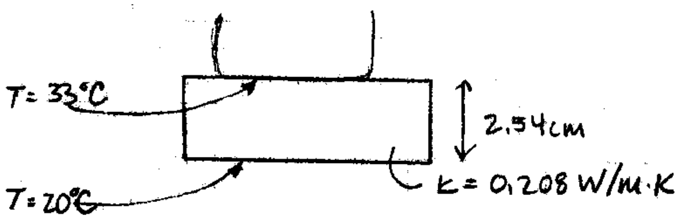
Problem 2.7.2

KNOWN: Temperature of body and chair. Chair dimensions and thermal properties.

FIND: Determine the conductive heat flux from the body.

SCHEMATIC AND GIVEN DATA



STRATEGY:

ASSUMPTIONS:

(1) Steady-state 1-D heat transfer
(2) Thermal properties constant
(3) Temperature profile described by a linear approximation
(4) The chair surface immediately equilibrates to 33°C while other side is maintained at 20°C.

SOLUTION:

$$q'' = -k\frac{\Delta T}{\Delta x}$$
$$= -\left(0.208\frac{\text{W}}{\text{mK}}\right)\left(\frac{20-33}{2.54\times 10^{-2}\,\text{m}}\frac{\text{K}}{\text{m}}\right)$$
$$= 106.46 \quad \text{W/m}^2$$

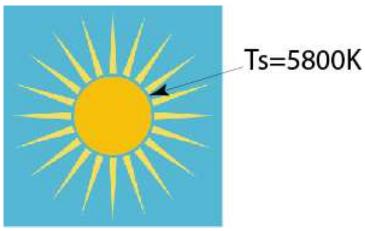
COMMENTS:

Problem 2.7.4

KNOWN: Average surface temperature of sun.

FIND: Determine the energy flux at the surface of the sun.

SCHEMATIC AND GIVEN DATA



STRATEGY:

ASSUMPTIONS:

(1) Sun is a perfect black body. $\varepsilon = 1$.
(2) Average surface temperature = 5800 K.

SOLUTION:

$$q'' = \sigma\varepsilon T^4$$
$$= 5.67\times10^{-8}\,\frac{\text{W}}{\text{m}^2\text{K}^4}(1)\left(5800\text{ K}\right)^4$$
$$= 6.416\times10^{7}\,\frac{\text{W}}{\text{m}^2}$$

COMMENTS:

(1) The energy flux is greater than the given flux of 1353 W/m^2 by 4 orders of magnitude. The decrease from sun surface to earth surface is due to the fact that the amount of energy leaving solar surface is going in all directions and only a fraction of this is intercepted by earth, due to large distance between the two and the relative sizes of the sun and the earth.

Problem 2.7.6

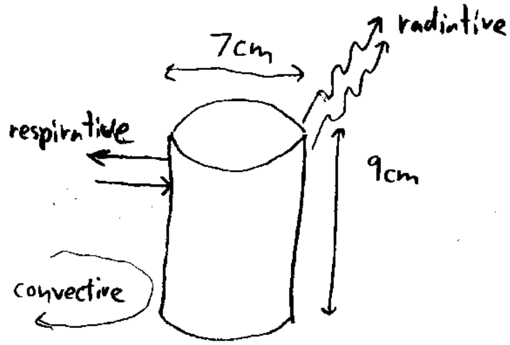
KNOWN:

T_{air} = 20°C
T_{canary} = 33°C
H_{canary} = 25.2 W/m^{2}K
ΔT (temp. diff. between exhaled and inhaled) = 4.3°C
Ventilation rate = 0.74 cc/s

$$c_{p,air} = 1.0066 \text{ kJ / kg·K}$$

$$\rho_{air} = 1.16 \text{ kg / m}^3$$

Canary can be modeled as a cylinder with a diameter of 7cm and height of 9cm
Heat gained by radiation from surrounding = 11.5 W



FIND:

1) Amount of heat lost by radiation

$$q = A\sigma T^4$$

$$A = 2\left(\pi\left(\frac{d}{2}\right)^2\right) + \pi dh$$

$$= 2\left(\pi\left(\frac{0.07\text{m}}{2}\right)^2\right) + \pi(0.07\text{m})(0.09\text{m})$$

$$= 0.0275 \text{ m}^2$$

$$q = \left(0.0275\text{m}^2\right)\left(5.67\times10^{-8}\frac{\text{W}}{\text{m}^2\text{K}^4}\right)(273.15\text{K} + 33\text{K})^4$$

$$= 13.7 \text{ W} \quad \text{gross heat lost}$$

Net heat lost = 13.7W – 11.5W = 2.2 W

2) Find amount of heat transferred by convection to surrounding air

$$Q = h\,A\,\left(T_{canary} - T_{air}\right)$$

$$= 25.2 \frac{\text{W}}{\text{m}^2\text{K}} (0.0275 \text{ m}^2) (33°\text{C} - 20°\text{C})$$
$$= 9.01 \text{ W}$$

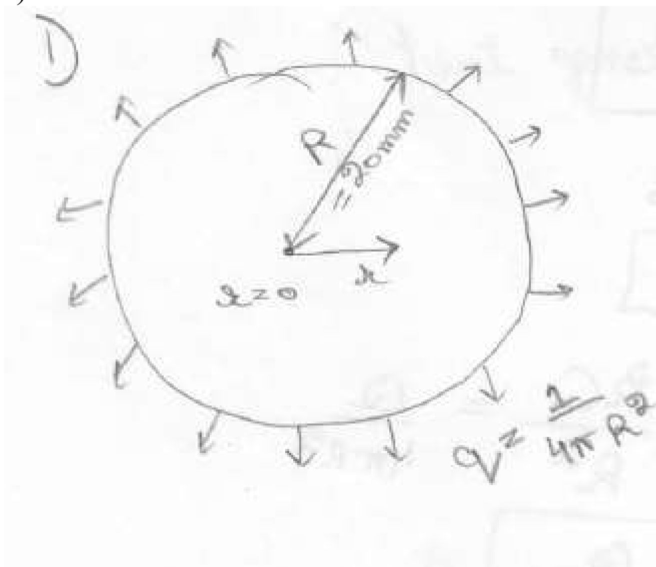
3) Find amount of heat transferred in the exhaled air

$$q = \dot{m}\, c_p \Delta T$$
$$= \dot{v} \rho c_p \Delta T$$
$$= (0.74 \text{ cc/s}) \left(\frac{1\text{m}^3}{1 \times 10^6 \text{ cc}} \right) \left(1.16 \frac{\text{kg}}{\text{m}^3} \right) \left(1.0066 \frac{\text{kJ}}{\text{kg} \bullet \text{K}} \right) (4.3°\text{C})$$
$$= 3.72 \times 10^{-3} \frac{\text{J}}{\text{s}}$$
$$= 3.72 \text{ mW}$$

4) Total power = $2.2\text{W} + 9.01\text{W} + 3.72 \times 10^{-3}\text{W} = 11.21 \text{ W}$

Problem 4.8.15

1) Schematic



2) Governing Equation in Spherical Co-ordinates:

$$\frac{k}{\rho c_p}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right]+\frac{Q}{\rho c_p}=\frac{\partial T}{\partial t}$$

Assumption:

1) No heat generation => $Q = 0$
2) Steady State => $\frac{dT}{dt} = 0$

Governing Equation $\frac{k}{\rho c_p}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right]=0$

3) Boundary Conditions

1) $q''_{r=R} = \frac{Q}{4\pi R^2}$; Q = total heat generated = 1 W

$$-k\left.\frac{\partial T}{\partial r}\right|_{r=R} = \frac{Q}{4\pi R^2}$$

2) $T(r=\infty) = T_\infty; T_\infty = 36°C = (273+36)K$

4) Solution

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=0$$

Integrating

$$\int\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=\int 0\partial r + C_1$$

Problem 4.8.28

Given: Diameter, thermal conductivity, blood perfusion rate, density, specific heat, temperature

Find: heat source term, governing equation, temperature profile, average temperature

Schematic:

$$d = 53\text{mm} = 0.053\text{m}$$

$$k = 0.5 \text{ W/m}\cdot\text{K}$$

$$V_b^v = 50\text{ml}/100g\cdot\text{min} = \frac{0.5}{60}\times 10^{-6}\text{m}^3/\text{g}\cdot\text{s}$$

$$= 0.0833\times 10^{-7}\text{m}^3/\text{g}\cdot\text{s}$$

$$\ell_{\text{tissue}} = 1.05\text{g/cm}^3 = 1.05\times 10^6\text{g/m}^3$$

$$V_b^v = 0.0833\times 10^{-7}\times 1.05\times 10^6$$

$$= 0.00875\text{m}^3 \text{ of blood/m}^3\text{of tissue}\cdot\text{s}$$

$$\ell_{\text{blood}} = 1050\text{kg/m}^3$$

$$c_b = 3800 \text{ J/kg}\cdot\text{K}$$

$$T_\infty = 30^\circ\text{C}$$

$$T_a = 37^\circ\text{C}$$

$$\text{Heat generation } Q = 10437 \text{ W/m}^3$$

Solution:

1)

$$\text{Heat due to blood flow } = \rho_b c_b V_b^v (T_a - T)$$

$$= 1050\times 3800\times 0.00875\times (37-30)$$

$$= 244387.5 \text{ W/m}^3$$

2) Though the heat generation calculated is higher than the metabolic heat generation, it cannot be ignored.

3)

$$\text{Constant heat generation due to blood flow } = \frac{244387.5}{2}$$

$$= 122193.75 \text{ W/m}^3$$

$$\text{Total heat generation } = 122193.75 + 10437 \text{ W/m}^3$$

4) Governing Equation (steady state)

Problem 4.8.28

$$\frac{k}{\rho c}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right]+\frac{\rho_b c_b V_b^v\left(T_a-T\right)}{\rho c}+\frac{Q}{\rho c_p}=0$$

$$k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right]+Q^{'}=0$$

$$Q' = \text{ Total heat source term}$$

5) Boundary conditions:

$$q''\big|_{r=0}=0\left(\text{No flux}\right)$$

$$T=30^\circ C=303K\left(\text{On the hemisperical surface}\right)$$

$$K\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right]+Q'=0$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=-\frac{Q'r^2}{k}$$

$$r^2\frac{\partial T}{\partial r}=\frac{-Q'r^3}{3k}+C_1 \quad \left(\text{Integrate with respect to r}\right)$$

$$\frac{\partial T}{\partial r}=-\frac{Q'r}{3k}+\frac{C_1}{r^2}$$

From BC 1: $q''\big|_{r=0}=0$

$$-k\frac{\partial T}{\partial r}=0$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=0}=0$$

$$c_1=0$$

$$T=-\frac{Q'r^2}{6k}+c_2$$

From BC 2: $T\big|_{r=R}=303K$

$$303=-\frac{Q'R^2}{6k}+c_2$$

$$c_2=303+\frac{Q'R^2}{6k}$$

6) Temperature profile is given by

$$T=303+\frac{Q'}{6k}\left(R^2-r^2\right)$$

Problem 4.8.31

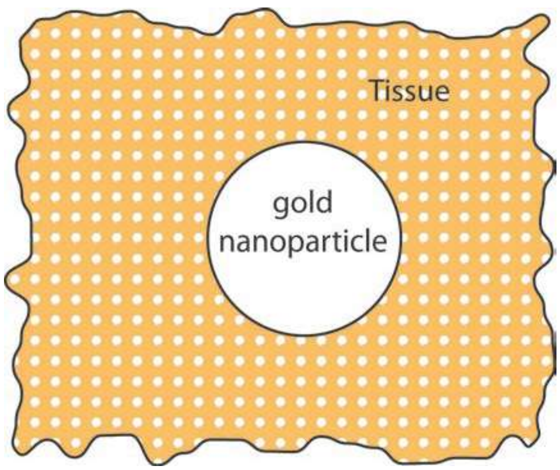
KNOWN:

T_{lethal}=323 K
k = 0.48 W/m·K
T_∞ = 310
$D_P = 6\times10^{-9}$ m

FIND:

The governing equation and boundary conditions. Then, T as a function of r. Temperature of shell such that at 5 nm from shell, T = 323 K. Energy absorbed per shell in W.

SCHEMATIC AND GIVEN DATA



STRATEGY:
This is a steady-state problem in spherical geometry. Since you are asked to derive the temperature profile, you have to start from the governing equation. Note that although heat is generated inside the gold nanoparticle, the entire sphere is at one temperature, thus we are obviously not solving for temperature inside the sphere. Our domain, therefore, is outside the sphere, starting from the surface of the sphere to infinity. We simply solve the equation with the boundary conditions of temperature given at the surface of sphere and body temperature at r going to infinity.

ASSUMPTIONS:

None

SOLUTION:

1. $T\big|_{r=R} = T_i$

$$T\big|_{r=\infty} = T_\infty$$

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) = 0$$

2.

$$\frac{dT}{dr} = \frac{c_1}{r^2}$$

$$T = -\frac{c_1}{r} + c_2$$

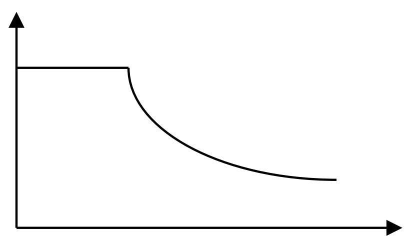
$$T_i = -\frac{c_1}{R} + c_2 \quad T_\infty = -\frac{c_1}{R_\infty} + c_2$$

$$c_2 = T_\infty$$

$$c_1 = R(T_\infty - T_i)$$

$$T = -\frac{R(T_\infty - T_i)}{r} + T_\infty$$

3.



4.

$$323 = -\frac{3\times10^{-9}\,m(310 - T_i)}{8\times10^{-9}\,m} + 310$$

$$T_i = 344.67$$

5. Energy absorbed per nanoparticle

$$q = -kA\frac{dT}{dr}\bigg|_{r=R} = -0.48(4\pi R^2)\frac{R(T_\infty - T_i)}{R^2} = -0.48(4\pi R)(T_\infty - T_i)$$

$$= -0.48\frac{\text{W}}{\text{mK}}(4\pi(3\times10^{-9}\,\text{m}))(310 - 344.67) = 6.27\times10^{-7}\,\text{W}$$

Problem 4.8.34

Known:

$L = 0.04m$

$r_i = 0.012m$

$r_o = 0.016m$

$h = 200\frac{W}{m^2K}$

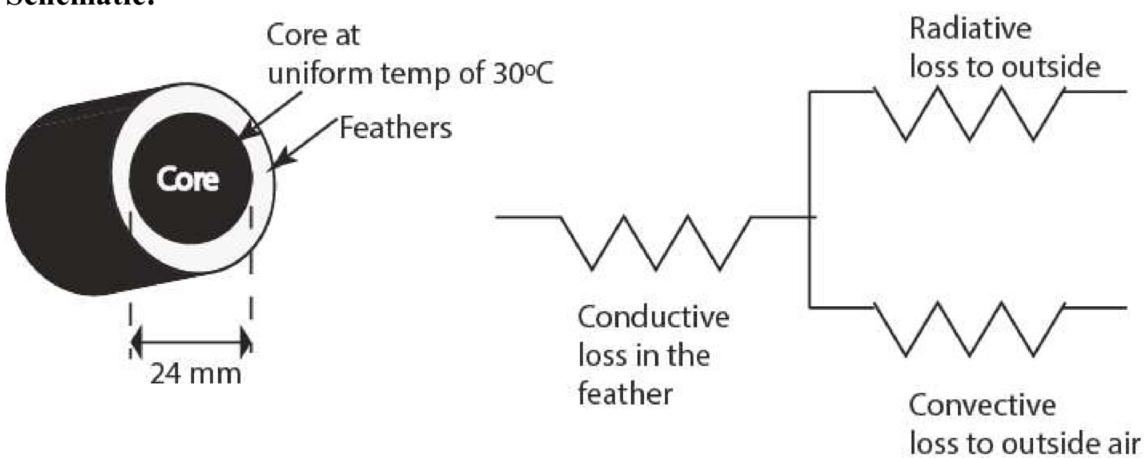
$\sigma = 5.67 \times 10^{-8}\frac{W}{m^2K^4}$

$k = 0.02\frac{W}{mK}$

$T_i = 30^\circ C$

$T_\infty = 0^\circ C$

Schematic:



Find:

All 3 resistances and metabolic heat generation

Strategy:

This is a steady state problem without heat generation, so it is a candidate for resistance formulation since we have not been asked to derive temperatures or heat flow from scratch. Notice three resistances (convective and radiative on the outside, and conductive in the feathers). Find the total resistance by combining the resistances in parallel and series. From the total resistance, find heat flow. This heat flow is equal to heat generation in the core as the process is at steady state. Note that we are not solving for anything in the core since the entire core is at 30°C.

Solution:

1

$$q'' = \frac{(T_s - T_\infty)}{\frac{1}{4\sigma T_\infty^3 A}} = \frac{(T_s - T_\infty)}{\frac{1}{4\sigma T_\infty^3 2\pi r_o L}}$$

$$R = \frac{1}{4\sigma T_\infty^3 (2\pi r_o L)}$$

2

We have resistance in parallel from the ambient to the surface and then resistance in series for conductive heat transfer and the combined parallel radiative and convective heat transfer.

$$R = R_{surface} + R_{feather}$$

$$\frac{1}{R_{surface}} = \frac{1}{R_{conv}} + \frac{1}{R_{rad}} \Rightarrow R_{surface} = \frac{R_{rad} R_{conv}}{R_{rad} + R_{conv}}$$

$$R = R_{surface} + R_{feather} = \frac{R_{rad} R_{conv}}{R_{rad} + R_{conv}} + R_{cond}$$

$$R_{rad} = \frac{1}{4\sigma T_\infty^3 (2\pi r_o L)}$$

$$R_{conv} = \frac{1}{hA} = \frac{1}{h(2\pi r_o L)}$$

$$R_{cond} = \frac{\ln \frac{r_o}{r_i}}{2\pi k L}$$

$$R = \frac{\frac{1}{4\sigma T_\infty^3 (2\pi r_o L)} \frac{1}{h(2\pi r_o L)}}{\frac{1}{4\sigma T_\infty^3 (2\pi r_o L)} + \frac{1}{h(2\pi r_o L)}} + \frac{\ln \frac{r_o}{r_i}}{2\pi k L} = \frac{\frac{1}{4\sigma T_s^3} \frac{1}{h(2\pi r_o L)}}{\frac{1}{4\sigma T_s^3} + \frac{1}{h}} + \frac{\ln \frac{r_o}{r_i}}{2\pi k L} = \frac{\frac{1}{\sigma T_\infty^3 h(8\pi r_o L)}}{\frac{1}{4\sigma T_\infty^3} + \frac{1}{h}} + \frac{\ln \frac{r_o}{r_i}}{2\pi k L}$$

also equals

$$R = \frac{1}{4\sigma T_\infty^3 (2\pi r_o L) + h(2\pi r_o L)} + \frac{\ln \frac{r_o}{r_i}}{2\pi k L}$$

3