2000 SOLUTIONS, PART II

1. If there are at most 44 colors and at most 44 cans of the same color, then the total number of cans is $44 \times 44 = 1936$. This is a contradiction.

2. The answer is NO. Here is an example. The sides of the first triangle are 1, 1.5, $1.5 \times 1.5 = 2.25$. The sides of the second triangle are 1.5, $1.5 \times 1.5 = 2.25$, $1.5 \times 1.5 \times 1.5 = 3.375$. In both cases the triangle inequality is satisfied, so the triangles exist. The triangles are similar. The measures of the 3 angles of the first triangle are all different from each other but are the same as the measures of the corresponding angles of the second triangle.

3. $(a_{n+1})^2 = (a_n)^2 + 2 + (a_n)^2 > 2n + 2$. Therefore, by induction, $(a_n)^2 > 2n$ for all $n > 2$, so $(a_{10000})^2 > 20000 > 141^2$, so $a_{10000} > 141$.

4. Consider the 250 disks, each of radius 1/10 that are centered at each of the points. The sum of the areas of these disks is 2.5pi, and the union of the disks is contained inside a disk of radius 1.1. Since $2.5\pi > 2(1.1)^2\pi$, there is a point P (not necessarily in the chosen set) that is contained in at least 3 of the small disks. Thus, the disk of radius 1/10, centered at P contains at least three of the original points.

5. a. Given any five integers, either three of them have the same remainders when divided by 3 or three of them have all different remainders. In both cases, the sum of these three is a multiple of 3, say 3a. Take any five of the remaining 8 integers and select three with the sum 3b. Of the remaining 5 integers select three with the sum 3c. Two of the integers a, b, c are of the same parity, say, a and b. The sum $3a + 3b = 3(a + b)$ is divisible by 6.

b. Given 71 integers, select six with the sum $6a_1$ and repeat the procedure ten more times, every time at least 11 integers being available. We obtain eleven pairwise disjoint sixtuples with the sums $6a_1,..., 6a_{11}$. Of the eleven integers, $a_1$ through $a_{11}$, we can select six, say, $a_1$ through $a_6$, with the sum divisible by 6. The six sixtuples with the sums $6a_1,..., 6a_6$ are the required 36 integers with the sum $6(a_1 + ... + a_6)$ divisible by 36.