Problem 5.9.37

KNOWN:

Sphinx moth abdomen treated as a sphere and analyzed using lumped parameter approach.

FIND:

(1) Energy balance on the sphere over time Δt

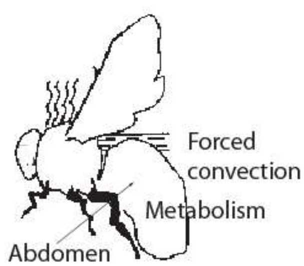
(2) Differential equation of energy balance

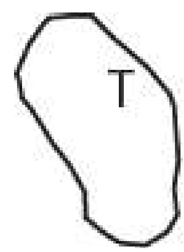
(3) Expression for temperature as a function of time

(4) Difference between steady state temperature of moth abdomen and ambient temperature

(5) Time constant

SCHEMATIC AND GIVEN DATA





STRATEGY:

Given that we need to use a lumped parameter approach, since we have heat generation, we cannot use the solution provided in the text. Thus, we need to start from a heat balance similar to what was done in the text for a lumped parameter approach.

ASSUMPTIONS:

Lumped parameter approach is valid

SOLUTION:

(1) $-mC_p\Delta T = hA\left(T - T_\infty\right)\Delta t - QV\ \Delta t$

(2) $-mC_p\dfrac{\Delta T}{\Delta t} = hA\left(T - T_\infty\right) - QV$

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$\Delta t \to 0,$

$$-mC_p \frac{dT}{dt} = hA\left(T - T_\infty\right) - QV$$

$$\frac{dT}{dt} = \frac{-hA}{m\,c_p}\left(T - T_\infty\right) + \frac{QV}{m\,c_p}$$

$$= \frac{-hA}{m\,c_p}\left(T - T_\infty - \frac{QV}{hA}\right)$$

$$\frac{dT}{dt} = -K_1\left(T - K_2\right) \text{ where } \quad K_1 = \frac{hA}{m\,c_p}, \quad K_2 = T_\infty + \frac{QV}{hA}$$

(3) Integrating,

$$\int_{T_i}^{T} \frac{dT}{\left(T - K_2\right)} = -\int_{O}^{t} k_1 dt \Rightarrow \ln\left(\frac{T - K_2}{T_i - K_2}\right) = -K_1 t$$

$$\frac{T - K_2}{T_i - K_2} = e^{-K_1 t}$$

$$T - K_2 = \left(T_i - K_2\right) e^{-K_1 t}$$

(4) At steady state, $t \to \infty$

$$T = T_\infty + \frac{QV}{hA}$$

Difference between body and ambient temperature is

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$$T - T_\infty = \frac{QV}{hA}$$

$$= \frac{14000 \text{ W/m}^3 \times \frac{4}{3}\pi \times \left(1.5\times10^{-3}\text{m}\right)^3}{50 \text{ W/m}^2\text{K} \times 4\pi \times \left(1.5\times10^{-3}\text{m}\right)^2}$$

$$= 0.14 \text{ K}$$

Small since the absolute size is small.

(5) Time constant

$$= \frac{1}{K_1} = \frac{m\, c_p}{hA}$$

$$= \frac{\frac{4}{3}\pi r^3 \rho\, c_p}{h\, 4\pi r^2}$$

$$= \frac{4}{3}\pi \frac{\left(1.5\times10^{-3}\text{m}\right)^3 \left(1000 \text{ kg/m}^3\right) \times 4200 \text{J/kg K}}{50\, W / \text{m}^2\text{K} \times \pi \left(1.5\times10^{-3}\text{m}\right)^2}$$

$$= 42 \text{ s}$$

COMMENTS:

Problem 5.9.40

KNOWN: skin surface temperature, time to reach temperature, constant surface heat flux

FIND: Time taken to reach another temperature

SOLUTION:

Temperature profile for <u>constant flux</u> is given by (use Eq. 5.32):

$$T - T_i = \frac{2}{k} q_s'' \left(\frac{\alpha t}{\pi}\right)^{\frac{1}{2}} \exp\left(\frac{-x^2}{4\alpha t}\right) - \frac{q_s'' x}{k}\left(1 - \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right)$$

Evaluate at $x = 0$

$$T - T_i = \frac{2}{k} q_s'' \left(\frac{\alpha t}{\pi}\right)^{\frac{1}{2}} (1)$$

$$= \frac{2}{k} q_s'' \left(\frac{\alpha t}{\pi}\right)^{\frac{1}{2}}$$

$$T - T_i = \frac{2}{k} q_s'' \sqrt{\frac{\alpha t}{\pi}}$$

$$T_1 - T_i = \frac{2}{k} q_s'' \sqrt{\frac{\alpha t_1}{\pi}}$$

$$T_2 - T_i = \frac{2}{k} q_s'' \sqrt{\frac{\alpha t_2}{\pi}}$$

$$\frac{T_1 - T_i}{T_2 - T_i} = \sqrt{\frac{t_1}{t_2}}$$

$$\frac{30 - 25}{43 - 25} = \sqrt{\frac{t_1}{t_2}} = \frac{5}{18}$$

$$\frac{t_1}{t_2} = \frac{25}{324}$$

$$t_2 = \frac{324 t_1}{25}$$

$t_1 = 25$ secs. Therefore, $t_2 = 324$ secs.

Note: Eq. 5.28 will give the same answer, but is wrong.

Problem 5.9.48

Known:
Fish swimming speed and physical properties. Thermal properties of the fish are known
Find:
Thermal properties of water at the film temperature. The correct thermal model based on internal resistance's importance and 'length of time.'
Schematic and Given Data:

$$h \to \infty$$

$$T_i = 16^\circ C$$

$$T_\infty = 2^\circ C$$

$$T = 4^\circ C$$

$$k = 0.53 \frac{W}{mK}$$

$$c_p = 2.97 \frac{kJ}{kgK}$$

$$L = 0.05m$$

$$m = 10kg$$

Assumptions:
Assume long time first (Fo>0.2)
Strategy:
Going down the decision tree, for a slab, we need to check Biot number to see if lumped parameter solution may be valid. Write the appropriate solution in terms of temperature vs. time and compare with the one given.
Solution:
1)

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$Bi = \frac{hL}{k} = \infty$$

$$T\left(0 < x < L, t = 0\right) = T_i$$

$$T\big|_{x=L} = T_s$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$$

2)
Since $h \to \infty, T_s = T_\infty$

$$\alpha = \frac{k}{\rho c_p} = 4.46 \times 10^{-7} \frac{m^2}{s}$$

$$\frac{T - T_s}{T_i - T_s} = \frac{4}{\pi} e^{-100\alpha\pi^2 t}$$

Plugging in T_i and T_s,

$$T = 17.8e^{-4.40\times10^{-4}t} + 2$$

3)

Our model (center of a slab):

$$\frac{T - T_s}{T_i - T_s} = \frac{4}{\pi}e^{-\alpha\left(\frac{\pi}{2L}\right)^2 t}$$

Their model:

$$\frac{T - T_s}{T_i - T_s} = Ae^{-Rt}$$

Therefore:

$$A = \frac{4}{\pi}$$

$$R = \alpha\left(\frac{\pi}{2L}\right)^2$$

4)

From their model:

$$m = 100\times25\times10\times10^{-6}\times400 = 10\text{kg}$$

From the given equation, $R = 4.19\times10^{-4}t$

$$\ln\left(\frac{T - T_\infty}{T_i - T_\infty}\right) = \ln\left(\frac{4-2}{16-2}\right) = -4.19\times10^{-4}t$$

$t = 4644$ s

Our model:

$$at \quad T = 4 \quad t = 4971s$$

Long time assumption is valid:

$$Fo = \frac{\alpha t}{L^2} = \frac{4.46\times10^{-7}\times4971}{0.05^2} = 0.89 > 2$$

There error is 7% by:

$$100\times\left(\frac{4971-4644}{4644}\right) = 7.053\%$$

5)

If a quick calculation is needed for design purposes to have a good first guess, the theoretical model is excellent. If data is unavailable, the empirical is quickest and possibly best. From a fundamental understanding the theoretical is best as it provides understanding in terms of the physics and assists in improving future designs.

Problem 5.9.50

Known:

$T_i = 35^\circ C$

$T_\infty = 60^\circ C$

$\rho = 2200 \frac{kg}{m^3}$

$Cp = 1260 \frac{J}{kgK}$

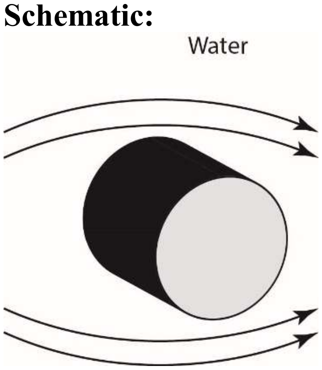
$h = 349 \frac{W}{m^2K}$

$\alpha = 2.27 \times 10^{-7} \frac{m^2}{s}$

$D = 0.0072m$

$t = 30s$

Schematic:



Find:

GE and BC. Temperature at a location after 60 s.

Strategy:

This is obviously a transient problem. If we are talking about the center temperature changing, center being the farthest point, it is definitely not a semi-infinite problem. One can check if it is a lumped parameter problem, by calculating hD/k but mass and surface areas (needed to use the lumped parameter solution) cannot be calculated from the given information. Thus, our choice is series solution or Heisler chart. If the time is early, given by $\alpha t/R^2<0.2$, we cannot use Heisler chart, but that is not the case. So, we have a choice of using Heisler chart or its equivalent series expression. We choose to use the chart since that is faster.

Solution:

1

$$\rho Cp\frac{dT}{dt}=\frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{dT}{dr}\right)$$

$$T_i=35^{\circ}C$$

$$-k\frac{dT}{dr}\bigg|_{r=0}=0$$

$$-k\frac{dT}{dr}\bigg|_{r=0.0036}=h(T-T_{\infty})$$

2

$$n=\frac{r}{R}=\frac{1}{3}$$

$$m=\frac{k}{hR}=\frac{\rho Cp\alpha}{hR}=\frac{2200\times1260\times2.27\times10^{-7}}{349\times0.0036}=0.5$$

$$Fo=\frac{t\alpha}{R^2}=\frac{30\times2.27\times10^{-7}}{0.0036^2}=0.53$$

$$\frac{T-T_{\infty}}{T_i-T_{\infty}}=0.3$$

$$T=52.5^{\circ}C$$

3

$$n=\frac{x}{L}=\frac{1}{3}$$

$$m=\frac{k}{hL}=\frac{\rho Cp\alpha}{hL}=\frac{2200\times1260\times2.27\times10^{-7}}{349\times0.0036}=0.5$$

$$Fo=\frac{t\alpha}{L^2}=\frac{30\times2.27\times10^{-7}}{0.0036^2}=0.53$$

$$\frac{T-T_{\infty}}{T_i-T_{\infty}}=0.6$$

$$T=45^{\circ}C$$

4) From the analysis, the temperature of the slab is closer to the initial temperature. To explain this, we revisit the familiar concept of energy balance, i.e. energy (in - out) + generation = Change in storage. which translates to the following equation

$$\rho V C_p \frac{\partial T}{\partial t} = hA(T_\infty - T_{surface})$$

$$\Rightarrow \frac{\partial T}{\partial t} = \left(\frac{h}{\rho C_p}\right) * \left(\frac{A}{V}\right) * \left(T_\infty - T_{surface}\right)$$

It is evident that for larger value of $\frac{\partial T}{\partial t}$ the temperature at any point will be larger. So if we compare the terms in the left hand side we can see that $\left(\frac{h}{\rho C_p}\right)$ and $\left(T_\infty - T_{surface}\right)$ are same for both cylinder and slab. Now if we evaluate $\left(\frac{A}{V}\right)$ for both cylinder (with Diameter D) and slab (with thickness D) then we get

$$\left(\frac{A}{V}\right)_{Slab} = \frac{1}{D} \ and \ \left(\frac{A}{V}\right)_{cylinder} = \frac{4}{D}$$

So we can see that $\frac{\partial T}{\partial t}$ is more for cylinder than slab (exactly 4 times), which explains higher temperature in the cylinder compared to slab.