on the Reynolds number. If there is no bulk motion of the surrounding fluid, Re = 0 and the heat transfer is only due to the conduction within the surrounding fluid. In that case the Nu shall be 2, which was what we obtained above, when we considered heat transfer from the sphere to the surrounding fluid as only due to the surrounding fluid conduction.

Problem 6.10.14

KNOWN:

FIND:

SCHEMATIC AND GIVEN DATA

STRATEGY:

ASSUMPTIONS:

SOLUTION:

$$\int_{\theta_i}^{\theta}\frac{d\theta}{\theta}=\int_0^t -\frac{hA}{mc_\rho}dt \qquad \theta=T_{av}-T_\infty$$

$$\ln\frac{T_{av}-T_\infty}{T_i-T_\infty}=-\frac{A}{mc_\rho}\int_0^t 200\, e^{-t/600}dt$$

$$=+\frac{(A)(200)}{mc_\rho}(600)\left(e^{-t/600}-1\right)$$

$$\ln\frac{115-121}{30-121}=\frac{0.04\left[m^2\right](200)(600)\left(e^{-t/600}-1\right)}{200\times10^{-3}\left[kg\right]4.19\times10^{3}\,\frac{J}{kg\ K}}\left[W/m^2K\right]\left[s\right]$$

$$\ln\left[\frac{-6}{-91}\right]=5.7279\left(e^{-t/600}-1\right)$$

$$-2.7191=5.7279\left(e^{-t/600}-1\right)$$

$$-0.4747=e^{-t/600}-1$$

$$e^{-t/600}=0.5252885$$

$$\frac{-t}{600}=-0.6438$$

$$t=386\ s\cong 6.4\ min$$

COMMENTS:

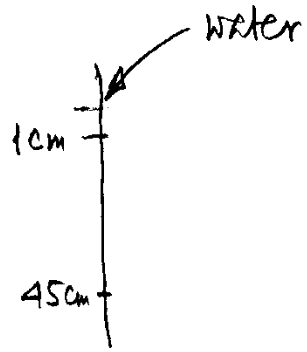
1. Heat transfer coefficient reduces with time since the flow is driven by the temperature difference between the fluid (T_{av}) and boundary, T_∞. This temperature difference reduces with time.
2. Viscosity in the fluid can decrease considerably at higher temperatures, which will increase the heat transfer coefficient (Re is increased).

Problem 6.10.17

KNOWN:
Two locations along the flow over a flat plate

FIND:
The ratio of heat transfer coefficients at these two locations.

SCHEMATIC AND GIVEN DATA



STRATEGY:
The problem description already mentions flow over a flat plate. Heat flux is given by $h(T_s - T_\infty)$, where only the h is varying with location along the flow. Thus, if we find ratio of h at the two locations, we would have found the ratio of heat fluxes at the two locations.

ASSUMPTIONS:
Laminar flow over a flat plate

SOLUTION:

Flat plate, forced convection. Water flows down your back due to gravity.

$$\frac{hx}{k} = Nu_x = 0.332\ \mathrm{Pr}^{1/3}\ \mathrm{Re}_x^{\ 1/2} \qquad \mathrm{Re}_x = \frac{\mu_\infty x \rho}{\mu}$$

$$h = 0.332\ \mathrm{Pr}^{1/3}\left(\frac{\mu_\infty x \rho}{\mu}\right)^{1/2}\frac{k}{x} = 0.332\ \mathrm{Pr}^{1/3}\left(\frac{\mu_\infty \rho}{u}\right)^{1/2} k\ x^{-1/2}$$

$$\frac{h_{1cm}}{h_{45cm}} = \frac{0.332\ \mathrm{Pr}^{1/3}\left(\frac{\mu_\infty \rho}{\mu}\right)^{1/2} k\left(0.01\mathrm{m}\right)^{-1/2}}{0.332\ \mathrm{Pr}^{1/3}\left(\frac{\mu_\infty \rho}{\mu}\right)^{1/2} k\left(0.45\right)^{-1/2}} = \frac{0.01^{-1/2}}{0.45^{-1/2}}$$

$$= 6.71$$

Problem 6.10.17

COMMENTS:

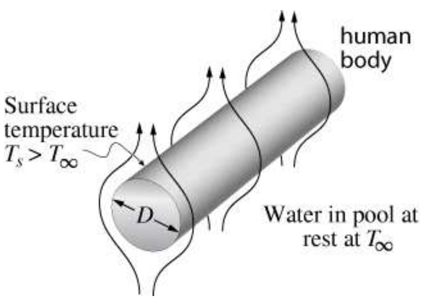
You would feel 6.71 times colder at 1 cm from where water hits vs. at 45 cm.

Problem 6.10.22

KNOWN: Water temperature T_w = 20°C = 293K
Skin surface temperature T_s = 34°C
Diameter of body = D = 0.3m

FIND: Convective heat transfer coefficient

SCHEMATIC:



STRATEGY:
This problem is straightforward calculation of convective heat transfer coefficient. There is no velocity (of water) given as well as the computation is to be compared with the equation provided that has *Gr* that is relevant in natural convection. Thus, it is a natural convection problem and over a horizontal cylinder (the subject stayed horizontal).

SOLUTION:
T_w = 20°C T_s = 34°C
$T = \frac{T_w+T_s}{2}$ = 27°C = 300K

Find properties of water at 300K:
$\rho = 1000\ kg/m^3$
$\beta = 2.761 \times 10^{-4}\ K^{-1}$
Pr = 5.83
$\mu = 855 \times 10^{-6} N\text{-}s/m^2$
D = 0.3m
k = 0.613W/m·K

a) Grashof Number: $Gr = \frac{\beta g \rho^2 D^3 \Delta T}{\mu^2}$

$$= \frac{276.1 \times 10^{-6} \times 9.8 \times (1000)^2 \times (0.3)^3 \times (34-20)}{(855 \times 10^{-6})^2}$$

$Gr = 1.4 \times 10^9$

Rayleigh Number: $Ra = Gr \times Pr$
$= 1.4 \times 10^9 \times 5.83$
$= 8.16 \times 10^9$

which satisfies the Rayleigh number restriction for the formula $\left(10^{-5} < Ra_D < 10^{12}\right)$.

Nusselt Number: $Nu_D = \left(0.6 + \frac{0.387 Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right)^2$

$= 281.76$

$\frac{hD}{k} = 281.76$

$h = 575.73\ \text{W/m}^2\cdot\text{K}$

b) Using equation provided:

$h = 0.09 \times (\text{GrPr})^{0.275}$

$h = 47.85\text{W/m}^2\cdot\text{K}$

The experimental value obtained is much lower.

c) For arm of diameter = 0.075m (Note that when one parameter such as the diameter changes, we do not have to compute some of the quantities from scratch as below, instead, can simply scale using that parameter.

Grashof Number: $\text{Gr} = \frac{\beta g \rho^2 D^3 \Delta T}{\mu^2}$

$= \frac{276.1 \times 10^{-6} \times 9.8 \times (1000)^2 \times (0.075)^3 \times (34-20)}{(855 \times 10^{-6})^2}$

$= 2.186 \times 10^7$

Rayleigh Number: $\text{Ra} = \text{Gr} \times \text{Pr}$

$= 1.27 \times 10^8$

Nusselt Number: $Nu_D = \left(0.6 + \frac{0.387 Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right)^2$

$= 75.57$

$\frac{hD}{k} = 75.57$

$h = \frac{75.57 \times 0.613}{0.075}$

$h = 617.63\text{W/m}^2\cdot\text{K}$

From the solution we see that the value of h increases as the diameter is decreased. Thus, in parts of body with smaller diameter, the value of h is high (more convection occurs).

d) It is not reasonable to assume that skin surface temperature will continue to be at 34°C because as the time for which the body is kept in water increases, the body surface temperature is no longer at 34°C .

e) The value of h increases as a result of shivering as it would create local turbulence.

f) The value of h calculated in step 1 includes heat conduction as well because Nu involves fluid thermal conductivity.