BEE 3500 Quiz 1
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1. Math Review

Name ____________________

Date ____________________

Choose the closest answer.

1) Write the limit $\underset{\Delta r \to 0}{Lt} \frac{(r+\Delta r)T_{r+\Delta r} - rT_r}{\Delta r} =$

$\frac{d(Tr)}{dr}$ (If T is function of r only)

or, $\frac{\partial(Tr)}{\partial r}$ (If T is function of more than one variable)

2) Solve $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$

A) $\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0 \times r \Rightarrow r\frac{dT}{dr} = \int 0\, dr = C_1 \Rightarrow \int \frac{dT}{dr}dr = \int \frac{C_1}{r}dr$

$\Rightarrow T = C_1 \ln r + C_2$

$\Rightarrow T = \ln(rC)^{C'}$

3) Solve $\nabla^2 T = 0$ in Cartesian coordinate, one-dimension (x only)

Given $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ in Cartesian

$= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}$ in Cylindrical

A. Since Cartesian co-ordinate, $\nabla^2 T = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(T)$

Since, only one dimension, $\nabla_x^2 T = \frac{\partial^2}{\partial x^2}(T) = \frac{d^2T}{dx^2}$

So, $\nabla^2 T = 0 \Rightarrow \frac{d^2T}{dx^2} = 0 \Rightarrow \frac{dT}{dx} = \int 0\, dx = C_1$

$\Rightarrow T = \int C_1 dx = C_1 x + C_2$

4) What will be the form of a particular solution *T(x)* to the equation

$\frac{d^2T}{dx^2} + cT = 0$ (Here c is a non-zero constant)

(No need for detailed solution—just mention the type of function *T(x)* that will satisfy the above equation)

If

$c > 0$. $T = A\sin(\sqrt{c}\,x) + B\cos(\sqrt{c}\,x)$

$c < 0$ $T = Ae^{\sqrt{|c|}\,x} + Be^{-\sqrt{|c|}\,x}$. [$|\,|$ → Mod or absolute value operator, e.g. $|-2| = 2$]

$= C\sinh(\sqrt{|c|}\,x) + D\cosh(\sqrt{|c|}\,x)$ [Sinh → Sin hyperbolic, Cosh → Cos hyperbolic]

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