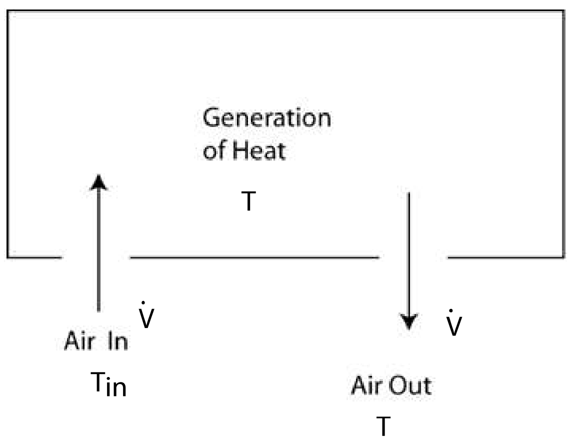
Problem 1.9.2

KNOWN:
Temperature of air coming in, generation of heat inside the room, volume of the room

FIND:
Temperature of the room (same as temperature of air going out)

SCHEMATIC AND GIVEN DATA



STRATEGY: Since the problem even mentions energy balance, we obviously need to apply energy conservation (see equation below). A change in storage (therefore a change in temperature) is caused by various amounts of energies coming in and out, and generation (body heat) within the room.

ASSUMPTIONS: No heat loss through the walls, other obvious assumptions.

SOLUTION:

We start from energy conservation

$$\text{In} - \text{Out} + \text{Gen} = \text{Storage}$$

Energy in during $\Delta t = \dot{V}\left[\frac{\text{m}^3}{\text{s}}\right]\rho_{air}\left[\frac{\text{kg}}{\text{m}^3}\right]c_{p,air}\left[\frac{\text{J}}{\text{kg}\bullet\text{K}}\right]T_{in}\,[\text{K}]\,\Delta t\,[\text{s}],$ in J

Energy out $= \dot{V}\rho_{air}c_{p_{air}}T\,\Delta t$ (ignoring property changes due to temp change)

Energy generation = 80 × 60× Δt

Change in storage $= 5\times3\times3\left[\text{m}^3\right]\times\rho_{air}\left[\frac{\text{kg}}{\text{m}^3}\right]c_{p_{air}}\left[\frac{\text{J}}{\text{kg}\,°\text{C}}\right]\Delta T\left[°\text{C}\right]$, in J

Substituting in conservation equation

$$\dot{V}\,\rho\,c_p\,T_{in}\Delta t - \dot{V}\,\rho\,c_p\,T\Delta t + 4800\Delta t = 45\,\rho\,c_p\,\Delta T$$

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$$\frac{\Delta T}{\Delta t} = \left(\dot{V}\rho\, c_p\, T_{in} + 4800 - \dot{V}\ \rho\, c_p\, T\right)/\,45\rho\, c_p$$

$$\frac{dT}{dt} = \left(\dot{V}\ \rho\, c_p\, T_{in} + 4800 - \dot{V}\ \rho\, c_p\, T\right)/\,45\rho\, c_p$$

$$= -\frac{\dot{V}}{45}\left(T - \frac{\dot{V}\rho\, c_p T_{in} + 4800}{\dot{V}\rho\, c_p}\right)$$

To solve temperature vs. time, we need initial condition, i.e., temperature at time t=0. Assume $T\ (t = 0) = T_i$. Substituting into the above equation,

$$\int_{T_i}^{T} \frac{dT}{T-a} = -\frac{\dot{V}}{45}\int_0^t dt \text{ where } a = \frac{\dot{V}\rho\, C_p T_{in} + 4800}{\dot{V}\rho\, C_p}$$

$$\ln\frac{T-a}{T_i - a} = -\frac{\dot{V}}{45}t$$

$$\frac{T-a}{T_i-a} = e^{-\frac{\dot{V}}{45}t}$$

Substituting for a,

$$T - \frac{\dot{V}\rho\, c_p T_{in} + 4800}{\dot{V}\rho\, c_p} = \left(T_i - \frac{\dot{V}\rho\, c_p T_{in} + 4800}{\dot{V}\rho\, c_p}\right)e^{-\frac{\dot{V}}{45}t}$$

$$\rho_{air} \text{ at } 25°C = 1.1769\frac{\text{kg}}{\text{m}^3}$$

$$\dot{V} = \underbrace{0.1\frac{\text{m}}{\text{s}}}_{\text{velocity}} \times \underbrace{1\text{m}\times 2.5\text{m}}_{\text{door opening}} = 0.25\frac{\text{m}^3}{\text{s}}$$

$$c_{p_{air}} \text{ at } 25°C = 1006\frac{\text{J}}{\text{kg } K}$$

$$\dot{V}\rho\, c_p T_{in} = 0.25\ \frac{\text{m}^3}{\text{s}}\times 1.1769\ \frac{\text{kg}}{\text{m}^3}\times 1006\ \frac{\text{J}}{\text{kg°C}}\times 25°\text{C}$$

$$= 7399.76\frac{\text{J}}{\text{s}}$$

T_{in}=25°C

$$\dot{V}\rho\, c_p = 295.99\frac{\text{J}}{\text{s °C}}$$

Plugging in

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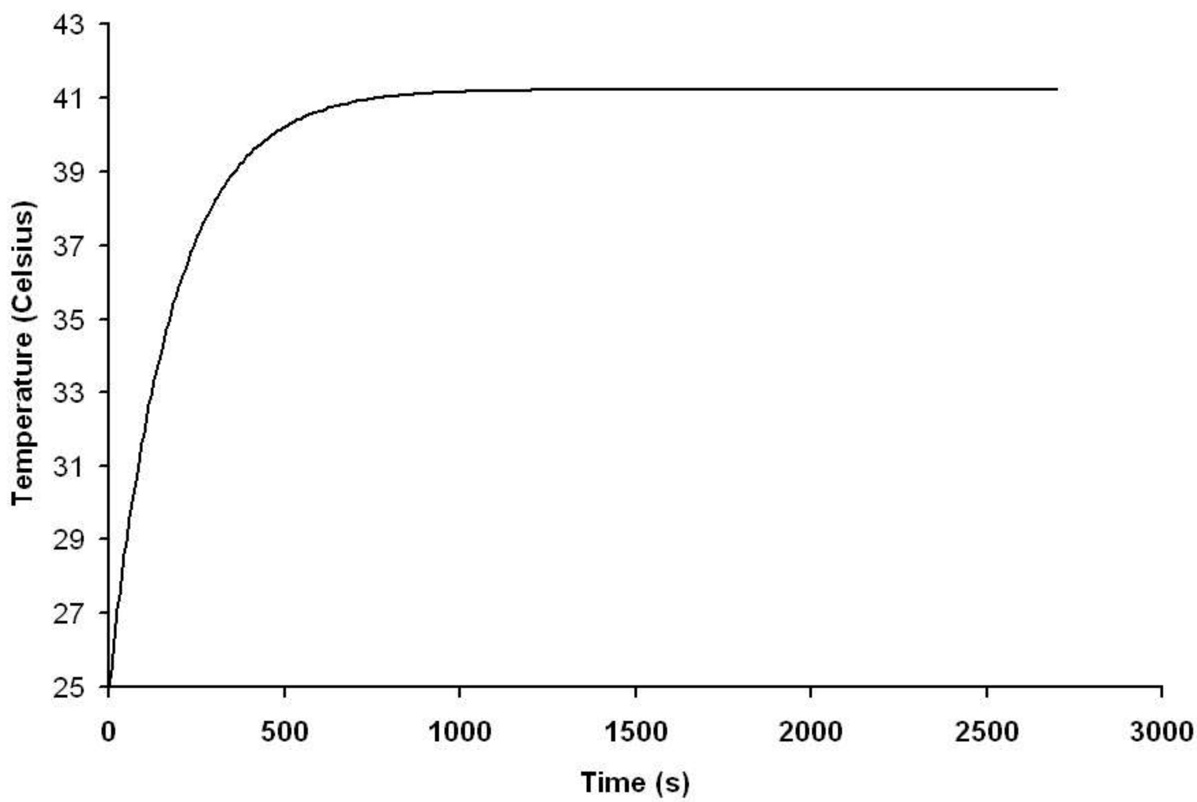
$$T - 41.22 = (25 - 41.22)e^{-\frac{0.25}{45}t}$$

$$T = 41.22 - 16.22\, e^{-0.00556t}$$

At the end of lecture $\left(t = 45\ \text{min} \times \frac{60\ s}{\text{min}}\right)$, $T = 41.22\ °\text{C}$



COMMENTS:

After about 900 seconds, temperature reaches almost a steady state.

Problem 1.9.4

Given: Mass of player, surface area, speed of running, air and wall temperature, convective heat transfer coefficient

Find: Heat balance, body surface temperature, effective heat transfer coefficient

Schematic

m = 80 kg
H = 180 cm
A = 2 m^2
v = 1.2 m/s
$T_\infty = T_w = 20°$ C
h = 20 $W/m^2 \cdot K$

$$h_{evap} = 0.124\sqrt{v}$$
$$= 0.124\sqrt{v} = 0.124\sqrt{1.2}$$
$$= 0.136 \ \text{W/m}^2\text{K}\cdot\text{Pa}$$

Solution

a) In – out + generation = storage

$$\Delta t\left(4mv - h\cdot A\cdot\left(T_s - T_\infty\right) - \sigma A\left(T_s^4 - T_w^4\right) - h_{evap}\cdot A\cdot p_s\right) = m\,c_p\Delta T$$

$$4mv - hA\left(T_s - T_\infty\right) - \sigma A\left(T_s^4 - T_w^4\right) - h_{evap}\cdot A\cdot p_s = m\,c_p\frac{\Delta T}{\Delta t}$$

$$\Delta t \to 0$$

$$4mv - hA\left(T_s - T_\infty\right) - \sigma A\left(T_s^4 - T_w^4\right) - h_{evap}\cdot A\cdot p_s = m\,c_p\frac{dT}{dt}$$

b) Steady state $\frac{dT}{dt} = 0$

$$4mv - hA\left(T_s - T_\infty\right) - \sigma A\left(T_s^4 - T_w^4\right) - h_{evap}\cdot A\cdot 13.33e^{20.386-5132/T_s} = 0$$

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Alternate answer: since at steady state, q (total loss) = q (total gain)

q (total loss) = $4mv$ = 4(80)(1.2)

= 384W

c) q (total loss) $= hA\left(T_s - T_\infty\right) + \sigma A\left(T_s^{\,4} - T_w^{\,4}\right) + h_{evap}\cdot A\cdot 13.33e^{20.386-5132/T_s}$

$$= (20)(2)(298.7-293) + \left(5.67\cdot 10^{-8}\right)(2)\left(298.7^4 - 293^4\right)$$

$$+ (0.136)(2)\left(13.33e^{20.386-5132/298.7}\right)$$

$$= 384W$$

q (total loss) $= heff\cdot A\left(T_s - T_\infty\right)$

$$heff = \frac{q\text{ (total loss)}}{A\cdot\left(T_s - T_\infty\right)} = \frac{384}{2(298.7-293)} = 33.7\ W/m^2\cdot K$$