Robust Formation Trajectory Tracking Control for Multiple Quadrotors
With Communication Delays

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Abstract—This brief presents a robust formation control method for a team of quadrotors with communication delays. The dynamics of each quadrotor is subject to nonlinearities, parametric perturbations, and external disturbances. A distributed formation controller is developed for the quadrotor team, which consists of a position controller to achieve the desired formation trajectories and patterns, and an attitude controller to regulate the attitude angles for each quadrotor. It is proven that the tracking errors can converge into a neighborhood of the origin bounded by a given constant in a finite time. Experimental results on multiple microquadrotors are provided to verify the effectiveness of the proposed control method.

Index Terms—Formation control, micro quadrotors, nonlinear system, robust control, uncertain system.

I. INTRODUCTION

In recent years, the formation control problem for a team of unmanned aerial vehicle (UAV) systems has been attracting increasing interest from the academic and industrial fields. The UAV formation has significant advantages over a single UAV in the case of performing complicated tasks and has various potential applications in the communication relay, search and rescue, and persistent reconnaissance, as illustrated in [1]–[5]. Over the past two decades, several classical formation control methods have been developed to achieve the formation control for multi-agent systems, such as the leader-follower method, the behavioral strategy, and the virtual structure approach (see e.g., [7]–[12]). Recently, a consensus-based approach has been developed, which serves as a promising solution to address the formation control problem of a team of UAVs, as shown in [16]–[24]. The leader-follower method, the behavioral strategy, and the virtual structure approach can be unified under a general framework of the consensus-based approach, as elaborated in [13].

Quadrotor UAVs served as popular aerial platforms possessing various advantages such as low-cost, simple structure, and vertical take-off and landing, as illustrated in [14] and [15].

Multiple works have been conducted for studying the formation control problems in a team of unmanned vehicles. An integrated optimal control framework was developed in [12] for fixed-wing UAVs to achieve formation trajectory tracking. Formation flight in [8] was realized by a potential field theory-based control scheme. In [2] and [4], formation control problems were discussed for a set of UAVs with simplified translational dynamics or rotational dynamics. In [7], a hybrid 3-D formation control framework was presented under the leader-follower context using UAVs with simplified translational dynamics. In [19], the dynamical model of each vehicle was considered as a second-order linear system and the formation experiments were implemented. The formation control problems for multiple vehicles modeled by second-order nonlinear systems were studied in [8] and [12]. The vehicle dynamics was simplified and the formation control problems for such under-actuated systems were not fully studied in [2], [4], [7], and [19].

Furthermore, the formation control problems for a team of uncertain systems were studied in [6], [10], [22], and [23]. In [6], an adaptive control approach was presented for a team of nonlinear systems to reduce the effects of parametric perturbations on the closed-loop control system. In [10], the formation control problem for a team of uncertain quadrotors was studied based on the classical leader-follower method and a suboptimal $\mathcal{H}_\infty$ control controller was developed to reduce the effects of model uncertainties and external disturbances. However, disturbances cannot be restrained by the $\mathcal{H}_\infty$ control approach in the full frequency range. In [22] and [23], a disturbance estimator was designed for multiple 3-DOF helicopters to restrain the influence of model uncertainties and external disturbances in the rotational dynamics, but the under-actuated 6-DOF aerial vehicle system was not discussed further.

Moreover, several works have been conducted to address the robust formation control problem for a team of unmanned vehicles subject to communication delays. In [9], a model predictive control method was studied to achieve the formation flight control for multiple quadrotors with communication delays. In [24], a formation control law based on a singularity-free extraction algorithm was developed to address the communication delay problem for a class of under-actuated systems. In [18], a formation control method was developed for the second-order agents to achieve the desired formation trajectory tracking, and a neighbor-based feedback control rule was designed to reduce the effects of communication delays between different agents. However, the effects of other uncertainties were not fully discussed in the stability property analysis of the constructed closed-loop control systems in [9], [18], and [24].

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In this brief, a robust formation controller is proposed for a team of microquadrotors subject to communication delays. The proposed formation controller consists of a position controller to achieve the desired formation trajectories and patterns and an attitude controller to regulate the attitude dynamics of the quadrotors. The position controller and the attitude controller are both developed based on the linear quadratic regulator (LQR) control method and the robust compensation theory. Compared to the existing literature, the main contributions of this brief can be summarized as follows:

First, the model of each quadrotor is an under-actuated nonlinear system involving three translational DOF and three rotational DOF, and experimental results on multiple microquadrotors are provided to demonstrate the advantages of the proposed formation control protocol. But, the formation control problems for such under-actuated 6-DOF systems with completely nonlinear and coupled terms were not further discussed in [7], [8], [13], [19], [22], and [23]. Second, the effects of parametric perturbations and external disturbances are considered in the dynamical model of each quadrotor. However, in [7]–[9], [18]–[21], and [24], the uncertainty rejection problems were not fully considered in the robust stability analysis of the constructed closed-loop control systems. Third, the influence of communication delays can be restrained by the proposed robust formation control protocol and the tracking errors can converge into a neighborhood of the origin bounded by a given constant in a finite time. The effects of communication delays on the global closed-loop control systems were not further studied in [6]–[8], [10]–[12] and [19]–[23]. The remaining parts of this brief are organized as follows. Preliminaries on the quadrotor model and the graph theory and the problem formulation are presented in Section II. The formation control protocol design is introduced in Section III. The robustness properties of the proposed global closed-loop control system are analyzed in Section IV. Experimental results on the formation for a team of quadrotors subject to uncertainties and communication delays are provided in Section V. Section VI concludes this brief.

Notations: Denoteas $I_n \in \mathbb{R}^{n \times n}$ an $n \times n$ identity matrix, $0_{n \times h} \in \mathbb{R}^{n \times h}$ a zero matrix, and $C_{n \times j} \in \mathbb{R}^{n \times 1}$ an $N \times 1$ column vector with 1 on the $j$th element and 0s elsewhere. Let $\otimes$ indicate the Kronecker product. Define $\|y\| = (y_1^2 + \cdots + y_N^2)^{1/2}$ and $\|x\|_\infty = \sup_{t \geq 0} \|x(t)\|$, where $y \in \mathbb{R}^{N \times 1}$ and $x(t) \in \mathbb{R}^{N \times 1}$.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. Graph Theory

The interaction relationship between a team of $N$ homogeneous quadrotors, labeled from 1 to $N$, is described by a directed graph $G = (\hat{V}, \hat{E}, \hat{W})$. Let $\Pi = \{1, 2, \ldots , N\}$. $\hat{V} = \{\hat{v}_1, \hat{v}_2, \ldots , \hat{v}_N\}$ indicates a set of $N$ nodes, where node $\hat{v}_i$ represents quadrotor $i$. A set of edges $\hat{E} \subseteq \{(\hat{v}_i, \hat{v}_j) : \hat{v}_i, \hat{v}_j \in \hat{V}\}$ represents the communication relationship between the quadrotors, and $\hat{E}_{ij} = (\hat{v}_i, \hat{v}_j) \in \hat{E}$ means that quadrotor $j$ can send information to quadrotor $i$ and quadrotor $j$ is a neighbor of quadrotor $i$. $\hat{N}_i = \{\hat{v}_j \in V : (\hat{v}_i, \hat{v}_j) \in \hat{E}\}$ is the set of all neighbors of node $\hat{v}_i$.

The weighted adjacency matrix $\hat{W} = [\hat{w}_{ij}] \in \mathbb{R}^{N \times N}$ represents the communication intensity among the quadrotors, where $\hat{w}_{ij} > 0$ if $(\hat{v}_i, \hat{v}_j) \in \hat{E}$, otherwise $\hat{w}_{ij} = 0$. Besides, $\hat{w}_{ii} = 0$ for $i \in \Pi$. Define the in-degree matrix as $\hat{D} = \hat{d} diag(\hat{d}_i) \in \mathbb{R}^{N \times N}$, where $\hat{d}_i = \sum_{j=1}^{N} \hat{w}_{ij}$. $\hat{L} = \hat{D} - \hat{W}$ is the weighted graph Laplacian matrix. There exists a spanning tree in the directed graph $\hat{G}$, if a node (called the root) has directed paths to all other nodes in $\hat{G}$.

B. Quadrotor Model

Consider the dynamical model of quadrotor $i$ as a rigid body. Let $\mathcal{I}$ indicate an earth-fixed inertial frame and $\mathcal{B}$ a body-fixed frame attached to quadrotor $i$. Let $p_i = [p_{xi}, p_{yi}, p_{zi}]^T \in \mathbb{R}^{3 \times 1}$ represent the position of quadrotor $i$ in $\mathcal{B}$. Denote $\Theta_i = [\phi_i, \theta_i, \psi_i]^T \in \mathbb{R}^{3 \times 1}$ as the attitude of quadrotor $i$, where $\phi_i$ represents the roll angle, $\theta_i$ the pitch angle, and $\psi_i$ the yaw angle. In order to avoid the singularity problem, the attitude of quadrotor $i$ is assumed to satisfy: $|\phi_i| < \pi/2$, $|\theta_i| < \pi/2$, and $|\psi_i| < \pi/2$, respectively. Denote $R_{bi} \in SO(3)$ as the rotation matrix, which can be described as

\[
R_{bi} = \begin{bmatrix}
\cos \phi_i \cos \psi_i & \cos \phi_i \sin \psi_i - \cos \psi_i \sin \theta_i & \sin \phi_i \\
- \sin \phi_i \cos \psi_i & \sin \phi_i \sin \psi_i + \cos \phi_i \sin \theta_i & \cos \phi_i \\
\sin \theta_i & - \cos \theta_i & 0
\end{bmatrix}
\]

As depicted in [14], the dynamical model of quadrotor $i$ can be given by

\[
m_i \ddot{p}_{bi} = R_{bi} f_{bi} + d_{pi}
\]

\[
J_{bi} \dot{\Theta}_i = T_f \Theta_i + C_i (\Theta_i, \dot{\Theta}_i) \dot{\Theta}_i + d_{lei}
\]

where $m_i$ and $J_i$ represent the mass and the inertia matrix of quadrotor $i$, respectively, and $C_i (\Theta_i, \dot{\Theta}_i)$ represents the Coriolis term, as illustrated in [14]. $d_{pi}$ and $d_{lei}$ are the external disturbances, which are assumed to be bounded. The external force $f_{bi}$ and the torque $T_{bi}$ can be described as

\[
f_{bi} = \begin{bmatrix}
0 \\
0 \\
T_f
\end{bmatrix} + R_{bi}^T \begin{bmatrix}
0 \\
0 \\
-m_{j bi}
\end{bmatrix}
\]

\[
T_{bi} = \begin{bmatrix}
l_{bi} k_{\sigma_i}(\sigma_{2,i}^2 - \sigma_{4,i}^2) \\
-l_{bi} k_{\sigma_i}(\sigma_{2,i}^2 - \sigma_{4,i}^2) \\
k_{\sigma_i} \sum_{r=1}^{4} (-1)^{r+1} \sigma_{r,i}^2
\end{bmatrix}
\]

where $g$ indicates the gravity constant, $k_{\sigma_i}$ and $k_{\sigma_i}$ are positive scale factors, $\sigma_{r,i}(r = 1, 2, 3, 4)$ represent the rotational velocity of rotor $r$, and $\Theta_{bi}$ indicates the distance between the center mass of quadrotor $i$ and each rotor. The total lift $T_f = k_{\sigma_i} \sum_{r=1}^{4} \sigma_{r,i}^2$ is assumed to be positive, because $\sigma_{r,i}$ is nonnegative and $T_f = 0$ results in a free state for quadrotor $i$. The control input commands to the four rotors can be written as follows:

\[
u_{c,i} = \sum_{r=1}^{4} \sigma_{r,i}^2, \quad u_{\Theta_{0,i}} = \sigma_{2,i}^2 - \sigma_{4,i}^2
\]

\[
u_{c,i} = \sigma_{1,i}^2 - \sigma_{2,i}^2, \quad u_{\Theta_{0,i}} = \sum_{r=1}^{4} (-1)^{r+1} \sigma_{r,i}^2
\]
Remark 1: It should be pointed out that quadrotor $i$ is an under-actuated system, which possesses 6-DOF (the height, the lateral and longitudinal positions, and three attitudes angles) and four control inputs $(u_{zi}, u_{\Theta_{1,i}}, u_{\Theta_{2,i}}, u_{\Theta_{3,i}})$. Furthermore, it can be observed that the dynamical model of quadrotor $i$ described in (1) is highly nonlinear and coupled.

C. Problem Formulation

The goal of this brief is to develop a distributed controller for a team of quadrotors to achieve the desired formation trajectories and patterns. Denote $\delta_{ij} = [\delta_{i,j}, \delta_{j,i}]^{T} \in \mathbb{R}^{3 \times 1}$ $(i, j \in \Pi)$ as the desired position deviation between quadrotor $i$ and quadrotor $j$, and $\delta_{i}$ denotes the formation pattern of the quadrotor team. Denote $p_{r} \in \mathbb{R}^{3 \times 1}$ as the desired trajectory of the formation center, which can also be considered as the virtual team leader. $p_{r}$ is differentiable. Let $\delta_{ij} = \delta_{i} - \delta_{j}(i, j \in \Pi)$, where $\delta_{i}$ indicates the desired position deviation between the virtual team leader and quadrotor $i$. $\Theta'$ represents the desired pitch angle and $\Theta''$ the roll angle references. The yaw angle needs to track the reference $\Theta'$. Let $\Theta' = [\Theta_{1}', \Theta_{2}', \Theta_{3}']^{T} \in \mathbb{R}^{3 \times 1}$. Then, one can obtain the position and attitude errors as $e_{pi} = p_{r} - \delta_{i} - p_{r} = [e_{pk,i}] \in \mathbb{R}^{3 \times 1}$, $e_{pi} = [e_{pk,i}]^{T} \in \mathbb{R}^{3 \times 1}$, $e_{\Theta_{i}} = \Theta_{i} - \Theta' = [e_{\Theta_{k,i}}]^{T} \in \mathbb{R}^{3 \times 1}$, and $e_{\Theta_{i}} = e_{\Theta_{k,i}}$ are differentiable. Let $B_{pi} = m^{-1}I_{3 \times 3}$ and $B_{\Theta_{i}} = J_{i}^{-1} diag([l_{i}k_{i}, l_{i}k_{i}, k_{i}]) (i \in \Pi)$. Define the communication delays between quadrotor $i$ and quadrotor $j$. The communication delays are assumed to be nonnegative and piecewise continuous. For quadrotor $i$, one can obtain that

$$\begin{align*}
\dot{\hat{p}}_{i} &= B_{pi}^{N}u_{pi} - g_{c,3,3} + \Delta p_{i} \\
\dot{\Theta}_{i} &= B_{\Theta_{i}}u_{\Theta_{i}} + (J_{i}^{-1})^{-1} C(\Theta_{i}, \Theta_{i}) \Theta_{i} + \Delta \Theta_{i} 
\end{align*}$$

where $u_{\Theta_{i}} = [u_{\Theta_{1,i}}, u_{\Theta_{2,i}}, u_{\Theta_{3,i}}]^{T}$, $u_{pi} = [u_{p1,i}, u_{p2,i}, u_{p3,i}]^{T}$ is the virtual position control input and satisfies

$$u_{zi} = u_{zi} \left[ \begin{array}{c}
sin \phi_{i} \sin \psi_{i} + \cos \phi_{i} \cos \phi_{i} \\
\cos \phi_{i} \sin \theta_{i} \sin \psi_{i} - \cos \psi_{i} \cos \phi_{i}
\end{array} \right].$$

$\Delta p_{i}$ and $\Delta \Theta_{i}$ are named equivalent disturbances, which include the parametric perturbations and external disturbances, and can be given by

$$\begin{align*}
\Delta p_{i} &= B_{pi}^{N} \tilde{u}_{pi} + m_{i}^{-1}d_{pi} \\
\Delta \Theta_{i} &= -(J_{i}^{-1})^{-1} C(\Theta_{i}, \Theta_{i}) \Theta_{i} + (J_{i}^{-1})^{-1} C(\Theta_{i}, \Theta_{i}) \Theta_{i} + B_{\Theta_{i}} u_{\Theta_{i}} + J_{i}^{-1} d_{\Theta_{i}} 
\end{align*}$$

where $\tilde{u}_{pi}$ is the force error and satisfies $\tilde{u}_{pi} = [\tilde{u}_{p1,i}, \tilde{u}_{p2,i}, \tilde{u}_{p3,i}]^{T} = u_{zi}(B_{pi}^{N} - B_{pi}R_{pi}c_{3,3} - u_{pi})$.

Remark 2: The model (2) represents the real dynamical model of each quadrotor. One can obtain the nominal model by removing the equivalent disturbances $\Delta p_{i}$ and $\Delta \Theta_{i}$. The complete model includes the nominal model and equivalent disturbances.

III. Controller Design

The formation controller is designed based on the LQR approach and the robust compensation theory, as illustrated in [25] and [26], which consists of a position controller and an attitude controller.

A. Position Controller Design

The virtual position control input $u_{pi}(t)$ consists of two parts

$$u_{pi}(t) = u_{pi}^{N}(t) + u_{pi}^{R}(t), \quad i \in \Pi$$

where $u_{pi}^{N}(t) \in \mathbb{R}^{3 \times 1}$ is the nominal control input and $u_{pi}^{R}(t) \in \mathbb{R}^{3 \times 1}$ is the robust compensating input. The nominal control input $u_{pi}^{N}(t)$ is designed to achieve the desired formation control of the nominal model and $u_{pi}^{R}(t)$ to restrain the influence of the equivalent disturbance on the real model.

When quadrotor $i$ receives the information (position and velocity) from its neighbor, i.e., quadrotor $j$, there exist communication delays in information. Let $p_{r}$ represent the communication delays between quadrotor $i$ and quadrotor $j$ or the virtual team leader. The communication delays are assumed to be nonnegative and piecewise continuous. The nominal control input $u_{pi}^{N}$ can be designed as follows:

$$u_{pi}^{N}(t) = -\mu \sum_{j \in N_{i}} \tilde{u}_{ij}(B_{pi}^{N})^{-1} K_{p}(p_{r}(t) - p_{j}(t - \tau_{i} - \pi_{ij}))$$

where $\mu \in \mathbb{R}^{3 \times 3}$ and $K_{p} \in \mathbb{R}^{3 \times 3}$ are the diagonal nominal controller gain matrices. $\tau_{i}$ is a constant, indicating the connection weight between the virtual team leader and quadrotor $i$: $\tau_{i} > 0$ represents that the virtual team leader can send information to quadrotor $i$, and $\tau_{i} = 0$ otherwise. Define $K_{L} = [K_{p}, K_{c}]$ as the LQR position controller gain. Let

$$A_{p} = \begin{bmatrix} 0_{3 \times 3} & I_{3} \\
0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}, \quad B_{Z} = \begin{bmatrix} 0_{3 \times 3} \\
0_{3 \times 3} \end{bmatrix}.$$  

Define $Q_{L} = Q_{T}^{T} \in \mathbb{R}^{6 \times 6}$ and $\Gamma_{L} = \Gamma_{L}^{T} \in \mathbb{R}^{3 \times 3}$ are positive definite and symmetric matrices. As depicted in [27], the LQR position controller gain can be given by $K_{L} = \Gamma_{L}^{-1} B_{Z}^{T} P_{L}$, where $P_{L}$ is the positive definite solution to the following Riccati equation:

$$A_{p}^{T} P_{L} + P_{L} A_{p} + Q_{L} - P_{L} B_{Z} \Gamma_{L}^{-1} B_{Z}^{T} P_{L} = 0.$$  

Let $\Delta_{i}(k = 1, 2, 3)$ be the equivalent disturbances involving the communication delays, which satisfy

$$\begin{align*}
\Delta_{pi}(t) &= \Delta_{pi}(t) + \Delta_{puj} (t - \tau_{i}) - \Delta_{puj}(t) \\
\Delta_{\Theta_{i}}(t) &= \Delta_{\Theta_{i}}(t)
\end{align*}$$

(6a)
where
\[ \Delta_{puj}(t) = \mu_p \sum_{j \in N_i} \hat{w}_{ij}(K_p p_j(t) + K_o \dot{p}_j(t)) + \mu_p \hat{a}_{pi}(K_p p_i(t) + K_o \dot{p}_i(t)). \]
The term \( \Delta_{puj}(t - \rho_1) - \Delta_{puj}(t) \) represents the mismatch resulting from the communication delays.

Remark 3: It should be pointed out that \( \rho_1 \) represents the communication delays from quadrotor \( i \) to its neighbor quadrotor \( j \), and the delays can be time-varying. It can be seen that the communication delay is involved in the position control input, and thereby the global control system. Moreover, the robust compensating input \( u_R^i(s) (i \in \Pi) \) can be constructed as
\[ u_R^i(s) = -(B_i^N)^{-1} F_i(s) \Delta_i(s) \]
where \( s \) indicates the Laplace operator and \( F_i(s) = diag(F_{p1,i}(s), F_{p2,i}(s), F_{p3,i}(s)). \) \( F_{p1,i}(s) = f_{p1,i}^2/(s + f_{p1,i}) \) \( (l = 1, 2, 3) \) are robust position filters, where \( f_{p1,i} \) has a larger bandwidth and \( u_R^i(s) \) becomes closer to \( -\Delta_i(s) \). The influence of the equivalent disturbance \( \Delta_i(s) \) can be restrained. However, it is not easy to obtain the equivalent disturbance \( \Delta_i(s) \) directly in practical applications. From (2), one can obtain that
\[ \Delta_{puj}(t) = \dot{p}_i(t) - B_i^N u_{pi}(t) + g c(s, 3). \]
Substituting (8) into (7), one can obtain that
\[ \begin{align*}
\dot{z}_{11}(t) &= -f_{p1} z_{11}(t) - f_{p2} z_{12}(t) + B_i^N u_{pi}(t) - g c(s, 3) \\
\dot{z}_{12}(t) &= -f_{p1} z_{21}(t) + 2 f_{p2} z_{12}(t) + z_{11}(t) \\
u_{pi}(t) &= (B_i^N)^{-2} f_{p2} (z_{12}(t) - p_i)
\end{align*} \]
where \( z_{11}(t), z_{12}(t) \in \mathbb{R}^{3 \times 1} \) are the states of robust position filters and \( f_{pi} = diag(f_{p1,i}, f_{p2,i}) \in \mathbb{R}^{3 \times 3} \) \( (i \in \Pi) \).

Furthermore, by solving the equation in (3), one can obtain the vertical control input \( u_{zi} \) and the pitch and roll angle references \( \theta_i^r(t) \) and \( \phi_i^r(t) \) \( (i \in \Pi) \) as follows:
\[ \begin{align*}
u_{zi}(t) &= u_{pzi}(t)/\cos \phi_i(t)/\cos \theta_i(t) \\
\theta_i^r(t) &= \sin^{-1}((u_{pzi}(t)/u_{zi}(t))/\cos \phi_i(t)/\cos \theta_i(t)) \\
\phi_i^r(t) &= \sin^{-1}((\cos \phi_i(t) \sin \theta_i(t))/\cos \phi_i(t)/\cos \theta_i(t)) - u_{pzi}(t)/u_{zi}(t)/\cos \phi_i(t)/\cos \theta_i(t)
\end{align*} \]

B. Attitude Controller Design

Similar to the virtual position controller design, the attitude control input \( u_{\Theta_i}(t) \) can be designed as follows:
\[ u_{\Theta_i}(t) = u_{N\Theta_i}(t) + u_{R\Theta_i}(t), \quad i \in \Pi. \]
The nominal control input \( u_{N\Theta_i}(t) \) can be designed as
\[ \begin{align*}
u_{N\Theta_i}(t) &= (B_i^N)^{-1} (-K_\Theta \Theta_i(t) - K_o e_{\Theta_i}(t)) \\
&+ (B_i^N)^{-1} (J_i^{-1} C(\Theta_i(t), \dot{\Theta}_i(t)) \dot{\Theta}_i(t) + \ddot{\Theta}_i(t))
\end{align*} \]
where \( K_\Theta, K_o \in \mathbb{R}^{3 \times 3} \) are nominal controller parameter matrices. Denote \( K_R = [K_\Theta, K_o] \) as the LQR attitude controller gain. Define
\[ A_R = \begin{bmatrix} 0_{3 \times 3} & I_3 \\
0_{3 \times 3} & 0_{3 \times 3}
\end{bmatrix}. \]
Define \( Q_R = Q_R^T \in \mathbb{R}^{6 \times 6} \) and \( \Gamma_R = \Gamma_R^T \in \mathbb{R}^{3 \times 3} \) as positive definite and symmetric matrices. The LQR attitude controller gain \( K_R \) can be obtained as \( K_R = \Gamma_R^{-1} B_R^T P_R \), where \( P_R \) is the positive definite solution to the following Riccati equation:
\[ A_R^T P_R + P_R A_R + Q_R - P_R B_R \Gamma_R^{-1} B_R^T P_R = 0. \]
Similarly, the robust compensating input can be given by
\[ u_{R\Theta_i}(s) = -(B_i^N)^{-1} F_i(s) \Delta_i(s) \]
where \( F_i(s) = diag(F_{\Theta_i}(s)) \in \mathbb{R}^{3 \times 3}, \) \( F_{\Theta_i}(s) = diag(f_{\Theta_i}^2/(s + f_{\Theta_i})) \in \mathbb{R}^{3 \times 3} \) \( (i \in \Pi) \) with positive parameter \( f_{\Theta_i} \) to be determined. Let \( f_{\Theta_i} = diag(f_{\Theta_i}) \in \mathbb{R}^{3 \times 3}. \)
Then, one can obtain the realization of \( u_{\Theta_i}(t) \) in a similar way.

Remark 4: It can be seen that the proposed robust formation controller is distributed and time-invariant, which means that the controller of quadrotor \( i \) depends only on the position and velocity information from itself and its neighbors.

C. Robustness Property Analysis

Define \( \tilde{X}_i(t) = [x_{T}(t), e_T^T(\Theta_i(t))]^T = [x_{pki}(t)] \in \mathbb{R}^{6 \times 1}, \)
\[ \dot{\tilde{X}}_i(t) = [e_T(\Theta_i(t)), e_T^T(\Theta_i(t))] = [x_{pki}(t)] \in \mathbb{R}^{6 \times 1}, \] and
\[ A_{\Theta} = \begin{bmatrix} 0_{3 \times 3} & I_3 \\
-K_\Theta & -K_o
\end{bmatrix}. \]
Combining (2), (5), (6), (6a), (9), and (10), one can obtain the whole closed-loop error system as
\[ \begin{align*}
\dot{\tilde{x}}_p(t) &= A_{pc} \tilde{x}_p(t) + B_{p\Delta} \dot{\tilde{x}}_p(t) \\
\dot{\tilde{x}}_{\Theta}(t) &= A_{\Theta} \tilde{x}_{\Theta}(t) + B_{\Theta\Delta} \dot{\tilde{x}}_{\Theta}(t)
\end{align*} \]
where \( \tilde{x}_p(t) = [\tilde{X}_p(t)] \in \mathbb{R}^{6N \times 1}, \) \( \tilde{x}_{\Theta}(t) = [\tilde{X}_{\Theta}(t)] \in \mathbb{R}^{6N \times 1}, \) \( \Delta_p = [B_i^N u_{pi}(t) + \Delta_{puj}(t)] \in \mathbb{R}^{3N \times 1}, \) \( \Delta_{\Theta} = [B_i^N u_R^i(t) + \Delta_{\Theta}(t)] \in \mathbb{R}^{3N \times 1}, \) \( B_L = diag[a_{pi}] \in \mathbb{R}^{N \times N}, \) and
\[ A_{pc} = I_N \otimes A_p - \mu \ell (L + B_L) \otimes B_Z K_L \\
A_{\Theta} = I_N \otimes A_\Theta, \quad B_{p\Delta} = B_{\Theta\Delta} = I_N \otimes B_Z. \]
It can be seen that all the eigenvalues of \( A_{\Theta} \) have negative real parts, and thereby \( A_{\Theta} \) is asymptotically stable. According to Theorem 1 in [27], if the root can receive the information from the virtual team leader and the graph \( \hat{G} \) has a spanning tree, then, if \( \mu \geq \hat{\psi}_{pr}^{min}/2, \) where \( \hat{\psi}_{pr}^{min} = \min_{r \in \Pi} \text{Re}(\hat{\psi}_{pr}) \) and \( \hat{\psi}_{pr} \) indicates the eigenvalues of \( \hat{L} + B_L \) and \( A_{pc} \) is asymptotically stable. The robust compensating inputs \( u_{R\Theta_i}(t) \) and \( u_R^i(t) \) are given in the state-space forms as
\[ \begin{align*}
\dot{X}_{R\Theta_i}(t) &= A_{R\Theta_i}(f_{kl,i}) X_{R\Theta_i}(t) + c_{21}(b_i^N)^{-1} \Delta_{\Theta_i} \\
u_{R\Theta_i}(t) &= -c_{22} f_{kl,i} \cdot X_{R\Theta_i}(t), \quad k = p, \Theta; \quad l = 1, 2, 3
\end{align*} \]
where
\[ A_{R\Theta_i}(f_{kl,i}) = \begin{bmatrix} -f_{kl,i} & 0 \\
f_{kl,i} & -f_{kl,i}
\end{bmatrix}. \]
\[
\begin{align*}
\dot{X}_k(t) &= \dot{A}_k X_k(t) + \dot{B}_k \Delta_k(t), \quad k = p, \Theta \tag{13}
\end{align*}
\]

where

\[
\dot{A}_k = \begin{bmatrix} A_{kd} & B_{kl} A_{k\Delta} \\ 0_{6N \times 6N} & B_{k\Delta} \end{bmatrix}, \quad \dot{B}_k = \begin{bmatrix} B_{k\Delta 1} \\ B_{k\Delta 2} \end{bmatrix}.
\]

Denote \(\lambda_{ekl}(t)\) and \(\lambda_{ukl}(t)\) (\(l = 0, 1\)) as continuous and uniformly positive functions with upper bounds \(\|\lambda_{ekl}\|_\infty\) and \(\|\lambda_{ukl}\|_\infty\). The equivalent disturbances \(\Delta'_{p}\) and \(\Delta'_{\Theta}\) are assumed to satisfy

\[
\|\Delta'_{k}(t)\| \leq \sum_{l=0}^{1} \lambda_{ekl} \|E(t - \rho_l(t))\| + \sum_{l=0}^{1} \lambda_{ukl} \|u_k(t - \rho_l(t))\| + \gamma_{kd} \quad k = p, \Theta
\]

where \(\lambda_{ekl} = \|\lambda_{ekl}\|_\infty\), \(\lambda_{ukl} = \|\lambda_{ukl}\|_\infty\), and \(\gamma_{kd}\) is a uniformly bounded positive function involving the external disturbance \(d_k\), and \(E(t) = [e_{p1}^T(t), e_{p2}^T(t), e_{\Theta1}^T(t), e_{\Theta2}^T(t)]^T\). From (5) and (9), the equivalent disturbances \(\Delta'_{p}\) and \(\Delta'_{\Theta}\) can be rewritten as

\[
\|\Delta'_{k}(t)\| \leq \sum_{l=0}^{1} \lambda_{ekl} \|E(t - \rho_l(t))\| + \sum_{l=0}^{1} \lambda_{ukl} (\lambda_{kE} \|E(t - \rho_l(t))\| + f_{km} \|X_{Rk}(t - \rho_l(t))\|) + \gamma_{kd}
\]

(14)

where \(\lambda_{kE} = \|\lambda_{kE}\|_\infty\) satisfies \(\|\lambda_{kE}\|_\infty < \infty\), and \(\tilde{p}_{de} = \max \|\tilde{p}_{de}\| < 1\). Then, \(\rho_0(t) = 0\). Denote \(P_k\) (\(k = p, \Theta\)) as the solution to the Lyapunov equation: \(\dot{P}_k A_k + A_k^T \dot{P}_k = -I_{12N}\).

From (13), one can obtain that the matrix \(A_k\) is Hurwitz, and thereby \(P_k\) is positive definite. There exists a positive constant \(\lambda_{Bk}\) that satisfies

\[
\|P_k \dot{B}_k\| \leq \lambda_{Bk} f_{km}^{-1}, \quad k = p, \Theta.
\]

Define \(\xi_{ek} = \sum_{l=0}^{1} \lambda_{Bk} \xi_{ekl}, \quad \xi_{efk} = \sum_{l=0}^{1} \lambda_{Bk} \xi_{efkl}, \quad \xi_{uk} = \sum_{l=0}^{1} \lambda_{Bk} \xi_{ukl}, \quad \xi_{uk} \Delta_k = \sum_{l=0}^{1} \lambda_{Bk} \xi_{ukl} \Delta_{kE}, \quad \xi_{esfk} = \sum_{l=0}^{1} \lambda_{Bk} \xi_{esfkl}, \quad \xi_{esk} = \sum_{l=0}^{1} \lambda_{Bk} \xi_{eskl} + 2 \sum_{l=0}^{1} \lambda_{Bk} \xi_{esk}.\]

Theorem 1: Consider the dynamical model of the quadrotor as depicted in (1) and the robust controller as designed in Section III. For a given bounded and piecewise continuous initial state \(E(t), t \in [0 - \tilde{p}_{de}, 0]\), and a given initial time \(t_0\), there exist constants \(f_{ki}(k = p, \Theta)\), such that for any \(f_{ki} > f_{ki}\), \(E_k(t)\) is uniformly bounded for \(t \geq t_0\).

Furthermore, for a given constant \(c\), there exists a constant \(T\) such that the state \(E(t)\) satisfies \(\|E(t)\| \leq c, \forall t \geq T\).

Proof: First, one can select the Lyapunov function candidate by neglecting the communication delays as

\[
V_1(X(t)) = \sum_{k=p, \Theta} \|X_k(t)\|^2 P_k X_k(t). \tag{15}
\]

By differentiating (15), one can obtain that

\[
\dot{V}_1(X(t)) = - \sum_{k=p, \Theta} \|E_k(t)\|^2 + \|X_{Rk}(t)\|^2 + \sum_{k=p, \Theta} 2X_k^T(t) P_k \dot{B}_k \Delta_k(t)
\]

\[
\leq - \sum_{k=p, \Theta} \|E_k(t)\|^2 + \|X_{Rk}(t)\|^2 + \sum_{k=p, \Theta} 2\|E_k(t)\| \|X_{Rk}(t)\| \lambda_{Bk} f_{km}^{-1} \|\Delta_k(t)\|.
\]

(16)

Substituting the inequality in (14) into (16), one can obtain that

\[
\dot{V}_1(X(t)) \leq - \sum_{k=p, \Theta} \left(1 - \xi_{ek} - \xi_{efk} f_{km}^{-1}\right) \|E_k(t)\|^2
\]

\[
- \sum_{k=p, \Theta} \left(1 - \xi_{uk} - \xi_{uk} \Delta_k \|E_k(t - \rho_l(t))\| + f_{km} \|X_{Rk}(t - \rho_l(t))\|\right)
\]

\[
+ \sum_{k=p, \Theta} \xi_{esfk} \|E_k(t - \rho_l(t))\|^2 f_{km}^{-1}
\]

\[
+ \sum_{k=p, \Theta} \xi_{esk} \|X_{Rk}(t - \rho_l(t))\|^2 + \sum_{k=p, \Theta} \xi_{esk} \|X_{Rk}(t)\|^2 f_{km}^{-1}.
\]

(17)

Then, by introducing the communication delays, one can obtain the final Lyapunov function candidate as

\[
\dot{V}(X(t), t) = \dot{V}_1(X(t)) + \sum_{k=p, \Theta} \int_{t - \rho_l(t)}^{t} \|E_k(\tau)\|^2 d\tau
\]

\[
+ \sum_{k=p, \Theta} \int_{0}^{t - \rho_l(t)} \|E_k(\tau)\|^2 d\tau
\]

\[
\leq - \sum_{k=p, \Theta} \sum_{k=p, \Theta} \pi_{ek} \|E_k(t)\|^2 - \sum_{k=p, \Theta} \pi_{Rk} \|X_{Rk}(t)\|^2
\]

\[
- \sum_{k=p, \Theta} \sum_{k=p, \Theta} \pi_{Rk} \|X_{Rk}(t - \rho_l(t))\|^2 - \sum_{k=p, \Theta} \pi_{esk} \|E_k(t - \rho_l(t))\|^2
\]

\[
+ \sum_{k=p, \Theta} \pi_{esk} \|E_k(t)\|^2 + \sum_{k=p, \Theta} \pi_{esk} \|X_{Rk}(t)\|^2.
\]

(18)
It can be seen that if the robust compensator parameters $f_{km}(k = p, \Theta)$ satisfy
\[
\begin{align*}
f_{km} &> \zeta_{efk} / (1 - 2\zeta_{NE} - \zeta_{ek}) \\
f_{km} &> \zeta_{ufk} / (1 - 2\zeta_{NR} - \zeta_{uk}) \\
f_{km} &> \zeta_{esfk} / (1 - \bar{\rho}d)\zeta_{NE}, \quad k = p, \Theta
\end{align*}
\]
then, $\pi_{ek}$ and $\pi_{Rk}$ are positive, $\pi_{Rsk}$, $\pi_{esk}$, and $\pi_{\gamma k}$ are nonnegative. From (18), one can obtain that
\[
\dot{V}(X(t), t) \leq -\sum_{k=p,\Theta} \pi_{ek}\|E_k(t)\|^2 + \sum_{k=p,\Theta} \pi_{\gamma k}\|E_k(t)\|^2 \\
- \sum_{k=p,\Theta} \pi_{Rk}\|X_{Rk}(t)\|^2 + \sum_{k=p,\Theta} \pi_{\gamma k}\|X_{Rk}(t)\|^2 \\
\leq -\sum_{k=p,\Theta} (\pi_{ek} - \pi_{\gamma k})\|E_k(t)\|^2 \\
- \sum_{k=p,\Theta} (\pi_{Rk} - \pi_{\gamma k})\|X_{Rk}(t)\|^2 + \pi_{\gamma k}/2. \quad (19)
\]
From (19), one can obtain that the attractive radius of $\mathcal{E}(t)$ is determined by $\pi_{\gamma k}/2$. In fact, if $f_{km}$ are sufficiently large, $\pi_{\gamma k}$ can be made as small as possible and $\pi_{ek} - \pi_{\gamma k} > 0$ and $\pi_{Rk} - \pi_{\gamma k} > 0$. Therefore, one can obtain that the Theorem 1 holds.

Remark 5: It should be noted that the robust compensator parameters $f_{ki}(k = p, \Theta)$ determined by Theorem 1 are conservative, which means that the theoretical value of $f_{ki}$ may be much larger than its real one. Therefore, in practical applications, one can determine the robust compensator parameters using an online tuning method: first, set $f_{ki}(k = p, \Theta)$ with initial small values; second, increase $f_{ki}(k = p, \Theta)$ until the desired tracking performance of the proposed closed-loop control system is obtained.

IV. EXPERIMENTAL RESULTS
In this section, the experimental results of a team of microquadrotors are provided to validate the effectiveness of the proposed robust formation control approach. The main onboard sensors installed at the quadrotor consists of an inertial measurement unit (IMU) module, including a 3-D gyroscope, a 3-D accelerometer, and a 3-D magnetometer, a laser sensor measuring the altitude of the quadrotors, a Flow Deck measuring movements in relation to the ground, and a decentralized localization system using ultrawideband radio triangulation (UWB deck). The data for the IMU and the magnetometer sensor are fused to estimate the attitude angles and angle rates of the quadrotors. The UWB positioning system is a local positioning system, which is used to provide the current position of the quadrotors. This positioning system consists of a set of anchors and tags. The anchors are used as the reference and the tags measure the distance from each anchor to the tags. All information needed to calculate the position is available in the tag which enables the position estimation on board of
The desired trajectory of the virtual team leader is given by the information among three vehicles, where \( \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3 \), \( \dot{E} = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3] \), and \( \dot{W} = [\dot{w}_{ij}] \) with \( \dot{w}_{ij} = 1 \) for \( (\dot{\theta}_i, \dot{\theta}_j) \in \dot{E} \) and 0 otherwise. Only quadrotor 1 can obtain the information from the virtual team leader, and therefore \( a_{p1} = 1 \), \( a_{p2} = 0 \), and \( a_{p3} = 0 \). The sampling rate of communication sensor is 20 Hz. Therefore, one can obtain that the communication delays satisfy \( \| p_k(t) \|_\infty < 0.05s \) (for \( \Theta \) and \( \tilde{p}_e \leq 0.05s \). The nominal controller parameters are selected by the following steps. First, \( p_i \), \( K_p \), and \( K_\theta \) are determined by using the LQR-based control method. Second, \( K_\Theta \) and \( K_o \) are chosen based on the LQR approach for better attitude performance of each microquadrotor. The nominal parameters are selected as: \( p_i = 3 \), \( K_p = \text{diag}(1, 1, 1) \), \( K_\Theta = \{0.5, 0.5, 0.5\} \), \( K_o = \{20, 20, 20\} \), and \( K_o = \text{diag}(15, 15, 15) \). The robust compensator parameters are selected as: \( f_{p,i} = \text{diag}(1, 1, 1.6) \) and \( f_{\Theta,i} = \text{diag}(25, 25, 25) \) (for \( i = 1, 2, 3 \), as shown in Remark 5. The desired trajectory of the virtual team leader is given by \( p_i(t) = [0.6\sin(\pi t / 600), 0.6\cos(\pi t / 600), 0.8]^T \) and the three microquadrotors are required to form a constant triangle formation pattern as: \( \delta_1 = [0, 0.6, 0]^T \), \( \delta_2 = [-0.346, 0.8, 0]^T \), and \( \delta_3 = [-0.346, -0.4, 0]^T \). The yaw angle of each quadrotor needs to stabilize at 0 deg.

In the experimental results, the red, green, and blue solid lines represent quadrotors 1–3, respectively, and the purple dotted line indicates the formation pattern. The 3-D trajectories and the trajectory tracking errors are depicted in Figs. 2 and 3, respectively. The responses of the attitude are depicted in Fig. 4. The experimental video of quadrotor formation by the proposed control method is uploaded at https://youtu.be/itI8fdxW6Nk. In contrast, Fig. 5 shows the trajectory tracking errors using the baseline controller developed from [18]. The trajectory tracking errors in the longitude, lateral, and vertical channels by the proposed robust controller and the baseline controller are about 0.08, 0.08, 0.04, 0.1, 0.16, and 0.09 m, respectively. It can be observed that the proposed global closed-loop control system can improve the formation tracking performance, compared to the baseline controller. Besides, the effects of nonlinear dynamics, parametric perturbations, external disturbances, and communication delays can be restrained by the proposed formation protocol.

V. CONCLUSION

This brief presents a robust formation trajectory tracking controller design method to address the formation trajectory tracking control problem for a team of uncertain quadrotors subject to communication delays. The proposed robust formation trajectory tracking controller includes a position controller to achieve the desired formation trajectories and patterns and an attitude controller to regulate the attitudes of quadrotors. Robustness property analysis shows that the effects of nonlinearities, parametric perturbations, external disturbances, and communication delays can be restrained. Experimental results are provided to show the effectiveness and the advantages of the proposed robust formation protocol.

REFERENCES


