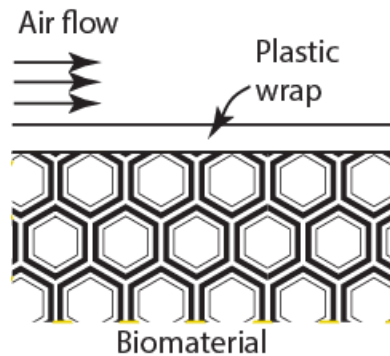


**Problem #1**

**Known:**

All required parameters.

**Schematic:**



**Find:**

Depth of freezing as function of time.

**Strategy:**

In these simplified analysis for freezing time, as was done in class for a slab geometry, we equate the rate of heat transfer through the already frozen layer to the rate at which heat needs to be removed to freeze an additional layer. The analysis here is exactly same as that done for a slab, with the only difference being the resistance here is not just of that of the frozen layer but also the plastic wrap in series with the frozen layer.

**Solution:**

1)

$$\frac{T_m - T_\infty}{\frac{L_{plastic}}{k_{plastic}A} + \frac{x}{k_{frozen}A} + \frac{1}{hA}} = \Delta H_f \rho A \frac{dx}{dt}$$

$$\frac{T_m - T_\infty}{\frac{L_{plastic}}{k_{plastic}} + \frac{x}{k_{frozen}} + \frac{1}{h}} = \Delta H_f \rho \frac{dx}{dt}$$

$$\frac{T_m - T_\infty}{\Delta H_f \rho} = \left( \frac{L_{plastic}}{k_{plastic}} + \frac{x}{k_{frozen}} + \frac{1}{h} \right) \frac{dx}{dt}$$

$$\int_0^t \frac{T_m - T_\infty}{\Delta H_f \rho} dt = \int_0^{L_{bio}} \left( \frac{L_{plastic}}{k_{plastic}} + \frac{x}{k_{frozen}} + \frac{1}{h} \right) dx$$

$$\frac{T_m - T_\infty}{\Delta H_f \rho} t = \frac{L_{plastic}}{k_{plastic}} x + \frac{x^2}{2k_{frozen}} + \frac{x}{h}$$

$$\frac{T_m - T_\infty}{\Delta H_f \rho} t = \frac{L_{plastic}}{k_{plastic}} L_{bio} + \frac{L_{bio}^2}{2k_{frozen}} + \frac{L_{bio}}{h}$$

$$t_{freeze} = \frac{\Delta H_f \rho}{T_m - T_\infty} \left( \frac{L_{plastic}}{k_{plastic}} L_{bio} + \frac{L_{bio}^2}{2k_{frozen}} + \frac{L_{bio}}{h} \right)$$

2)

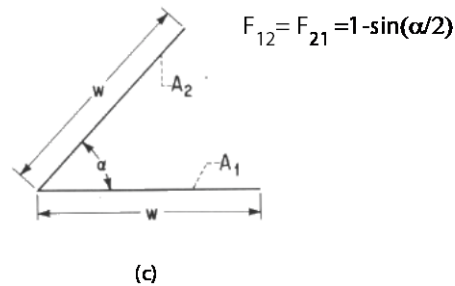
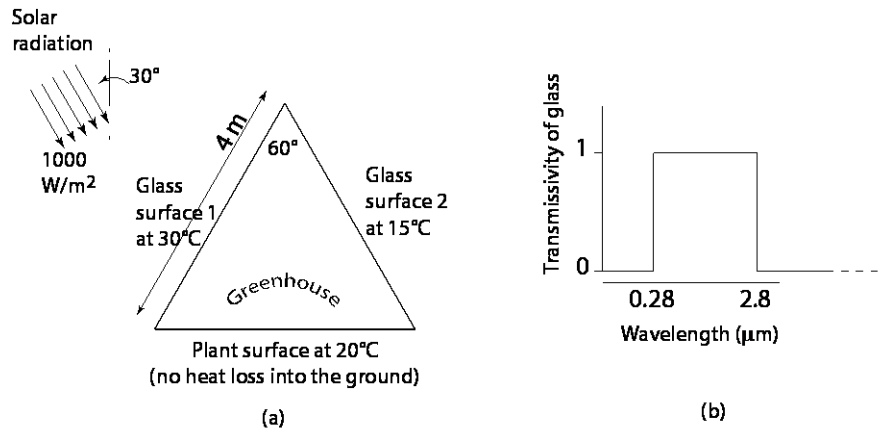
It will add an extra layer of conductive resistance that increases the time to freeze.

**Problem #2**

**Known:**

Reflectivity = 0;  $T_1=303$  K;  $T_2=288$  K;  $T_g=293$ K;  $T_\infty=283$ K;  $I = 1000$  W/m<sup>2</sup>;  $h_o=5$  W/m<sup>2</sup>K

**Schematic:**



**Find:**

$h_i, T_i$

**Strategy:**

There are two separate problems here. First, we need to find the inside heat transfer coefficient. To calculate convective heat transfer coefficient, we follow the steps provided in the coursenotes for this chapter. Although the surface is inclined, equation for vertical surface can be used, as mentioned.

For the second part of the problem, follow the steps provided in the coursenotes for solving a radiative energy balance problem. Note the transmissivity given for glass which would be needed for all radiative exchange terms.

**Solution:**

1)

$$Nu_L = \frac{hL}{k} = \left( 0.825 + \frac{0.387 Ra_L^{1/6}}{\left[ 1 + (0.492 / Pr)^{8/27} \right]^{8/27}} \right)^2$$

$$Ra_L = Gr \times Pr$$

$$Gr = \frac{\beta g \rho^2 L^3 \Delta T}{\mu^2}$$

$$\Delta T = 30 - 20 = 10K$$

$$\beta = \frac{1}{298} = 0.00336 \frac{1}{K}$$

$$L = 4m$$

Properties should be evaluated at  $\frac{1}{2}(30+20) = 25$ , (i.e. 298 K). Assume properties at 300 K for simplicity.

$$Pr = 0.708$$

$$\rho = 1.1769 \frac{kg}{m^3}$$

$$\mu = 1.8465 \times 10^{-5} \frac{kg}{m \cdot s}$$

$$k = 0.026 \frac{W}{mK}$$

$$Gr = \frac{0.00336 \cdot 9.81 \cdot 1.1769^2 \cdot 4^3 \cdot 10}{(1.8465 \times 10^{-5})^2} = 8.56 \times 10^{10}$$

$$Ra_L = 8.56 \times 10^{10} \times 0.708 = 6.06 \times 10^{10}$$

$$\frac{h \cdot 4}{0.026} = \left( 0.825 + \frac{0.387 \cdot (6.06 \times 10^{10})^{1/6}}{\left[ 1 + (0.492 / 0.708)^{9/16} \right]^{8/27}} \right)^2 = 447.5$$

$$h = 2.91 \frac{W}{m^2 K}$$

2)

$$F_{0-2.8-303} - F_{0-0.28-303} = F_{0-848} - F_{0-84.8} = 2 \times 10^{-5} - 0 \approx 0$$

$$\Rightarrow \tau = 0$$

$$\rho = 0, \quad \alpha + \tau + \rho = 1 \Rightarrow \alpha = 1 = \varepsilon$$

Hence, the glass can be considered a black body

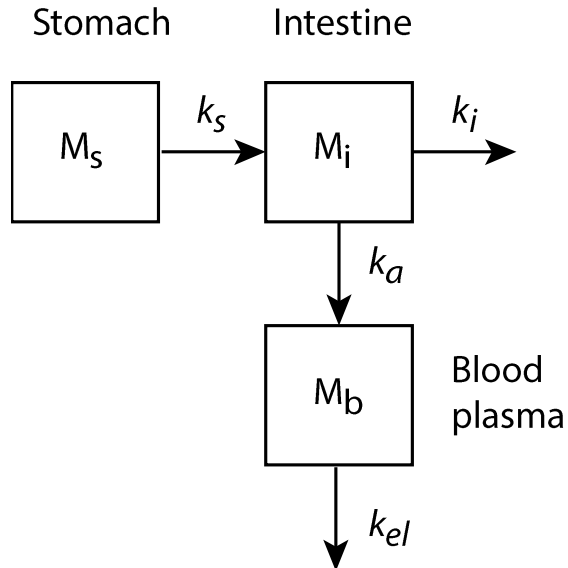


**Problem #3**

**Known:**

Various rates of generation and elimination

**Schematic:**



**Find:**

$M_s$  in stomach,  $M_i$  in intestine,  $M_b$  (just equation for  $M_b$ ) as function of time

**Strategy:**

Use mass balances

**Solution:**

Rate in – Rate out + Rate of Generation = Rate of change in Storage

1)

$$0 - k_s M_s + 0 = \frac{dM_s}{dt}$$

$$\ln(M_s) = -k_s t + C$$

$$M_s(t=0) = M_0$$

$$M_s = M_0 \exp(-k_s t)$$

2)

$$k_s M_s - k_i M_i - k_a M_i + 0 = \frac{dM_i}{dt}$$

$$k_s M_0 \exp(-k_s t) - (k_i + k_a) M_i + 0 = \frac{dM_i}{dt}$$

$$\frac{dM_i}{dt} + (k_i + k_a) M_i = k_s M_0 \exp(-k_s t)$$

$$c = M_i \quad P = (k_i + k_a) \quad Q = k_s M_0 \exp(-k_s t)$$

$$c = \left[ \exp\left(-\int P dt\right) \int Q \exp\left(\int P dt\right) dt \right] + D \exp\left(-\int P dt\right)$$

$$\exp\left(-\int P dt\right) = \exp\left(-\int (k_i + k_a) dt\right) = \exp\left(-(k_i + k_a)t\right)$$

$$\exp\left(\int P dt\right) = \exp\left(\int (k_i + k_a) dt\right) = \exp\left((k_i + k_a)t\right)$$

$$M_i = \left[ \exp\left(-(k_i + k_a)t\right) \int Q \exp\left((k_i + k_a)t\right) dt \right] + D \exp\left(-(k_i + k_a)t\right)$$

$$\int Q \exp\left((k_i + k_a)t\right) dt = \int k_s M_0 \exp(-k_s t) \exp\left((k_i + k_a)t\right) dt$$

$$= k_s M_0 \int \exp\left((k_i + k_a - k_s)t\right) dt$$

$$= \frac{k_s M_0 \exp\left((k_i + k_a - k_s)t\right)}{(k_i + k_a - k_s)}$$

$$M_i = \left[ \exp\left(-(k_i + k_a)t\right) \frac{k_s M_0 \exp\left((k_i + k_a - k_s)t\right)}{(k_i + k_a - k_s)} \right] + D \exp\left(-(k_i + k_a)t\right)$$

$$M_i = \left[ \frac{k_s M_0 \exp(-k_s t)}{(k_i + k_a - k_s)} \right] + D \exp\left(-(k_i + k_a)t\right)$$

$$M_i(0) = 0$$

$$0 = \frac{k_s M_0}{(k_i + k_a - k_s)} + D$$

$$D = \frac{-k_s M_0}{(k_i + k_a - k_s)}$$

$$M_i = \left[ \frac{k_s M_0 \exp(-k_s t)}{(k_i + k_a - k_s)} \right] + \frac{-k_s M_0}{(k_i + k_a - k_s)} \exp\left(-(k_i + k_a)t\right)$$

$$= \frac{k_s M_0}{(k_i + k_a - k_s)} \left[ \exp(-k_s t) - \exp\left(-(k_i + k_a)t\right) \right]$$

3)

$$Fk_a M_i - k_{el} M_b + 0 = \frac{dM_b}{dt}$$

$$Fk_a \frac{k_s M_0}{(k_i + k_a - k_s)} \left[ \exp(-k_s t) - \exp(-(k_i + k_a) t) \right] - k_{el} M_b + 0 - 0 = \frac{dM_b}{dt}$$



**Problem #4**

**#1**

Henry

**#2**

At higher altitudes, there is less oxygen since lower pressure. Therefore there is lower amount of oxygen in lungs due to a small ambient concentration

**#3**

As the temperature increases, the solubility of carbon dioxide decreases. Therefore, the CO<sub>2</sub> comes out of the solution and the concentration builds up in the head space, increasing the pressure until the can explodes.

**#4**

Hydrophobic since Partition coefficient =  $C_{\text{drug in solid (tissue)}}/C_{\text{drug in water}}$

**#5**

There is less moisture in the air in the winter so the skin surface coming to equilibrium with air also has less water.

**#6**

$$C_f = C_0 \exp\left(k_0 \exp\left(-\frac{E}{RT_1}\right)t_1\right) = C_0 \exp\left(k_0 \exp\left(-\frac{E}{RT_2}\right)t_2\right)$$

$$\exp\left(-\frac{E}{RT_1}\right)t_1 = \exp\left(-\frac{E}{RT_2}\right)t_2$$

$$\frac{t_2}{t_1} = \exp\left(-\frac{E}{RT_1}\right) / \exp\left(-\frac{E}{RT_2}\right) = \exp\left(-\frac{E}{RT_1} + \frac{E}{RT_2}\right)$$

$$\ln \frac{t_2}{t_1} = \frac{E}{R} \left(-\frac{1}{T_1} + \frac{1}{T_2}\right)$$

**#7**

$$P_{O_2} = H_{O_2}x_{O_2}$$

$$P_{CO_2} = H_{CO_2}x_{CO_2}$$

Compare xO<sub>2</sub> and xCO<sub>2</sub> to see which one is more soluble.

**#8**

At the interface of the two phases, the equilibrium is reached first. This is really fast, can be assumed instantaneous for our purpose.

**#9**

$$c_i = \frac{P_i}{RT}$$

**#10**

b) harder