Problem 5.9.50

Known:

$T_i = 35^\circ C$

$T_\infty = 60^\circ C$

$\rho = 2200 \frac{kg}{m^3}$

$Cp = 1260 \frac{J}{kgK}$

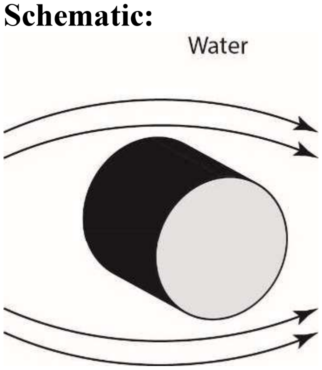
$h = 349 \frac{W}{m^2K}$

$\alpha = 2.27 \times 10^{-7} \frac{m^2}{s}$

$D = 0.0072m$

$t = 30s$

Schematic:



Find:

GE and BC. Temperature at a location after 60 s.

Strategy:

This is obviously a transient problem. If we are talking about the center temperature changing, center being the farthest point, it is definitely not a semi-infinite problem. One can check if it is a lumped parameter problem, by calculating hD/k but mass and surface areas (needed to use the lumped parameter solution) cannot be calculated from the given information. Thus, our choice is series solution or Heisler chart. If the time is early, given by $\alpha t/R^2 < 0.2$, we cannot use Heisler chart, but that is not the case. So, we have a choice of using Heisler chart or its equivalent series expression. We choose to use the chart since that is faster.

Solution:

1

$$\rho Cp\frac{dT}{dt}=\frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{dT}{dr}\right)$$

$$T_i=35^\circ C$$

$$-k\frac{dT}{dr}\bigg|_{r=0}=0$$

$$-k\frac{dT}{dr}\bigg|_{r=0.0036}=h(T-T_\infty)$$

2

$$n=\frac{r}{R}=\frac{1}{3}$$

$$m=\frac{k}{hR}=\frac{\rho Cp\alpha}{hR}=\frac{2200\times1260\times2.27\times10^{-7}}{349\times0.0036}=0.5$$

$$Fo=\frac{t\alpha}{R^2}=\frac{30\times2.27\times10^{-7}}{0.0036^2}=0.53$$

$$\frac{T-T_\infty}{T_i-T_\infty}=0.3$$

$$T=52.5^\circ C$$

3

$$n=\frac{x}{L}=\frac{1}{3}$$

$$m=\frac{k}{hL}=\frac{\rho Cp\alpha}{hL}=\frac{2200\times1260\times2.27\times10^{-7}}{349\times0.0036}=0.5$$

$$Fo=\frac{t\alpha}{L^2}=\frac{30\times2.27\times10^{-7}}{0.0036^2}=0.53$$

$$\frac{T-T_\infty}{T_i-T_\infty}=0.6$$

$$T=45^\circ C$$

4) From the analysis, the temperature of the slab is closer to the initial temperature. To explain this, we revisit the familiar concept of energy balance, i.e. energy (in - out) + generation = Change in storage. which translates to the following equation

$$\rho V C_p \frac{\partial T}{\partial t} = hA(T_\infty - T_{surface})$$

$$\Rightarrow \frac{\partial T}{\partial t} = \left(\frac{h}{\rho C_p}\right) * \left(\frac{A}{V}\right) * \left(T_\infty - T_{surface}\right)$$

It is evident that for larger value of $\frac{\partial T}{\partial t}$ the temperature at any point will be larger. So if we compare the terms in the left hand side we can see that $\left(\frac{h}{\rho C_p}\right)$ and $\left(T_\infty - T_{surface}\right)$ are same for both cylinder and slab. Now if we evaluate $\left(\frac{A}{V}\right)$ for both cylinder (with Diameter D) and slab (with thickness D) then we get

$$\left(\frac{A}{V}\right)_{Slab} = \frac{1}{D} \; and \; \left(\frac{A}{V}\right)_{cylinder} = \frac{4}{D}$$

So we can see that $\frac{\partial T}{\partial t}$ is more for cylinder than slab (exactly 4 times), which explains higher temperature in the cylinder compared to slab.

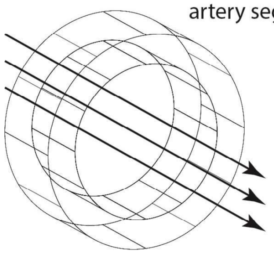
Problem #2

Known:

$T_i = 30.5^\circ C$

$T_\infty = 32.7^\circ C$

Schematic:



Find:

Temperature versus time uses energy balance.

Strategy:

Write an energy balance and solve the differential. Then, we will rearrange terms to easily measure experimental parameters.

Solution:

1

$$V\rho C_p \frac{dT}{dt} = -hA(T - T_b)$$

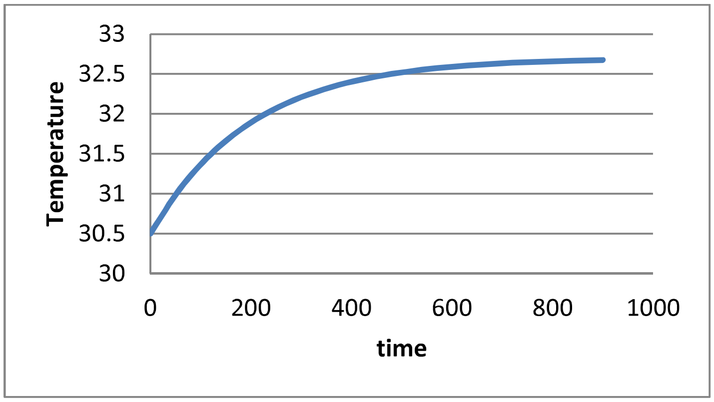
2

$$\int_{T_i}^{T} \frac{dT}{(T - T_b)} = -\int_{t=0}^{t} \frac{hA}{V\rho C_p} dt$$

$$\ln(T - T_b) - \ln(T_i - T_b) = \ln\left(\frac{(T - T_b)}{(T_i - T_b)}\right) = -\frac{hA}{V\rho C_p} t$$

$$T = T_b + (T_i - T_b)\exp\left(-\frac{hA}{V\rho C_p} t\right)$$

3



4

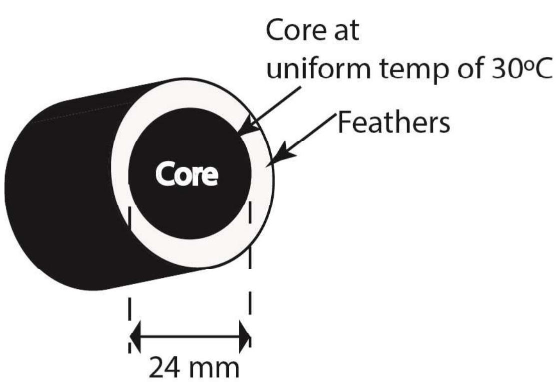
Plot $\ln\left(\frac{(T - T_b)}{(T_i - T_b)}\right)$ versus time and you will get a line that the slope $= -\frac{hA}{V\rho C_p}$ which can be used to determine h

Problem #3

Known:

$$L = 0.04m$$
$$r_i = 0.012m$$
$$r_o = 0.016m$$
$$h = 200\frac{W}{m^2K}$$
$$\sigma = 5.67\times10^{-8}\frac{W}{m^2K^4}$$
$$k = 0.02\frac{W}{mK}$$
$$T_i = 30^\circ C$$
$$T_\infty = 0^\circ C$$

Schematic:



Find:

All 3 resistances and metabolic heat generation

Strategy:

Use circuit analysis in series and parallel to find total resistance. Find surface temperature by setting ambient resistance equal to conductive resistance. Then, set conductive flux equal to metabolic heat generation times volume over area to make into flux.

Solution:

1

$$q" = \frac{(T_s - T_\infty)}{\frac{1}{4\sigma T_\infty^3 A}} = \frac{(T_s - T_\infty)}{\frac{1}{4\sigma T_\infty^3 2\pi r_o L}}$$

$$R = \frac{1}{4\sigma T_\infty^3 (2\pi r_o L)}$$

2

We have resistance in parallel from the ambient to the surface and then resistance in series for conductive heat transfer and the combined parallel radiative and convective heat transfer.

$$R = R_{surface} + R_{feather}$$

$$\frac{1}{R_{surface}} = \frac{1}{R_{conv}} + \frac{1}{R_{rad}} \Rightarrow R_{surface} = \frac{R_{rad} R_{conv}}{R_{rad} + R_{conv}}$$

$$R = R_{surface} + R_{feather} = \frac{R_{rad} R_{conv}}{R_{rad} + R_{conv}} + R_{cond}$$

$$R_{rad} = \frac{1}{4\sigma T_\infty^3 (2\pi r_o L)}$$

$$R_{conv} = \frac{1}{hA} = \frac{1}{h(2\pi r_o L)}$$

$$R_{cond} = \frac{\ln \frac{r_o}{r_i}}{2\pi k L}$$

$$R = \frac{\frac{1}{4\sigma T_\infty^3 (2\pi r_o L)} \frac{1}{h(2\pi r_o L)}}{\frac{1}{4\sigma T_\infty^3 (2\pi r_o L)} + \frac{1}{h(2\pi r_o L)}} + \frac{\ln \frac{r_o}{r_i}}{2\pi k L} = \frac{\frac{1}{4\sigma T_s^3} \frac{1}{h(2\pi r_o L)}}{\frac{1}{4\sigma T_s^3} + \frac{1}{h}} + \frac{\ln \frac{r_o}{r_i}}{2\pi k L} = \frac{\frac{1}{\sigma T_\infty^3 h(8\pi r_o L)}}{\frac{1}{4\sigma T_\infty^3} + \frac{1}{h}} + \frac{\ln \frac{r_o}{r_i}}{2\pi k L}$$

also equals

$$R = \frac{1}{4\sigma T_\infty^3 (2\pi r_o L) + h(2\pi r_o L)} + \frac{\ln \frac{r_o}{r_i}}{2\pi k L}$$

3

$$R_{rad}=\frac{1}{4\sigma T_s^3\left(2\pi r_o L\right)}=\frac{1}{4\left(5.67\times10^{-8}\right)273^3\left(2\pi 0.016\times0.04\right)}=53.9$$

$$R_{conv}=\frac{1}{hA}=\frac{1}{20\left(2\pi 0.016\times0.04\right)}=1.24$$

$$R_{cond}=\frac{\ln\frac{0.016}{0.012}}{2\pi 0.02\times0.04}=57.23$$

$$\frac{R_{rad}R_{conv}}{R_{rad}+R_{conv}}=\frac{1.24\times53.9}{1.24+53.9}=1.22$$

$$\frac{T_i-T_s}{57.23}=\frac{T_s-T_\infty}{1.22}\Rightarrow\frac{303-T_s}{57.23}=\frac{T_s-273}{1.22}\Rightarrow T_s=273.6K$$

$$Q=\frac{303-273.6}{57.23}=\frac{303-273}{57.23+1.22}=0.51\,W$$

$$0.82Q=0.51\,W\Rightarrow Q=0.62\,W$$

4

$$R_{rad}=\frac{1}{4\sigma T_s^3\left(2\pi r_o L\right)}=\frac{1}{4\left(5.67\times10^{-8}\right)T_s^3\left(2\pi 0.014\times0.04\right)}=61.6$$

$$R_{conv}=\frac{1}{hA}=\frac{1}{20\left(2\pi 0.014\times0.04\right)}=1.42$$

$$R_{cond}=\frac{\ln\frac{0.014}{0.012}}{2\pi 0.02\times0.04}=30.67$$

$$\frac{R_{rad}R_{conv}}{R_{rad}+R_{conv}}=\frac{1.42\times61.6}{1.42+61.6}=1.39$$

$$\frac{T_i-T_s}{30.67}=\frac{T_s-T_\infty}{1.39}\Rightarrow\frac{303-T_s}{30.67}=\frac{T_s-273}{1.39}\Rightarrow T_s=274.3K$$

$$Q=\frac{303-274.3}{30.67}=\frac{303-273}{1.39+30.67}\Rightarrow Q_{noevap}=0.94\,W$$

$$0.82Q=0.94\,W\Rightarrow Q=1.14\,W$$

Problem #4

Known:

$T_g = 15^\circ C$

$T_\infty = 25^\circ C / 5^\circ C$

$k = 0.67 \frac{W}{mK}$

$h = 5 \frac{W}{m^2 K}$

$Q = 2 \frac{W}{m^3}$

$L = 5m$

Schematic:

Outside is convection from the air at 25/5°C

Slab with heat generation

Bottom at ground temperature of 15°C

Find:

Write GE and BC and solve. Find location of max and find value. Calculate ratio.

Strategy:

Write GE and BC and solve. Take the derivative to find location of max and find value. Calculate ratio.

Solution:

1

$$0 = k\frac{d^2T}{dx^2} + Q$$

$$T\big|_{x=0} = T_g$$

$$-k\frac{dT}{dx}\bigg|_{x=5} = h\left(T\big|_{x=5} - T_\infty\right)$$

2

$$-\frac{Q}{k} = \frac{d^2T}{dx^2}$$

$$-\frac{Q}{k}x + c_1 = \frac{dT}{dx}$$

$$-\frac{Q}{2k}x^2 + c_1 x + c_2 = T$$

$$-\frac{Q}{2k}0^2 + c_1 0 + c_2 = T_g$$

$$c_2 = T_g$$

$$-\frac{Q}{2k}x^2 + c_1 x + T_g = T$$

$$-k\frac{dT}{dx}\bigg|_{x=L} = h\left(T\big|_{x=L} - T_\infty\right)$$

$$-k\left(-\frac{Q}{k}L + c_1\right) = h\left(-\frac{Q}{2k}L^2 + c_1 L + T_g - T_\infty\right)$$

$$QL - kc_1 = h\left(-\frac{Q}{2k}L^2 + T_g - T_\infty\right) + hc_1 L$$

$$QL - h\left(-\frac{Q}{2k}L^2 + T_g - T_\infty\right) = hc_1 L + kc_1 = c_1(hL + k)$$

$$c_1 = \frac{QL - h\left(-\frac{Q}{2k}L^2 + T_g - T_\infty\right)}{(hL + k)}$$

$$T = -\frac{Q}{2k}x^2 + \left(\frac{QL - h\left(-\frac{Q}{2k}L^2 + T_g - T_\infty\right)}{(hL + k)}\right)x + T_g$$

Max occurs when dT/dx=0

$$-\frac{Q}{k}x + c_1 = \frac{dT}{dx} = 0$$

$$c_1 = \frac{Q}{k}x$$

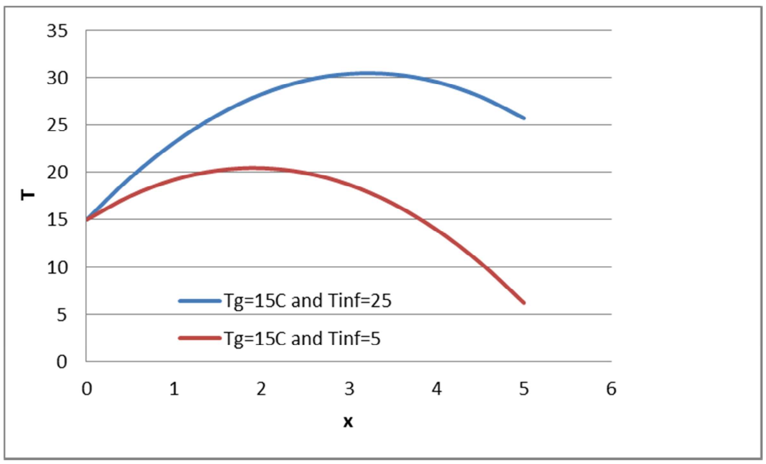
$$\frac{QL - h\left(-\frac{Q}{2k}L^2 + T_g - T_\infty\right)}{(hL + k)} = \frac{Q}{k}x$$

$$x = 3.22m \quad T_\infty = 25^\circ C$$

$$x = 1.91m \quad T_\infty = 5^\circ C$$

$$T_{\max} = 30.45^\circ C \quad T_\infty = 25^\circ C$$

$$T_{\max} = 20.5^\circ C \quad T_\infty = 5^\circ C$$



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The maximum temperature would then be on the boundary. It would be at the air or the ground depending on which was cooler.

$$0 = \frac{d^2T}{dx^2}$$

$$c_1 = \frac{dT}{dx}$$

$$c_1 x + c_2 = T$$

$$c_1 0 + c_2 = T_g$$

$$c_2 = T_g$$

$$c_1 x + T_g = T$$

$$-k\left.\frac{dT}{dx}\right|_{x=L} = h\left(T\big|_{x=L} - T_\infty\right)$$

$$-k(c_1) = h(c_1 L + T_g - T_\infty)$$

$$-kc_1 = h(T_g - T_\infty) + hc_1 L$$

$$-h(T_g - T_\infty) = hc_1 L + kc_1 = c_1(hL + k)$$

$$c_1 = \frac{-h(T_g - T_\infty)}{(hL + k)}$$

$$T = -\frac{h(T_g - T_\infty)}{(hL + k)} x + T_g$$

The maximum temperature would be on one of the boundaries and would be:

For $T_g < T_\infty \quad T_{\max} = -\frac{h(T_g - T_\infty)}{(hL + k)} L + T_g$

Or $T_g > T_\infty \quad T_{\max} = T_g$