BEE 3500 Prelim 1; Oct. 1, 2013; 7:30-9:30 PM

Name_________________________

Please do not look inside until asked

1. Note multiple parts of a question
2. Be brief. Space provided for a problem may not be indicative of the desired length of answer

Problem	1	2	3	4	Total
Points	18	18	20	16	72

Problem 1 (18 points): Heat transfer in tooth during drinking of warm fluid

We wish to model how the temperature of a single tooth will change as we drink hot water. This information can be useful for a sensitive tooth, for example. The tooth can be considered a long homogeneous cylinder (Figure 2) with a diameter of 7.2 mm, thermal diffusivity of $2.27 \times 10^{-7} \mathrm{m}^2/\mathrm{s}$, specific heat of 1.26×10^3 J/kg·K, and density of $2.20 \times 10^3 \mathrm{kg/m}^3$. Initial temperature of the tooth is 35°C, the tooth is heated symmetrically with water temperature kept constant at 60°C, and the heat transfer coefficient for flow of water over tooth is $3.49 \times 10^2 \mathrm{W/m}^2 \cdot \mathrm{K}$.

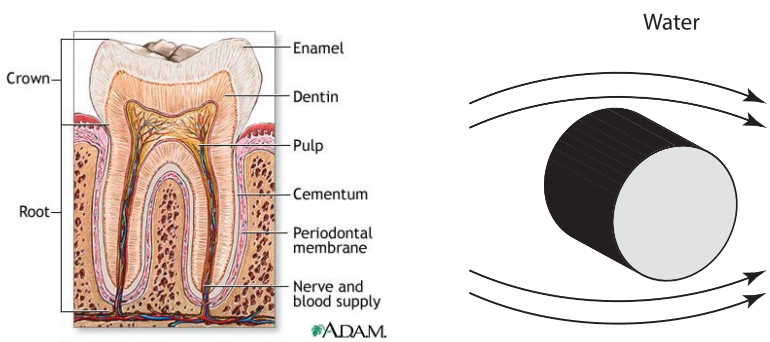

Figure 2: Single tooth with water flowing over it

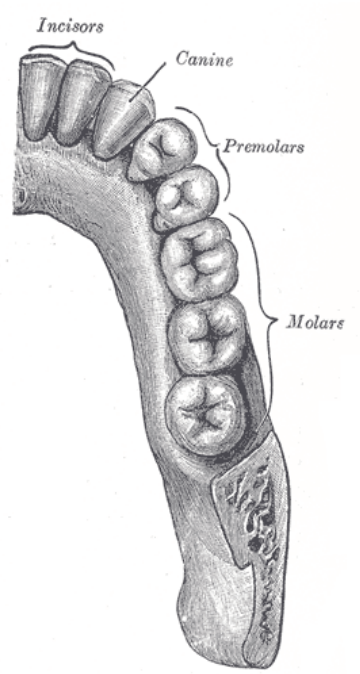

Figure 1: Permanent teeth of right half of lower dental arch; From http://en.wikipedia.org/wiki/File:Gray997.p

1) Write the governing equation and boundary conditions for this problem. Do not try to solve. ◄ *3 points* ►

2) Assuming the edge of the nerve that senses temperature is located at 1/3 radial distance from the center to the surface of the tooth (Figure 2), what is temperature at this location after 30 seconds? ◄ *8 points* ►

3) Due to its tight placement (Figure 1), the tooth array can also be treated as a slab (7.2 mm thick), heated from both sides. If the nerve can still be considered at 1/3 distance from the center of the slab to the side, what is the temperature at the nerve location after 30 seconds? ◄ *5 points* ►

4) Provide a short physical reasoning as to why the temperature in the case of a slab should be lower/higher than that in a cylindrical geometry. ◄ *2 points* ►

Problem 2 (18 points): Heat transfer over epicardium

In cardiac surgery, surgeons can obtain important information about coronary blood flow from thermographic measurements of epicardial temperature over time. Cold saline was introduced to an epicardial artery that brings down its temperature and the rewarming of the artery due to blood flow is to be modeled. The arterial wall has a surface area, A, and volume, V, exposed to blood, a specific heat of c_p and a density of ρ. The constant temperature of the flowing blood is T_b and the heat transfer coefficient between blood and the arterial wall is h. Ignore any heat loss of artery to surroundings (on the outside). Also, ignore any metabolic heat generation in the artery.

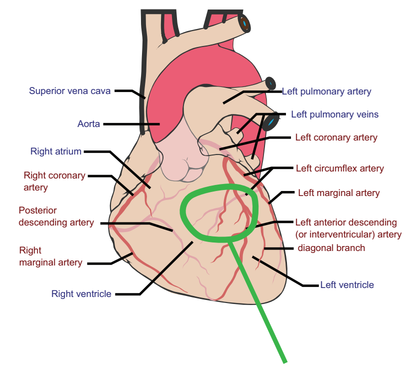

Figure 3: Epicardial arteries

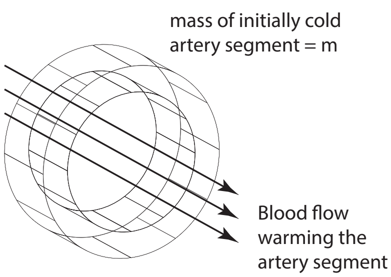

Figure 4: Segment of one epicardial artery on which heat balance will be performed

1) Thin arterial wall allowed a lumped parameter analysis of the artery segment (Medical and Engineering Physics, Volume 20). Perform a lumped heat balance on the segment shown in Figure 4, due to heat exchange with blood, to obtain a differential equation for the artery segment temperature with time. ◄ *6 points* ►

2) Solve this differential equation to obtain temperature of the artery segment as function of time. ◄ *6 points* ►

3) If the saline water had cooled the artery to an initial temperature of 30.5°C and the blood temperature is 32.7°C, sketch approximately the temperature vs. time. ◄ *3 points* ►

4) If experimentally measured temperature vs. time were available, how can you use it to estimate h using the model that you just developed? Explain in words. ◄ *3 points* ►

Problem 3 (20 points): Thermoregulation in arctic shore birds

For shore birds that live in the cold regions of the world, chicks can experience thermoregulatory issues because of their small size and poor insulation. We wish to do a heat balance on such a chick. The chick's body can be approximated as a cylinder, as shown in Figure 5, with feathers surrounding it. The external resistance between the feather outer surface and the surroundings consists of two resistances in parallel—convective and radiative.

Figure 5: Arctic shore bird Ruddy Turnstones

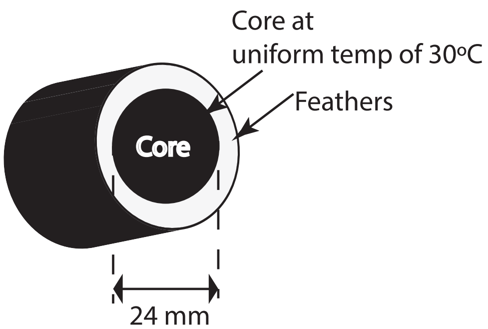

Figure 6: Schematic: A crude approximation for heat transfer analysis

1) If the radiative heat transfer from the outer surface of the feathers can be approximated as $q = 4\sigma T_\infty^3 A(T_s - T_\infty)$, where T_s is the temperature at the outer surface of the feathers and T_∞ is the ambient and air temperature, write an expression for the radiative resistance, making sure variables used are explained. ◄ *2 points* ►

2) Develop an expression for total resistance between the inner surface of the feathers and the surroundings at T_∞ that includes both convective and radiative heat loss on the outside and the conductive heat loss through the feathers. ◄ *8 points* ►

3) In addition to heat loss corresponding to step 2), 18% of the total metabolic heat is lost through evaporation. What is the total metabolic heat generation needed by the chick to maintain the core temperature of 30°C? The diameter of the core in Figure 6 is 24 mm and the height of the cylinder is 4 cm, the thickness of the feathers is 4 mm, outside air temperature is 0°C, outside heat transfer coefficient is 200 $\text{W/m}^2 \cdot \text{K}$, $\sigma = 5.670 \times 10^{-8} \text{W/m}^2\text{K}^4$, thermal conductivity of the feather is 0.02 $\text{W/m} \cdot \text{K}$. ◄ *7 points* ►

4) For the situation in 3), if the thickness of the insulation is halved, what would have to be the new rate of heat generation to keep the core temperature at 30°C? Assume evaporation is to remain constant. ◄ *3 points* ►

Problem 4 (16 points): Heat generation in waste as bioreactor

Bioreactor landfills aim to optimize the conditions within waste to enhance waste stabilization. Consider constant distributed heat generation at steady state in a waste pile of height L whose bottom is kept at the ground temperature of T_g and air at temperature T_∞ is blowing at the top surface with a heat transfer coefficient h. The uniform rate of heat generation in the waste is Q $\mathrm{W/m^3}$.

1) Write the governing equation and boundary conditions for the problem. ◄ *3 points* ►

2) Solve to provide temperature as a function of position in the waste. Sketch the solution and mention whether the maximum temperature will be in the middle. ◄ *9 points* ►

3) In the winter, heat generation ceases when the surrounding temperature becomes low enough. If we assume all heat generation has ceased (but all other conditions stay the same), what would be the new steady-state maximum temperature and where would it be located? ◄ *4 points* ►